Grammar Verification

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GdT INFINI
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Grammar Engineering

Klint et al. [2005]

- grammars as specifications
  - rich toolsets (parsers, pretty printers, etc.)
- grammars as concrete objects
  - methodology
  - testing
  - verification

Case Study: Modular Syntax

Modules

- meaningful subsets
- reusable
- composable

Comparing parsers

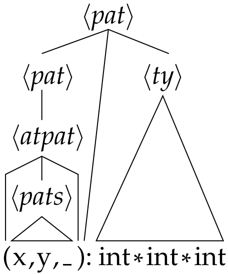
- two grammar fragments $\mathcal{G}_1$ and $\mathcal{G}_2$
- composition $\mathcal{G}_1 \cup \mathcal{G}_2$ using
  - LR parsers
  - GLR parsers

SML Pattern Syntax

Standard ML [Milner et al., 1997]

$$
\begin{array}{rcl}
\langle pat \rangle & \rightarrow & \langle atpat \rangle \mid \langle pat \rangle \; : \; \langle ty \rangle \\
\langle atpat \rangle & \rightarrow & vid \mid \_ \mid (\; \langle pats \rangle \;) \mid (\;) \\
\langle pats \rangle & \rightarrow & \langle pat \rangle \mid \langle pats \rangle \; , \; \langle pat \rangle
\end{array}
\qquad (\mathcal{G}_1)
$$

Example



SML Layered Pattern Syntax

Standard ML [Milner et al., 1997]

$$\langle pat \rangle \quad \rightarrow \quad vid : \langle ty \rangle \; as \; \langle pat \rangle \,|\, vid \; as \; \langle pat \rangle \qquad (\mathcal{G}_2)$$

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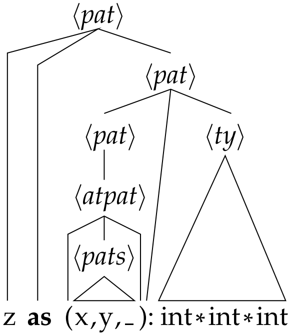

LR Parsers

Crespi Reghizzi and Psaila [1998]

- parser for a LR(k) grammar, works in $\mathcal{O}(n)$
- but DCFL not closed under union:

$$\textbf{val}\ f\ = \textbf{fn}\ z : \text{int} * \text{int} * \text{int}$$

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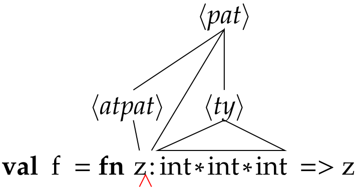

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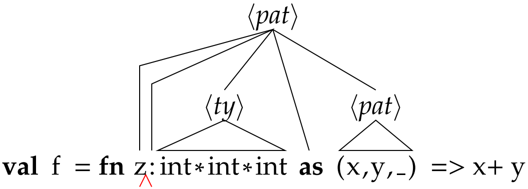

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Generalized LR Parsers

e.g. SDF2 [Visser, 1997]

- parser for any CFG, works in $\mathcal{O}(2^{|G|}\, n^3)$
- CFL closed under union

```
module G1
exports
  sorts PAT ATPAT
  context-free syntax
    ATPAT                 -> PAT
    PAT ":" TY            -> PAT
    "(" {PAT ","}* ")"    -> ATPAT
    "_"                   -> ATPAT
    VID                   -> ATPAT
```

```
module G1UG2
imports
  G1
exports
  context-free syntax
    VID (":" TY)? "as" PAT -> PAT
```

- but UCFL isn't!

