Distributed Synthesis with Delayed Signals

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Interacting under uncertainty

Uncertainty in distributed systems:

- omnipresent: private implementation details, imperfect signals
- computationally intractable, in general

Goal: identify types of uncertainty that

- can be handled computationally,
- cover realistic scenarios

Distributed games

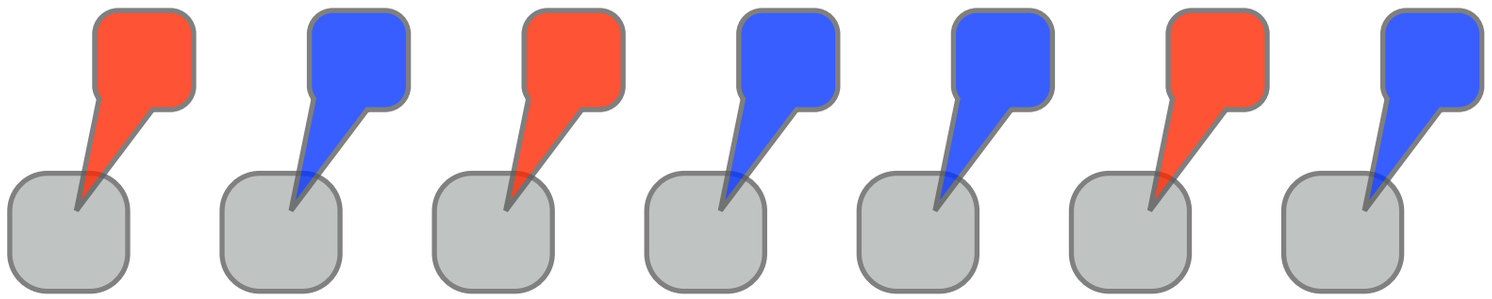
Model: state-based, n players, concurrent transitions

Basic setting – instant signals

- each player chooses an action
- joint action determines possible successor states, Nature picks one
- each player receives an (imperfect) signal about the new state

Question: can the grand coalition coordinate on a good run against Nature?

Instant signals:

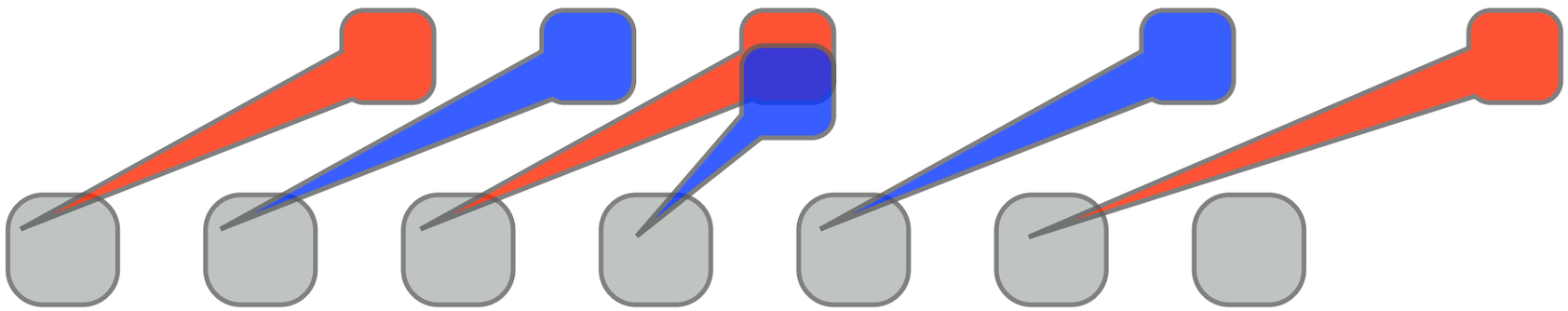


Distributed games with delayed signals

Capture uncertainty from unreliable information transfer
Same model except: signals are not delivered instantly, but within a bounded delay

- delays may be different for every player
- several signals may arrive at the same time
- signals are timestamped with emission date

Delayed signals:



(Representation in the state-transition system possible but difficult)

Our result: strategy transfer

Theorem

For a particular class of winning conditions:

the solution of a game can be adapted to the variant where signals are delivered with a bounded delay.

Key: delayed-response strategies for repeated games

Theorem [Fudenberg, Ishii, Kominers (2014)].

Every equilibrium outcome of a repeated mean payoff game
can be supported in the associated game with bounded observation lags.

