Temporal logics for multi-agent systems

Expressiveness and algorithms

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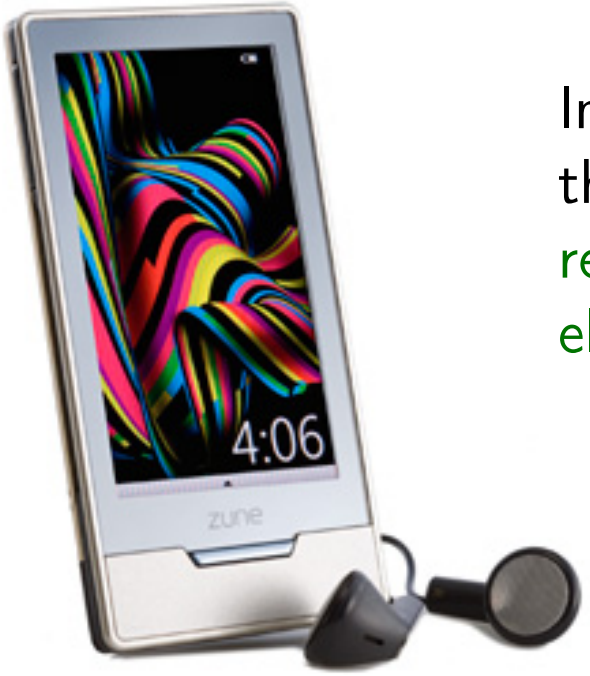
Verification of computerized systems

Verification of computerized systems – The Zune bug



In order to compute the current date, the Zune player has to compute the current year, given the total number of days elapsed since January 1st, 1980.

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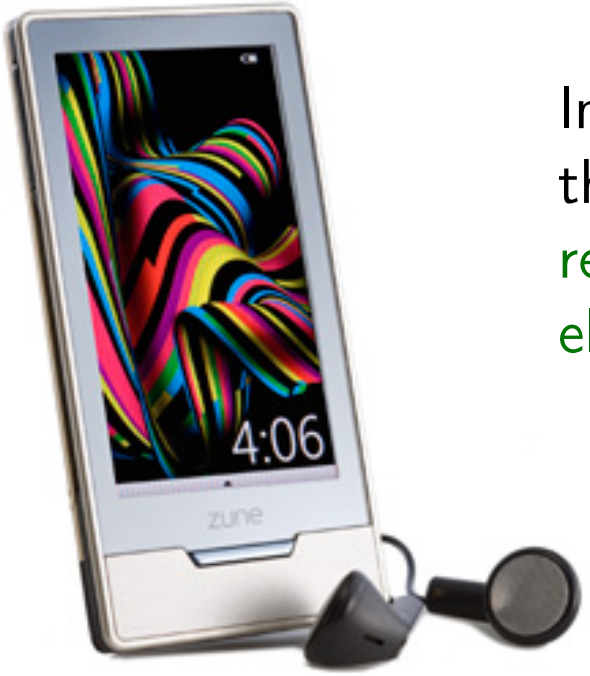

In order to compute the current date, the Zune player has to compute the current year, given the total number of days elapsed since January 1st, 1980.