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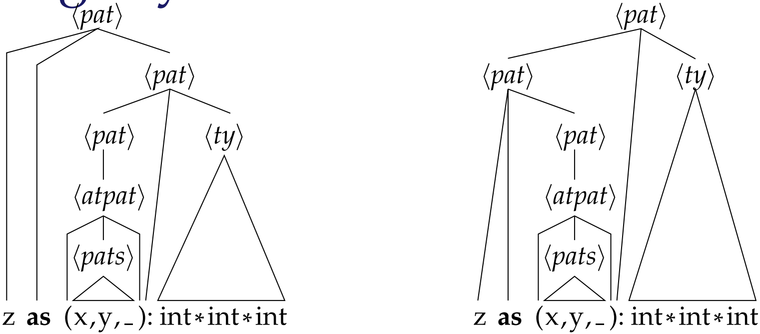

- run-time error:

```
sglr:error: Ambiguity in input, line 1, col 0:
  PAT  ":"  TY -> PAT;VID  (":" TY )?  "as"  PAT -> PAT
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- choose between alternative parses:

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VID (":" TY)? "as" PAT        -> PAT {prefer}
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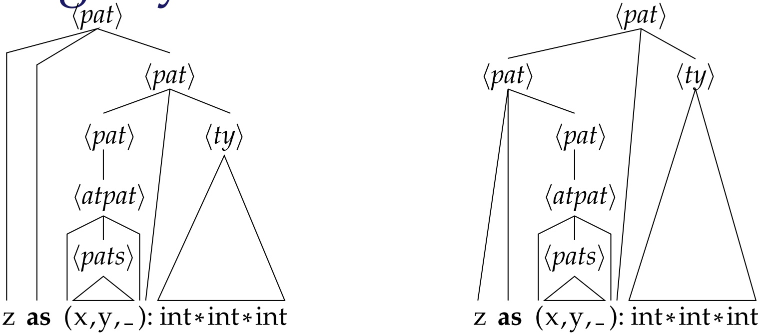

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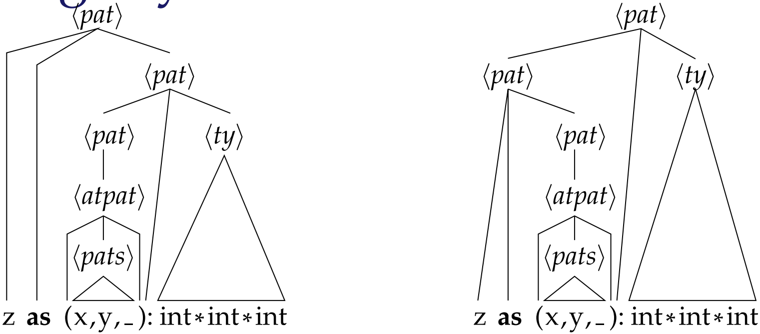

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Bracketed Grammars

$$\mathcal{G} = \langle N, T, P, S \rangle,\ V = N \cup T$$

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\begin{array}{rcl}
\langle pat \rangle & \xrightarrow{1} & \langle atpat \rangle \\
\langle pat \rangle & \xrightarrow{2} & \langle pat \rangle \ : \ \langle ty \rangle \\
\langle atpat \rangle & \xrightarrow{3} & vid \\
\langle atpat \rangle & \xrightarrow{4} & \_ \\
\langle atpat \rangle & \xrightarrow{5} & (\ \langle pats \rangle \ ) \\
\langle atpat \rangle & \xrightarrow{6} & (\ ) \\
\langle pats \rangle & \xrightarrow{7} & \langle pat \rangle \\
\langle pats \rangle & \xrightarrow{8} & \langle pats \rangle \ , \ \langle pat \rangle \\
\langle pat \rangle & \xrightarrow{9} & vid \ : \ \langle ty \rangle \ \text{as} \ \langle pat \rangle \\
\langle pat \rangle & \xrightarrow{10} & vid \ \text{as} \ \langle pat \rangle
\end{array}
$$

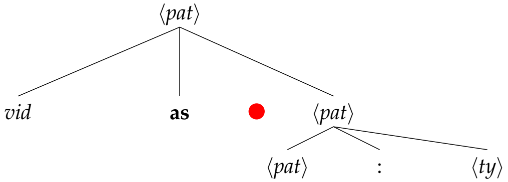
Bracketed Grammars

$$\mathcal{G}_b = \langle N, T_b, P_b, S \rangle, \; V_b = N \cup T_b$$

$$
\begin{array}{rcl}
\langle pat \rangle & \xrightarrow{1} & d_1 \; \langle atpat \rangle \; r_1 \\
\langle pat \rangle & \xrightarrow{2} & d_2 \; \langle pat \rangle \; : \; \langle ty \rangle \; r_2 \\
\langle atpat \rangle & \xrightarrow{3} & d_3 \; vid \; r_3 \\
\langle atpat \rangle & \xrightarrow{4} & d_4 \; \_ \; r_4 \\