Different setting:

- Fudenberg & al: repeated games with discounted mean payoff conditions
  - no states:
  - discounted mean payoff: temporal flexibility / patience
  - equilibrium support: no Nature, but one player may split off coalition

- Here: games on graphs
  - state transitions – less homogeneous
  - regular winning conditions – meaning of discounting?
  - distributed winning strategies against Nature

Repeated games with regular winning conditions:

- Players 1, 2 in coalition against Nature
  - sets of actions A_i, signals B_i – private to player i
  - set of consequences C – global (not observable)
  - consequence function: $\pi : A_1 \times A_2 \to \mathcal{P}(C)$
  - observation function: $\beta_i : C \to B_i$
- Play, infinite sequence of rounds
  - players choose simultaneously action profile $a = (a_1, a_2)$,
  - Nature picks a consequence $c \in \pi(a)$
  - each player i receives signal $\beta_i(c)$
- Outcome: infinite sequence of consequences
- Winning condition: ω-regular set of outcomes

General enough to describe games on graphs with imperfect information.

Delayed signals

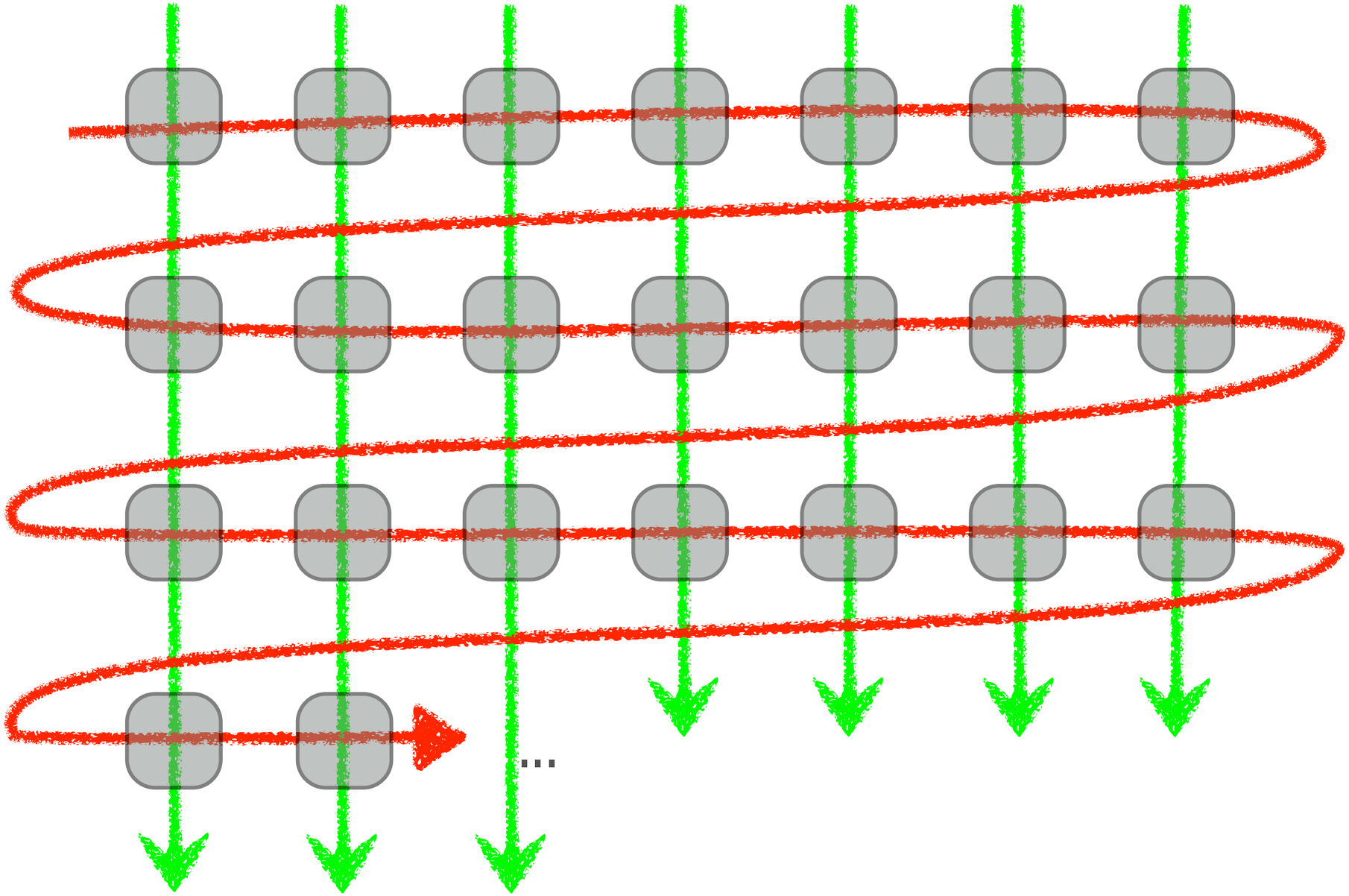
Associate to game $\mathcal{G}$, the delayed variant $\mathcal{G}^k$:

- Signals are received with delay up to a bound of k many steps
- timestamped with emission round

Depending on the winning condition, timely signals may be crucial. Nevertheless, some conditions may be permissive enough.