For instance, if 777 days elapsed since January 1st, 1980, then

- 366 days elapsed in 1980;
- 365 days elapsed in 1981;
- 46 days elapsed since January 1st, 1982.

Verification of computerized systems – The Zune bug

```
function current_year(int days)
 year = 1980;

 while (days > 365) {
     if (IsLeapYear(year)) {
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Example

on Feb. 15th, 1982:

$$\boxed{\begin{array}{c} \texttt{days} = 777 \\ \texttt{year} = 1980 \end{array}}$$

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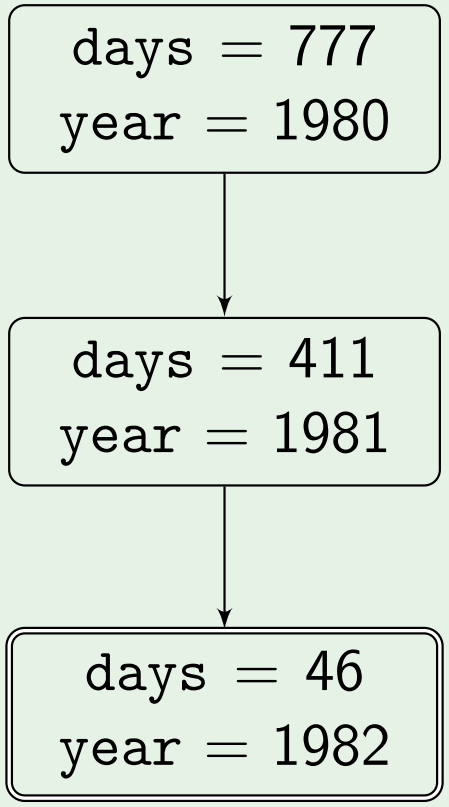