\langle atpat \rangle & \xrightarrow{5} & d_5 \; (\; \langle pats \rangle \;) \; r_5 \\
\langle atpat \rangle & \xrightarrow{6} & d_6 \; (\;) \; r_6 \\
\langle pats \rangle & \xrightarrow{7} & d_7 \; \langle pat \rangle \; r_7 \\
\langle pats \rangle & \xrightarrow{8} & d_8 \; \langle pats \rangle \; , \; \langle pat \rangle \; r_8 \\
\langle pat \rangle & \xrightarrow{9} & d_9 \; vid \; : \; \langle ty \rangle \; \text{as} \; \langle pat \rangle \; r_9 \\
\langle pat \rangle & \xrightarrow{10} & d_{10} \; vid \; \text{as} \; \langle pat \rangle \; r_{10}
\end{array}
$$

Position Graph Γ

Left-to-right Walks in Trees



$$d_{10}\ \mathit{vid}\ \mathbf{as} \bullet\ d_2\ \langle \mathit{pat} \rangle\ :\ \langle \mathit{ty} \rangle\ r_2\ r_{10}$$

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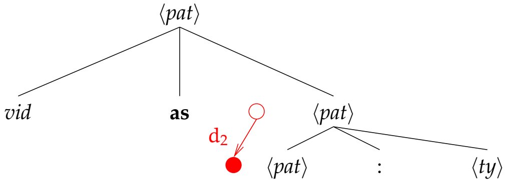

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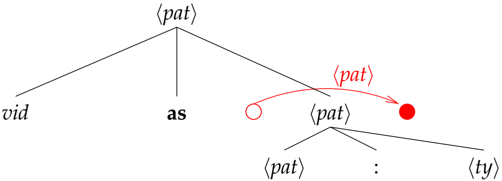

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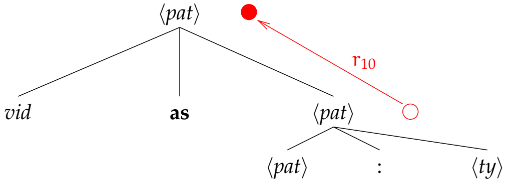


$$\mathrm{d}_{10}\ \textit{vid}\ \mathbf{as}\ \mathrm{d}_2\ \langle \textit{pat} \rangle\ :\ \langle \textit{ty} \rangle\ \mathrm{r}_2\ \mathrm{r}_{10}\ \bullet$$

Position Automaton $\Gamma/\equiv$

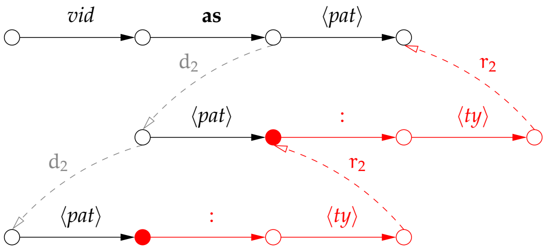
Definition

$\Gamma/\equiv$ is the quotient of Γ by an equivalence relation $\equiv$ between positions.

Language over-approximation

$$\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\Gamma/\equiv) \cap T_b^*$$

Example: item_0 Equivalence



- equivalence class $[\langle pat\rangle \xrightarrow{2} \langle pat\rangle \bullet\ :\ \langle ty\rangle]$
- LR(0) items
- Γ/item_0: nondeterministic LR(0) automaton

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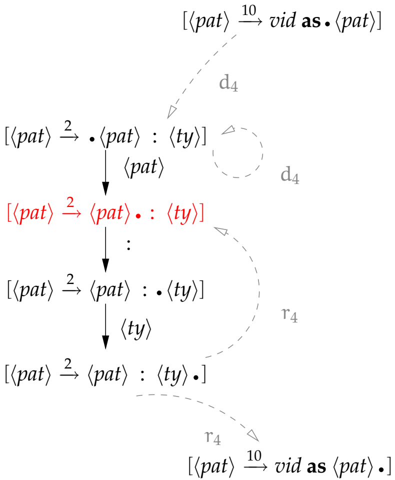
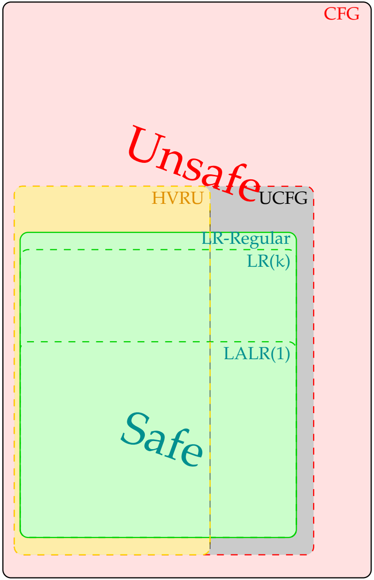


Unambiguous Grammar Classes

- LR(k) [Knuth, 1965]
- LR-Regular [Čulik and Cohen, 1973]
- Horizontal and vertical unambiguity test [Brabrand et al., 2007]
