TD 1

1 Models

Exercise 1 (Rendez-vous with Data). Consider the synchronization of transition systems with variables through a rendez-vous mechanism. Such a system is of form $M = (S, \Sigma, \mathcal{V}, (D_v)_{v\in\mathcal{V}}, T, I, \mathrm{AP}, l)$ where $\mathcal{V}$ the set of (typed) variables v, each with domain D_v.

We want to extend the rendez-vous mechanism between systems with variables with the ability to exchange data values. For instance, a system M_i may transmit a value m with guard g and label a by performing

$$s_i \xrightarrow{!m} s_i' \ ,$$

only if some system M_j is ready to receive the message, i.e. to perform

$$s_j \xrightarrow{?v} s_j' \ ,$$

where v is a variable of M_j and m is in D_v. Of course the synchronization is also possible if M_j performs instead

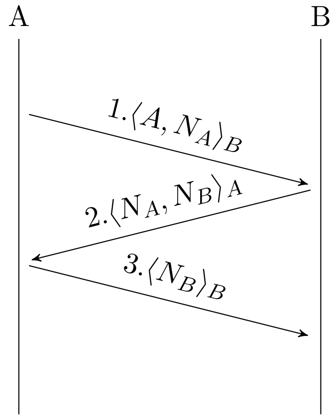
$$s_j \xrightarrow{?m} s_j' \ .$$

Propose Structural Operational Semantics for the rendez-vous with data synchronization.

Exercise 2 (Needham-Schroeder Protocol). We consider the analysis of a public-key authentication protocol proposed by Needham and Schroeder in 1978. The protocol relies on

- the generation of *nounces* N_C : random numbers that should only be used in a single session, and
- on public key encryption : we denote the encryption of message M using C's public key by $\langle M\rangle_C$.

A(lice) and B(ob) try to make sure of each other's identity by the following (very simplified) exchange :

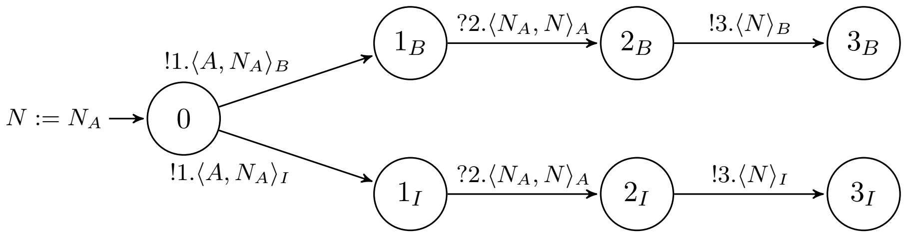


1. Alice first presents herself (the A part of the message) and challenges Bob with her nounce N_A. Assuming both cryptography and random number generation to be perfect, only Bob can decrypt $\langle A, N_A\rangle_B$ and find the correct number N_A.
2. Bob responds by proving his identity (the N_A part) and challenges Alice with his own nounce N_B.
3. Finally, Alice proves her identity by sending N_B.

The nounces N_A and N_B are used by Alice and Bob as secret keys for their communications.

In order to account for the insecure channel, we have to add an intruder I to the model, who has his own nounce N_I, and can read and send any message it fancies, but can only decrypt $\langle M\rangle_I$ messages and cannot guess the nounces generated by Alice and Bob.

We can model the behaviour of Alice as a transition system M_A with variables and rendez-vous with data, using a single variable N ranging over $\mathcal{N} = \{N_A, N_B, N_I\}$.



1. Provide a model M_B for Bob.
2. Provide a model M_I the intruder.
3. Give a LTL formula that states that the intruder cannot know N_A and N_B, nor make Bob believe he is Alice.
4. Unfold an execution path in the synchronized product of M_A, M_B, and M_I that unveils a flaw in the protocol.

2 Specification

Exercise 3 (LTL Formulæ)**.** We would like to verify the properties of a boolean circuit with input x, output y, and two registers r_1 and r_2. We define accordingly $\mathrm{AP} = \{x, y, r_1, r_2\}$ as our set of atomic propositions, and model check infinite runs $\sigma = s_0 s_1 s_2 \cdots$ from $(2^{\mathrm{AP}})^\omega$.

Translate the following properties in LTL and in FO($<$) :

1. "it is impossible to get two consecutive 1 as output"
2. "each time the input is 1, at most two ticks later the output will be 1"
3. "each time the input is 1, the register contents remains the same over the next tick"
4. "register r_1 is infinitely often 1"

3 Automata

Definition 1 (Büchi Automaton)**.** A *Büchi automaton* is a tuple $A = (Q, \Sigma, T, I, F)$ where Q is a finite set of states, Σ a finite alphabet, $T \subseteq Q \times \Sigma \times Q$ a transition relation, $I \subseteq$ a set of initial states, and $F \subseteq Q$ a set of accepting states.

An infinite run $\sigma = q_0 q_1 q_2 \cdots$ in Q^ω for an infinite word $w = a_0 a_1 a_2 \cdots$ in Σ^ω is *successful* if q_0 is in I, for each i (q_i, a_i, q_{i+1}) is in T, and σ visits F infinitely often, written $\mathsf{Inf}(\sigma) \cap F \neq \emptyset$. The language $L(A)$ of A is the set of words that have at least one successful run. We say that a language $L \subseteq \Sigma^\omega$ is *recognizable*, noted $L \in \mathsf{Rec}(\Sigma^\omega)$, if it is the language of some Büchi automaton.

Exercise 4 (Generalized Acceptance Condition)**.** A *generalized* Büchi automaton $A = (Q, \Sigma, T, I, (F_i)_i)$ has a finite set of accepting sets F_i. An infinite run is satisfies this generalized acceptance condition if

$$\bigwedge_i \mathsf{Inf}(\sigma) \cap F_i \neq \emptyset \ .$$

Show that for any generalized Büchi automaton, one can construct an equivalent (non generalized) Büchi automaton.

Exercise 5 (Basic Closure Properties)**.** Show that $\mathsf{Rec}(\Sigma^\omega)$ is closed under

1. finite union, and
2. finite intersection.

Exercise 6 (Ultimately Periodic Words)**.** An *ultimately periodic word* over Σ is a word of form $u \cdot v^\omega$ with u in Σ^* and v in Σ^+.

Prove that any nonempty recognizable language in $\mathsf{Rec}(\Sigma^\omega)$ contains an ultimately periodic word.

Exercise 7 (Rational Languages)**.** A *rational language* L of infinite words over Σ is a finite union

$$L = \bigcup X \cdot Y^\omega$$

where X is in $\mathsf{Rat}(\Sigma^*)$ and Y in $\mathsf{Rat}(\Sigma^+)$. We denote the set of rational languages of infinite words by $\mathsf{Rat}(\Sigma^\omega)$.

Show that $\mathsf{Rec}(\Sigma^\omega) = \mathsf{Rat}(\Sigma^\omega)$.

Exercise 8 (Deterministic Büchi Automata)**.** A Büchi automaton is *deterministic* if $|I| \leq 1$, and for each state q in Q and symbol a in Σ, $|\{(q, a, q') \in T \mid q' \in Q\}| \leq 1$.

1. Give a nondeterministic Büchi automaton for the language in $\{a, b\}^\omega$ described by the expression $(a + b)^* a^\omega$.
2. Show that there does not exist any deterministic Büchi automaton for this language.
3. Let $A = (Q, \Sigma, T, q_0, F)$ be a finite deterministic automaton that recognizes the language of finite words $L \subseteq \Sigma^*$. What is the language of infinite words recognized by A seen as a Büchi automaton ?