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Example

on Dec. 31st, 2008:

$$\boxed{\begin{array}{l} \texttt{days} = 9132 \\ \texttt{year} = 1980 \end{array}}$$

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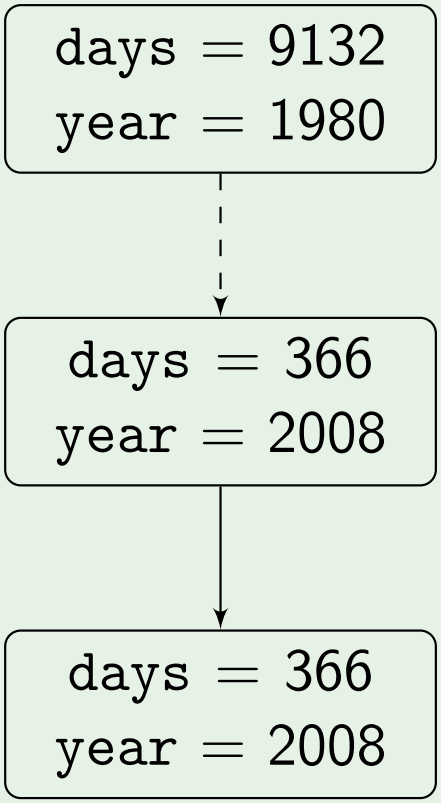

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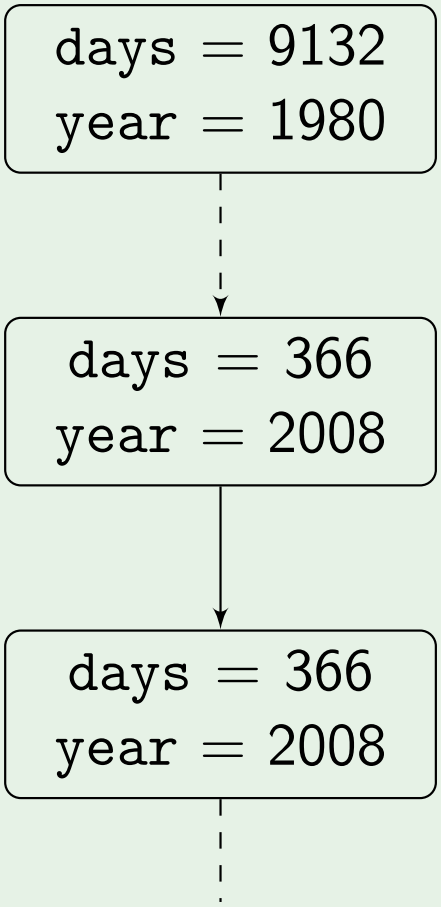

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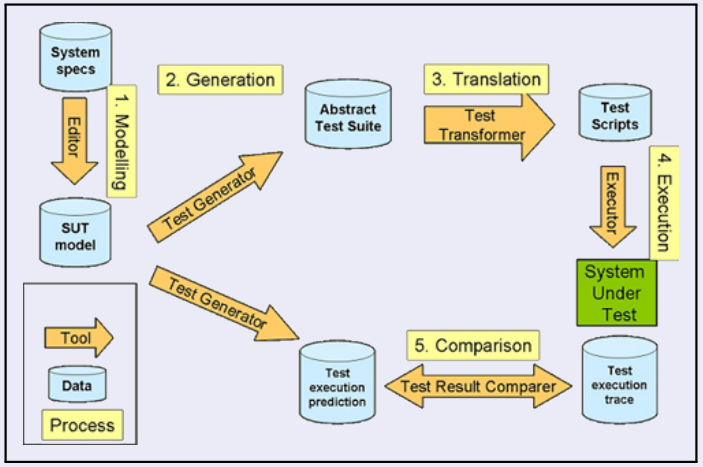

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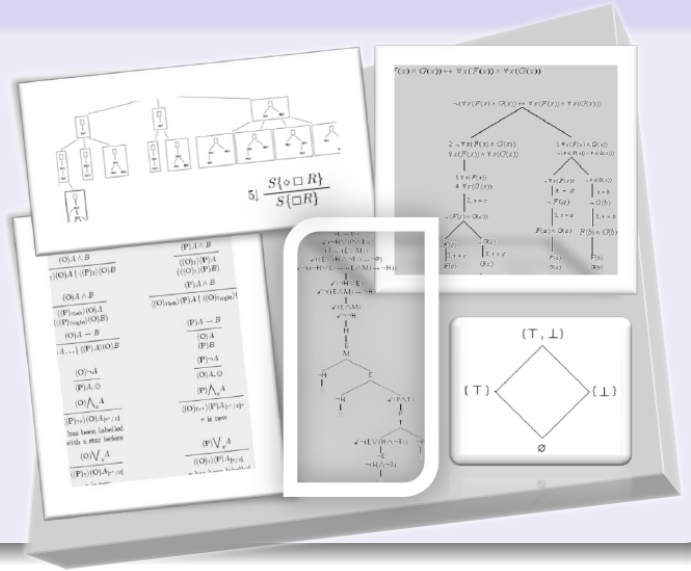

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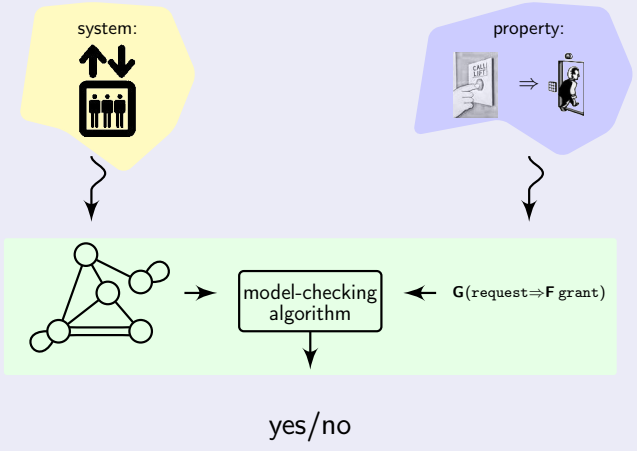
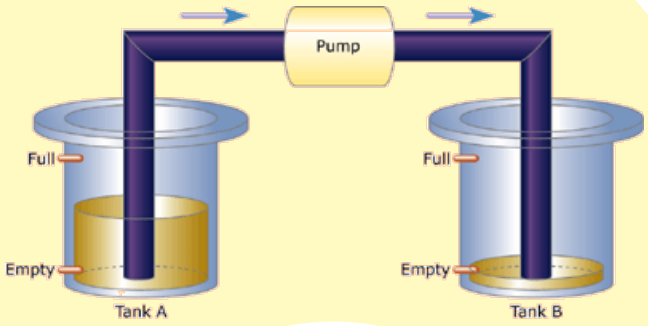