Convolution

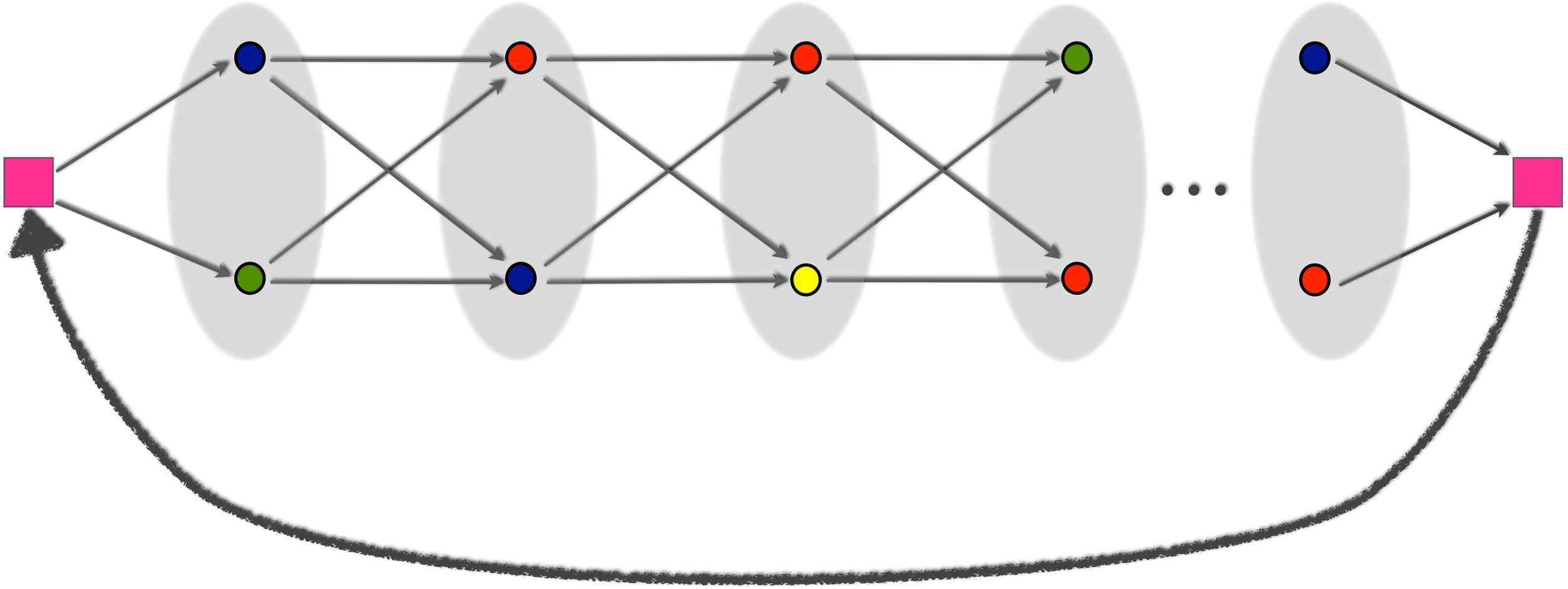
Two ways of traversing the space of histories:



Convolution

Examples:

- Each language is its own 1-fold convolution,
- Parity and Streett conditions are closed under k-fold convolution, for every k,
- Rabin and Muller are not, in general
- A cyclic play of period k is a convolution of k plays from successive states



Delayed response strategies

Theorem. Fix a repeated game $\mathcal{G}$ (with instant signals),

- W a winning condition for $\mathcal{G}$, and
- W^k the k-fold convolution of W.

Then, every winning strategy for $\mathcal{G}$ with condition W can be transformed into a winning strategy for the game $\mathcal{G}^k$ with condition W^k.

Idea

- players respond only to those signals that reached delivery deadline
- at the deadline, all players are certain about the signals issued k steps back
- the joint response is independent of (possibly) pending signals
- the outcome is a convolution of k virtual plays without delays, thus winning

Outlook

Delayed strategies are a worst-case solution.

If signals are delivered before the deadline, the coalition may also respond earlier.

- study measures for reactivity / quantitative objective

The set of plays in arbitrary game graphs is rarely a convolution.

- find more liberal conditions for the transfer