- Unambiguous CFGs [Cantor, 1962, Chomsky and Schützenberger, 1963]



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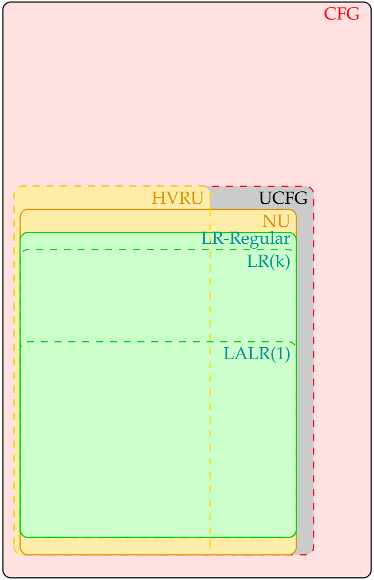

Principles

- a bracketed sentence = a derivation tree
- ambiguity = more than one tree with the same yield

$$\mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \mathrm{d}_2\ \mathrm{d}_1\ \mathrm{d}_3\ \mathit{vid}\ \mathrm{r}_3\ \mathrm{r}_1\ :\ \langle \mathit{ty} \rangle\ \mathrm{r}_2\ \mathrm{r}_{10}$$
$$\mathrm{d}_2\mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \mathrm{d}_1\ \mathrm{d}_3\ \mathit{vid}\ \mathrm{r}_3\ \mathrm{r}_1\ \mathrm{r}_{10}\ :\ \langle \mathit{ty} \rangle\ \mathrm{r}_2$$

- construct a FSA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\mathcal{A})$, and look for bracketed sentences with the same yield

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Regular Unambiguity

- h bracket erasing homomorphism
- $\mathcal{G}$ is *regular unambiguous* for $\equiv$ of *finite index*, if there does not exist $w_b \neq w'_b$ in $\mathcal{L}(\Gamma/{\equiv}) \cap T^*_b$ with $h(w_b) = h(w'_b)$

Squaring Algorithms

- NFA ambiguity [Even, 1965]
- functionality [Béal et al., 2003]
- mutual accessibility relations between pairs of states
- in $\mathcal{O}(|\Gamma/{\equiv}|^2)$

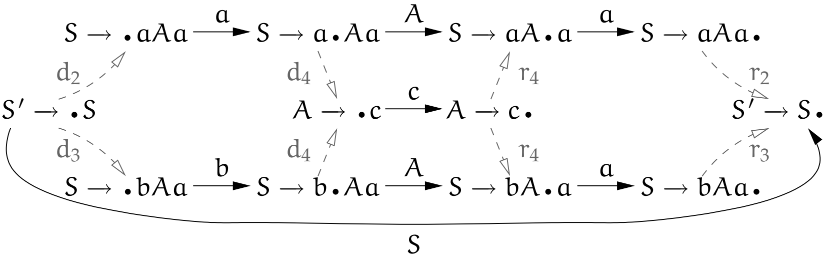
Horizontal and Vertical Ambiguity

Brabrand et al. [2007]

Definition

- $\mathcal{G}$ is *vertically unambiguous* iff $\forall A \to \alpha_1, A \to \alpha_2 \in P, \alpha_1 \neq \alpha_2, \mathcal{L}(\alpha_1) \cap \mathcal{L}(\alpha_2) = \emptyset$
- $\mathcal{G}$ is *horizontally unambiguous* iff $\forall A \to \alpha$ and $\forall \alpha_1, \alpha_2$ with $\alpha = \alpha_1\alpha_2, \mathcal{L}(\alpha_1) \mathbin{\mho} \mathcal{L}(\alpha_2) = \emptyset$
- $L_1 \mathbin{\mho} L_2 = \{xyz \mid x, xy \in L_1, y \in T^+, \text{ and } yz, z \in L_2\}$
- $\mathcal{O}(|\mathcal{G}|^5)$ algorithm
- $\mathrm{RU}(\equiv) \subset \mathrm{HVRU}(\equiv)$

LR(k) condition