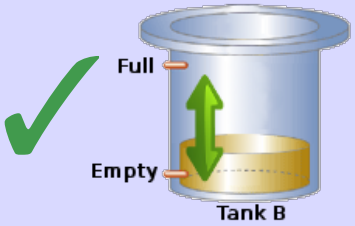
Model checking and synthesis

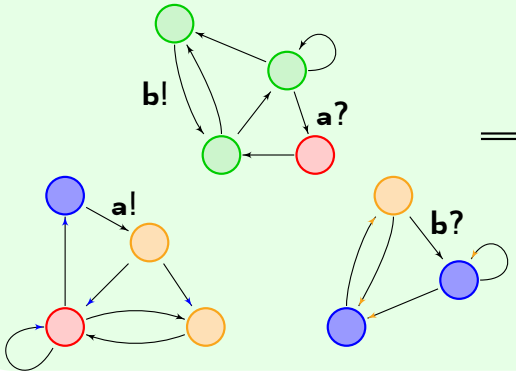
system:



[http://www.embedded.com]

property

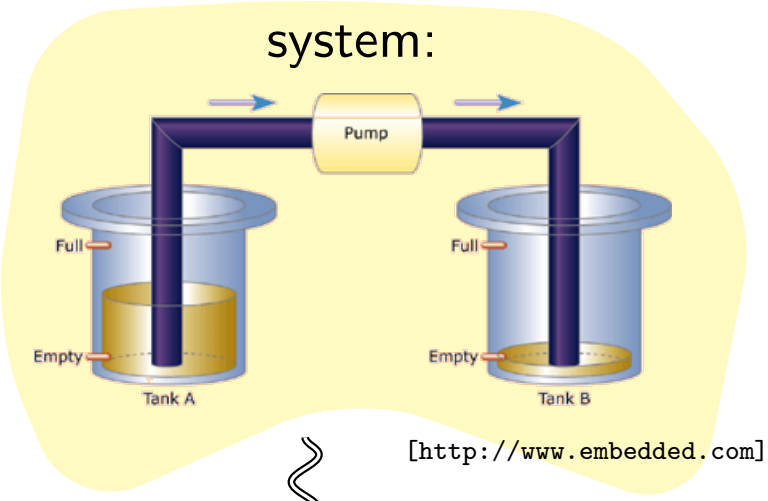


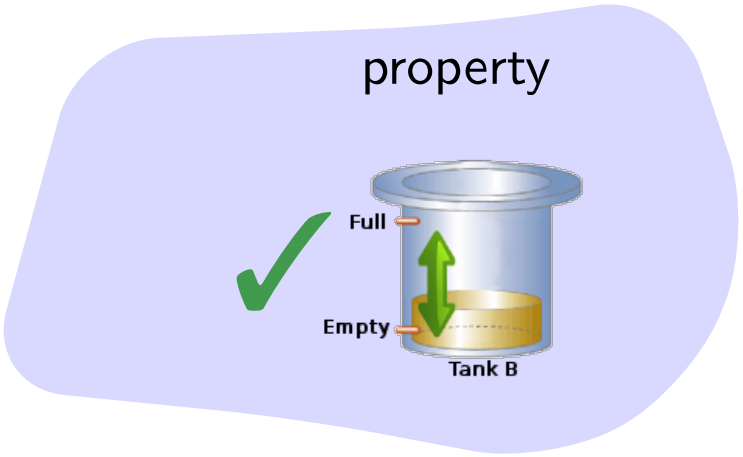


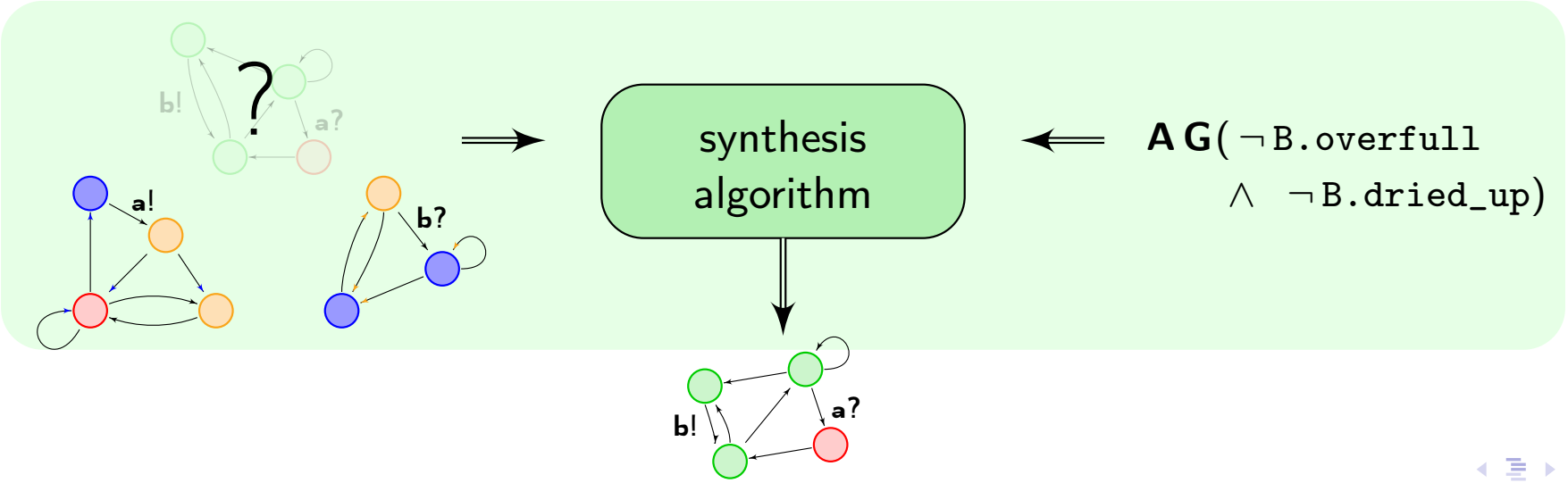
$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{A}\,\mathbf{G}(\neg\,\texttt{B.overfull} \wedge \neg\,\texttt{B.dried\_up})$

yes/no

Model checking and synthesis







Outline of the presentation

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2. Basics of temporal logics
   - convenient formalism for expressing properties of reactive systems
