TD 4: LTL Model-Checking

1 Synchronous Büchi Transducers

Exercise 1. Give synchronous Büchi transducers for the following formulæ:

1. $\mathsf{SG}\, q$ and $\mathsf{G}\, q$,
2. $\mathsf{G}(p \to \mathsf{F}\, q)$,
3. $\mathsf{G}_0\, q$ (recall from Exercise 3 of TD 3 that $w, i \models \mathsf{G}_0\, \varphi$ iff $\forall k \geq i, (k-i) \equiv 0 \bmod 2 \Rightarrow w, k \models \varphi$).

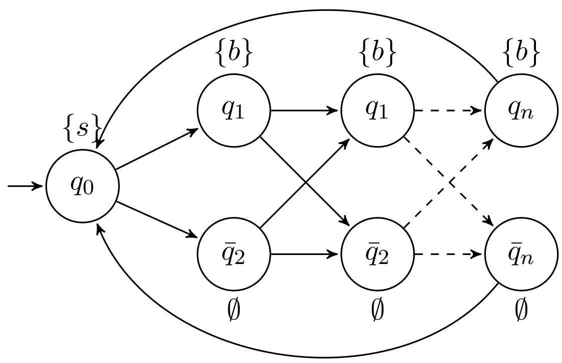
2 Complexity of LTL Model-Checking

Exercise 2 (Complexity of LTL(X))**.** We want to show that LTL(X) existential model checking is NP-complete (instead of PSPACE-complete for the full LTL(SU)).

1. Show that $\mathsf{MC}^{\exists}(\mathsf{X})$ is in NP.
2. Reduce $\mathsf{3SAT}$ to $\mathsf{MC}^{\exists}(\mathsf{X})$ in order to prove NP-hardness.

Exercise 3 (Hardness of LTL(X, F))**.** Adapt the proof given during the lecture to show that $\mathsf{MC}^{\exists}(\mathsf{X}, \mathsf{F})$ is PSPACE-hard.

As a preliminary question, consider the following Kripke structure M_n over AP $= \{s, b\}$:



Any infinite word σ generated by M_n is in $\big(\{s\}(\{b\}+\emptyset)^n\big)^\omega$, where each segment between two s's can be seen as describing a value from 0 to 2^n-1 encoded in binary. Provide an polynomial-sized LTL(X,F) formula φ that selects runs ρ where the successive values form the sequence 0, 1, …, 2^n-1, 0, 1, …, i.e. count modulo 2^n.

Exercise 4 (Stuttering and LTL(U))**.** In the time flow $(\mathbb{N},<)$, i.e. when working with words σ in Σ^ω, *stuttering* denotes the existence of consecutive symbols, like *aaaa* and *bb* in *baaaabb*. Concrete systems tend to stutter, and thus some argue that verification properties should be stutter invariant.

A *stuttering function* $f:\mathbb{N}\to\mathbb{N}_{>0}$ from the positive integers to the positive integers. Let $\sigma=a_0a_1\cdots$ be an infinite word of Σ^ω and f a stuttering function, we denote by $\sigma[f]$ the infinite word $a_0^{f(0)}a_1^{f(1)}\cdots$, i.e. where the i-th symbol of σ is repeated $f(i)$ times. A language $L\subseteq\Sigma^\omega$ is *stutter invariant* if, for all words σ in Σ^ω and all stuttering functions f,

$$\sigma\in L \text{ iff } \sigma[f]\in L\ .$$

1. Prove that if φ is a TL(AP, U) formula, then $L(\varphi)$ is stutter-invariant.

2. A word $\sigma=a_0a_1\cdots$ in Σ^ω is *stutter-free* if, for all i in $\mathbb{N}$, either $a_i\neq a_{i+1}$, or $a_i=a_j$ for all $j\geq i$. We note $\mathrm{sf}(L)$ for the set of stutter-free words in a language L.

   Show that, if L and L' are two stutter invariant languages, then $\mathrm{sf}(L)=\mathrm{sf}(L')$ iff $L=L'$.

3. Let φ be a TL(AP, X,U) formula such that $L(\varphi)$ is stutter invariant. Construct inductively a formula $\tau(\varphi)$ of TL(AP, U) such that $\mathrm{sf}(L(\varphi))=\mathrm{sf}(L(\tau(\varphi)))$, and thus such that $L(\varphi)=L(\tau(\varphi))$ according to the previous question. What is the size of $\tau(\varphi)$ (there exists a solution of size $O(|\varphi|\cdot 2^{|\varphi|})$)?

Exercise 5 (Complexity of LTL(U))**.** We want to prove that the model checking and satisfiability problems for LTL(U) formulæ are both PSPACE-complete.

1. Prove that $\mathsf{MC}^\exists(\mathsf{X},\mathsf{U})$ can be reduced to $\mathsf{MC}^\exists(\mathsf{U})$: given an instance (M,φ) of $\mathsf{MC}^\exists(\mathsf{X},\mathsf{U})$, construct a stutter-free Kripke structure M' and an LTL(U) formula $\tau'(\varphi)$. *Beware: the τ construction of the previous exercise does not yield a polynomial reduction!*

2. Show that $\mathsf{MC}^\exists(\mathsf{X},\mathsf{U})$ can be reduced to $\mathsf{SAT}(\mathsf{U})$.