- $S \xrightarrow{2} aAa$, $S \xrightarrow{3} bAa$, $A \xrightarrow{4} c$
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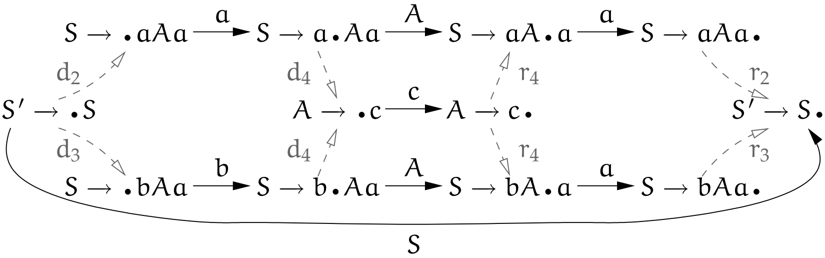

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Nonterminal Transitions

- $\mathcal{SF}(\mathcal{G}_b) \subseteq \mathcal{L}(\Gamma/\equiv)$
- look for two different bracketed sentential forms in $\mathcal{L}(\Gamma/\equiv)$

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Mutual Accessibility Relations

- between pairs of states of $\Gamma/\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

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- synchronized left-to-right walks from an initial pair (q_s, q_s)

$$d_{10}\ \mathit{vid}\ \mathbf{as}\ d_2\ \langle\mathit{pat}\rangle\ :\ \langle\mathit{ty}\rangle\ r_2\ r_{10}$$
$$d_2\ d_{10}\ \mathit{vid}\ \mathbf{as}\ \langle\mathit{pat}\rangle\ r_{10}\ :\ \langle\mathit{ty}\rangle\ r_2$$

conflict: mac

Mutual Accessibility Relations

- between pairs of states of $\Gamma/\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

$$\mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \mathrm{d}_2\ \langle \mathit{pat} \rangle\ :\ \langle \mathit{ty} \rangle\ \mathrm{r}_2\ \mathrm{r}_{10}$$
$$\mathrm{d}_2\ \mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \langle \mathit{pat} \rangle\ \mathrm{r}_{10}\ :\ \langle \mathit{ty} \rangle\ \mathrm{r}_2$$

shift: mas

Mutual Accessibility Relations

- between pairs of states of $\Gamma/\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

$$d_{10}\ \mathit{vid}\ \mathbf{as}\ d_2\ \langle \mathit{pat} \rangle\ :\ \langle \mathit{ty} \rangle\ r_2\ r_{10}$$
$$d_2\ d_{10}\ \mathit{vid}\ \mathbf{as}\ \langle \mathit{pat} \rangle\ r_{10}\ :\ \langle \mathit{ty} \rangle\ r_2$$

reduce: mar

Mutual Accessibility Relations

- between pairs of states of $\Gamma/\!\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

$$\mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \mathrm{d}_2\ \langle \mathit{pat} \rangle\ :\langle \mathit{ty} \rangle\ \mathrm{r}_2\ \mathrm{r}_{10}$$