   - efficient verification algorithms for finite-state models

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   - temporal logics for expressing controllability properties

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Modelling reactive systems: finite-state transition systems

We consider finite-state machines for modelling reactive systems.

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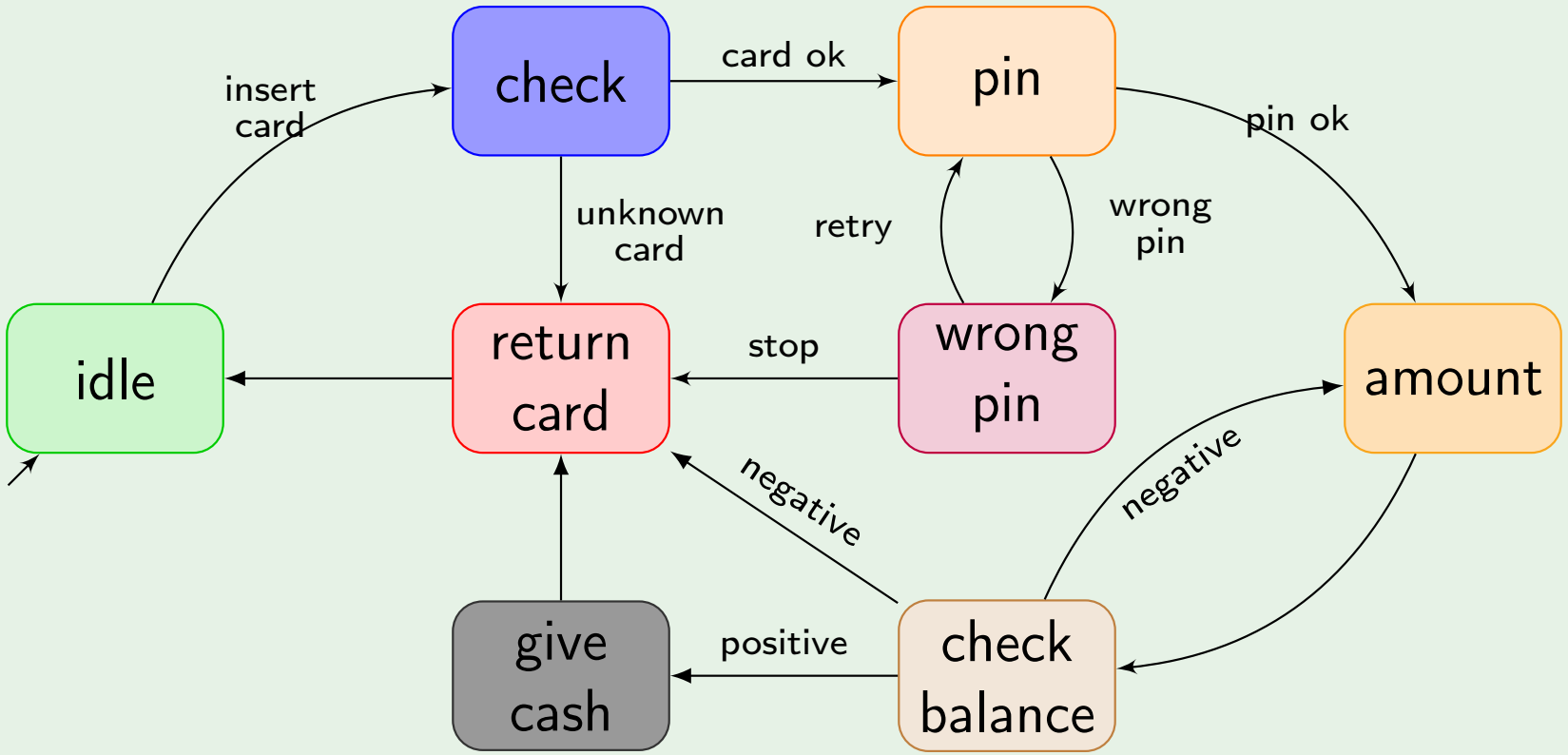
Example (An ATM)



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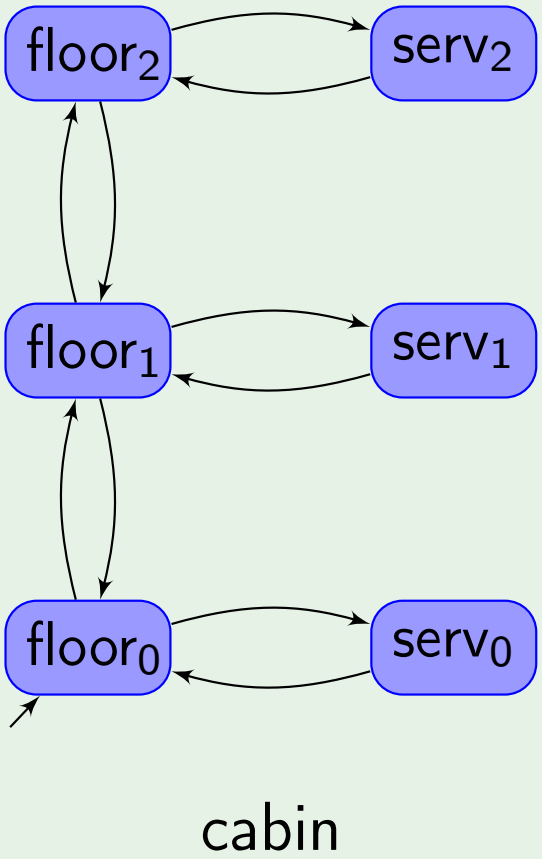

