

## Proposition de stage de Master

### Lieu :

Laboratoire Spécification et Vérification  
École Normale Supérieure de Cachan  
Cachan, France

Department of Computer Science  
University of Oxford  
Oxford, UK

### Titre : Post Embedding Problems, Dickson's Lemma, and Master Problems for Monotonic Counter Systems

### Description du stage :

**Lossy counter machines**, aka LCMs, are a variant of [Minsky counter machines](#) where integer counters are *unreliable* and can lose any part of their contents without any warning. They are a prototypical instance of well-structured transition systems, where the underlying well-quasi-ordering (wqo) is the product ordering  $(\mathbb{N}^k, \leq)$ , cf. Dickson's Lemma [[FS01](#), [Sch10a](#)].

It is now recognized that lossy counter machines correspond to a fundamental level in complexity : the Ackermann level in the Fast-Growing Hierarchy [[Sch10b](#), [FFSS11](#)]. This pivotal place is acknowledged by several works showing complexity lower bounds by reduction from LCM's. However, problems on LCM's are not as abstract and versatile as one could wish. For the corresponding complexity class, we lack a simple and elegant master problem like SAT (for NP) or Post Corresponding Problem (for  $\Sigma_1^0$ ).

The aim of this research internship is to discover and develop a new master problem equivalent to lossy counter systems reachability. The initial idea is to invent a variant of Post's Embedding Problem (see [[CS07](#), [CS08a](#), [CS10](#)]) that relies on Dickson's Lemma rather than Higman's Lemma. This should opens new avenues for research, and the work can be continued in at least two main directions : (1) investigating the new master problem, and (2) connecting it (via two-way reductions) to problems in other areas.

This research program is directly connected to [MPRI C2-9 course](#), on « Vérification de systèmes dynamiques et paramétrés », and is part of the [ReacHard](#) project (2011-2015) funded by ANR, on hard reachability problems. It should suit a theoretically-minded student with some taste for abstract algorithmic constructions like what is encountered in basic courses on recursion theory and computational complexity. The internship is an ideal opportunity for starting a PhD thesis funded by the [ReacHard](#) project.

### Personne(s) encadrant le stage :

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## Références

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