$$\mathrm{d}_2\ \mathrm{d}_{10}\ \mathit{vid}\ \mathbf{as}\ \langle \mathit{pat} \rangle\ \mathrm{r}_{10}\ :\langle \mathit{ty} \rangle\ \mathrm{r}_2$$

conflict: mac

NU(≡)

- ma=mas ∪ mae ∪ mac ∪ mar
- $\mathcal{G}$ is *noncanonically unambiguous* if there does not exist a relation (q_s, q_s) ma* (q_f, q_f) that uses mac at some step
- Computation in $\mathcal{O}(|\Gamma/{\equiv}|^2)$

Comparisons

- Regular Unambiguity RU($\equiv$)
- Bounded-length detection schemes
- LR(k) and LR-Regular (LR(Π))
- Horizontal and vertical ambiguity (HVRU($\equiv$))

Bounded-length detection

[Gorn, 1963, Cheung and Uzgalis, 1995, Schröer, 2001, Jampana, 2005]

- generate sentences
- not conservative
- prefix_m prevents from false positives in sentences of length $< m$
- need to generate a^{2^n+1} to find $\mathcal{G}_4^n$ ambiguous, but $\mathcal{G}_4^n \notin \mathrm{NU}(\mathsf{item}_0)$

$$S \to A \mid B_n a,\ A \to Aaa \mid a,$$
$$B_1 \to aa,\ B_2 \to B_1 B_1, \ldots,\ B_n \to B_{n-1} B_{n-1} \qquad (\mathcal{G}_4^n)$$

LR(k) and LR-Regular

[Knuth, 1965, Hunt III et al., 1975, Čulik and Cohen, 1973, Heilbrunner, 1983]

- conservative tests
- define item_Π s.t. $\mathrm{LR}(\Pi) \subset \mathrm{NU}(\mathsf{item}_\Pi)$
- need a LR(2^n) test to prove $\mathcal{G}_3^n$ unambiguous, but $\mathcal{G}_3^n \in \mathrm{NU}(\mathsf{item}_0)$

$$S \rightarrow A \,|\, B_n,\; A \rightarrow Aaa \,|\, a,$$
$$B_1 \rightarrow aa,\; B_2 \rightarrow B_1B_1, \ldots,\; B_n \rightarrow B_{n-1}B_{n-1} \qquad (\mathcal{G}_3^n)$$

Implementation

- For the whole SML grammar:
  - conflicts in the LALR(1) parser

    `sml.y: conflicts: 223 shift/reduce, 35 reduce/reduce`
  - Our tool:

    `89 potential ambiguities with LR(1) precision detected`

- For the SML grammar fragment:
- Benchmark: NU(item_1) correctly identifies 87% of our unambiguous grammars—73% of the non-LALR(1) ones

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