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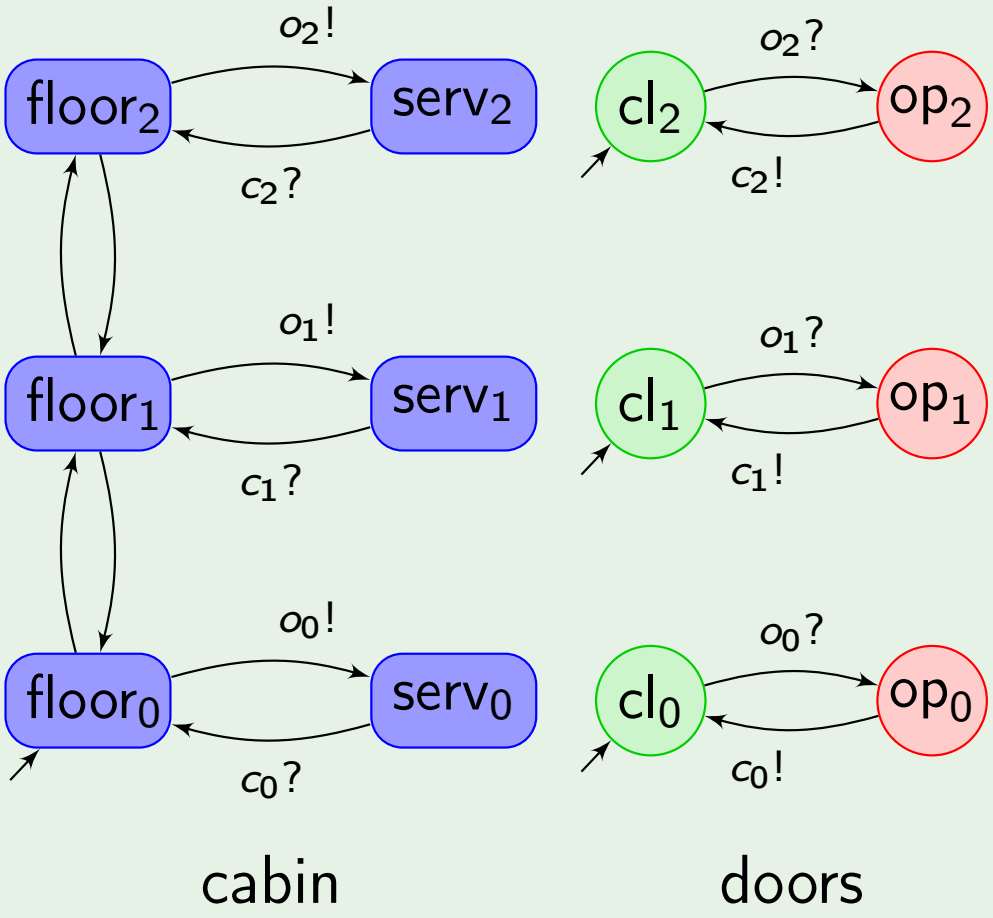

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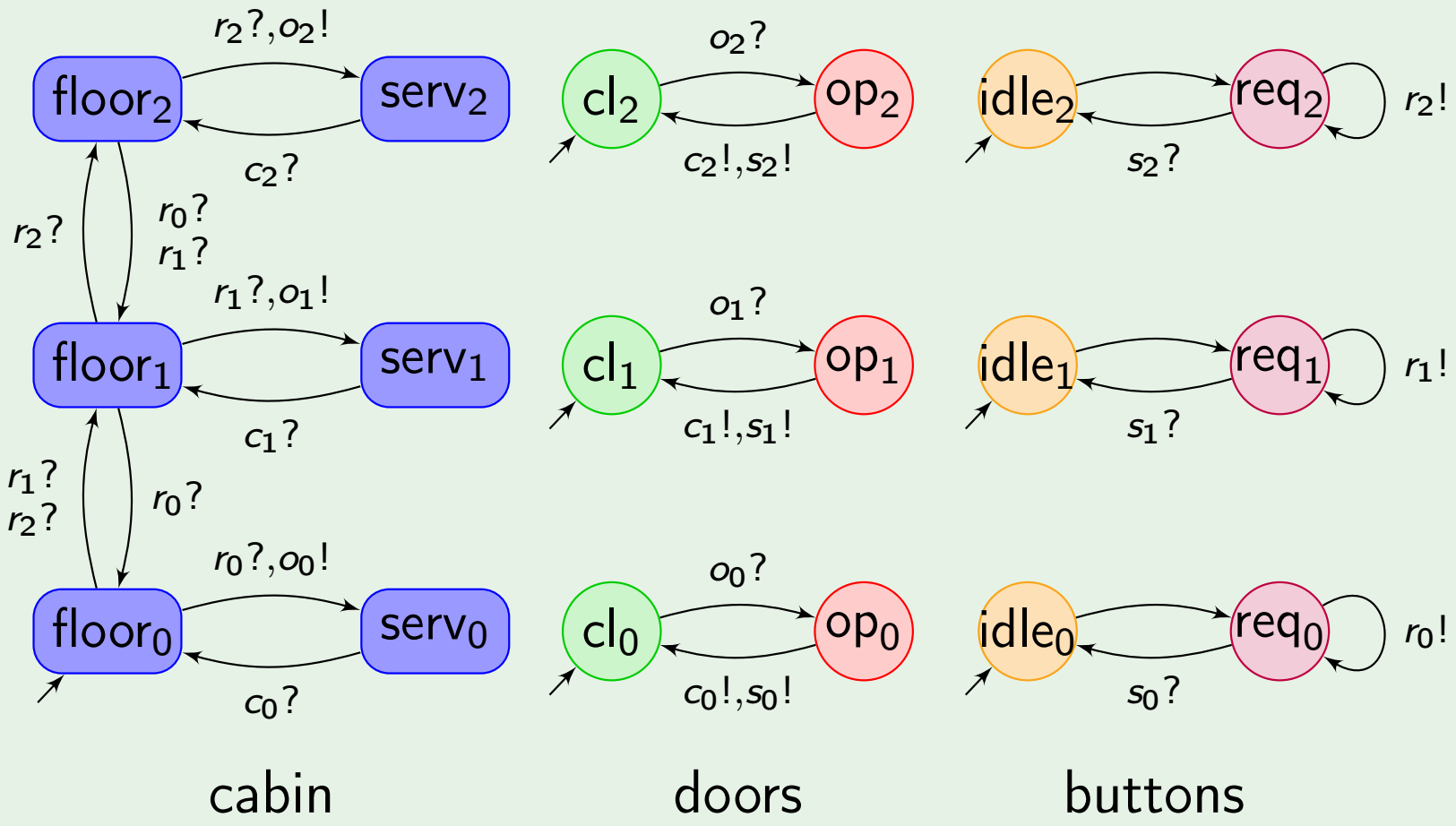

Modelling reactive systems: finite-state transition systems

Formally:

Definition

A finite-state machine (a.k.a. Kripke structure) is a 4-tuple $\mathcal{M} = \langle Q, q_0, R, \ell \rangle$ where

- Q is a finite (or countable) set of states;
- $q_0 \in Q$ is the initial state;
- $R \subseteq Q \times Q$ is a transition relation satisfying
$$\forall q \in Q.\ \exists q' \in Q.\ (q, q') \in R$$
- $\ell \colon Q \to 2^{\mathsf{AP}}$ labels states with atomic propositions.

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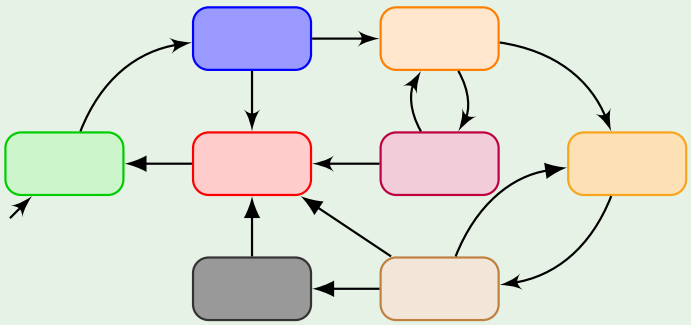

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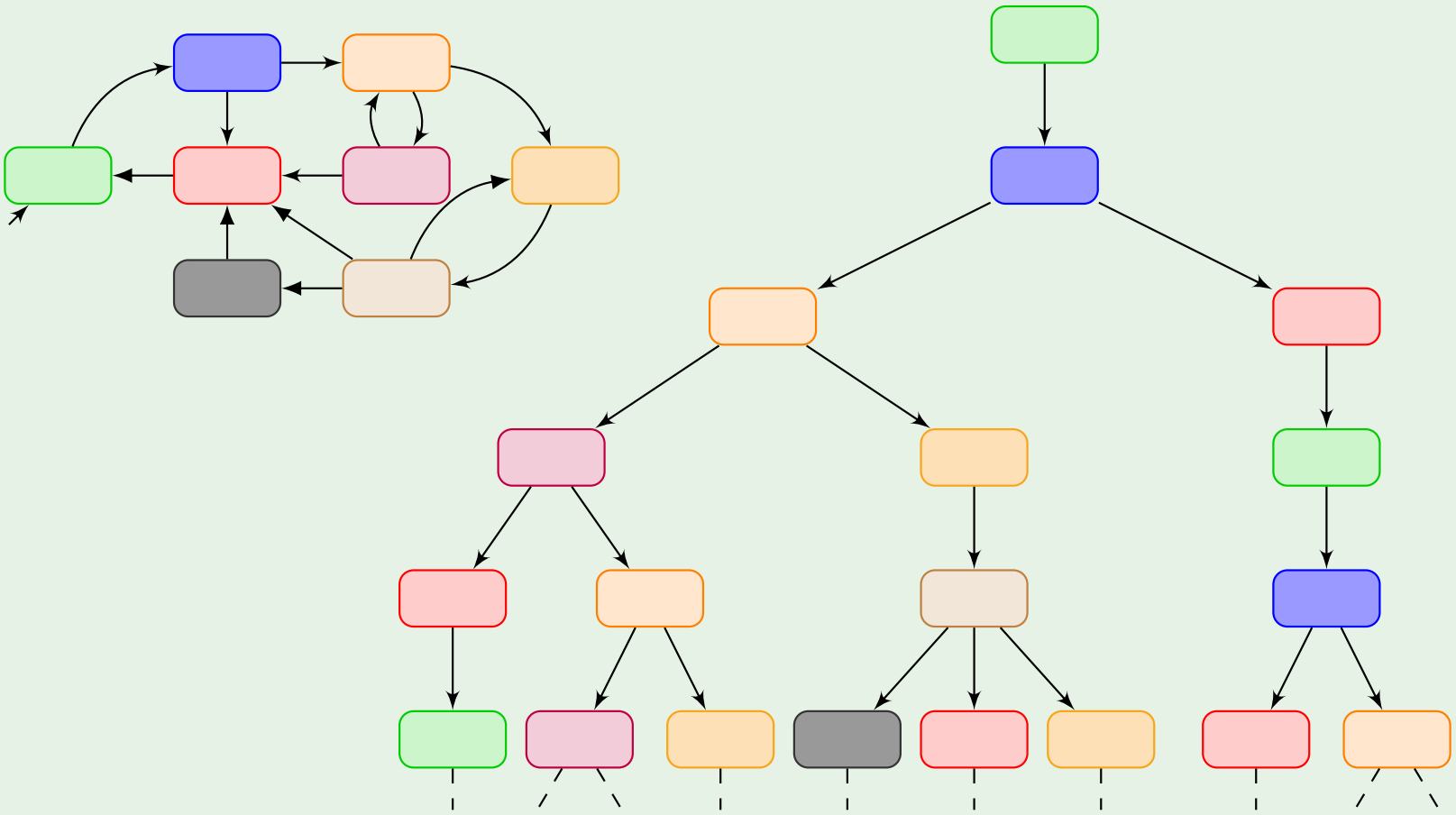

Modelling reactive systems: finite-state transition systems

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Definition

- A finite run of $\mathcal{M}$ from q is a pair (w, ℓ) where
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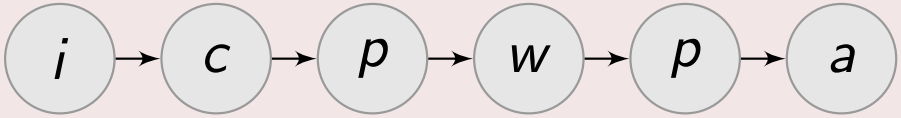

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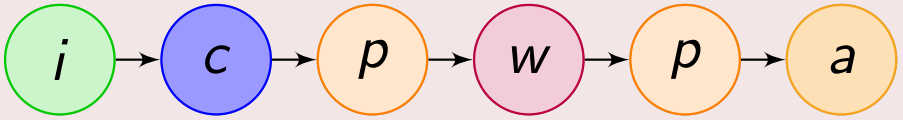

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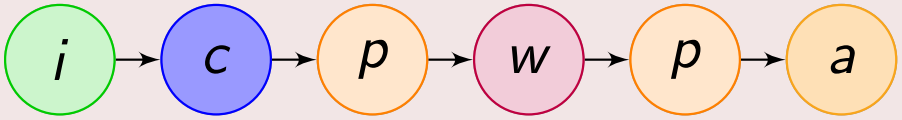

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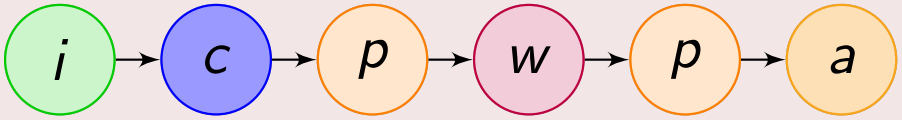

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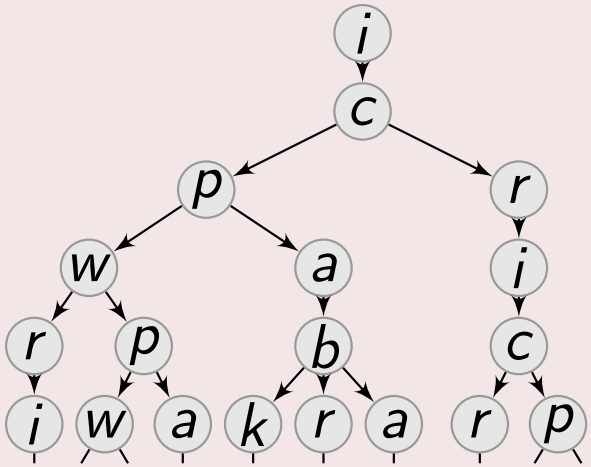

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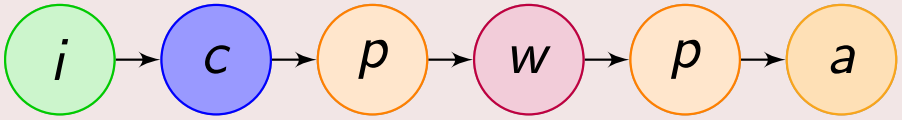

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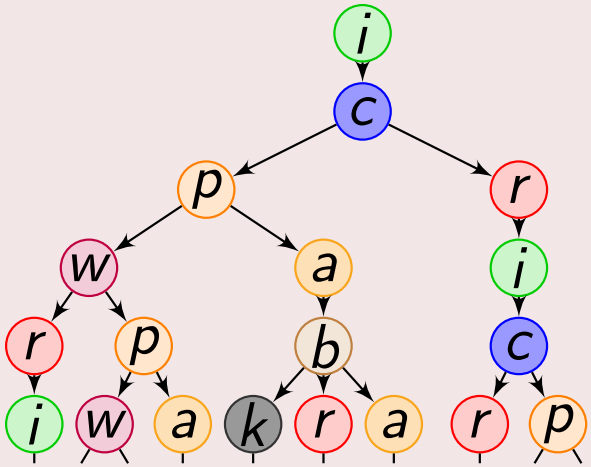

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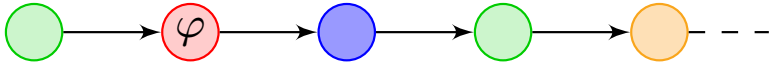

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$\varphi\ \mathbf{U}\ \psi$	$\varphi \to \varphi \to \psi \to \bullet \to \bullet \dashrightarrow$	"φ until ψ"
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$\neg\ \mathbf{F}\ \neg\varphi \equiv \mathbf{G}\,\varphi$	$\varphi \to \varphi \to \varphi \to \varphi \to \varphi \dashrightarrow$	"always φ"

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$\varphi\ \mathbf{U}\ \psi$	$\bigcirc^{\varphi} \to \bigcirc^{\varphi} \to \bigcirc^{\psi} \to \bigcirc \to \bigcirc \cdots$	"φ until ψ"
$\texttt{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$	$\bigcirc \to \bigcirc \to \bigcirc \to \bigcirc^{\varphi} \to \bigcirc \cdots$	"eventually φ"
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A Kripke structure satisfies $\varphi \in$ LTL if all its infinite executions do:

$$\mathcal{M} \models \varphi \quad \Longleftrightarrow \quad \forall \pi \in \mathcal{T}_{\mathcal{M}}.\ \pi \models \varphi.$$

Examples of LTL formulas

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Satisfiability and model checking

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  - Input: a formula φ in LTL;
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  - Input: a formula φ in LTL, and a Kripke structure $\mathcal{M}$;
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Theorem ([SC85])

LTL satisfiability and model checking are logspace-equivalent.

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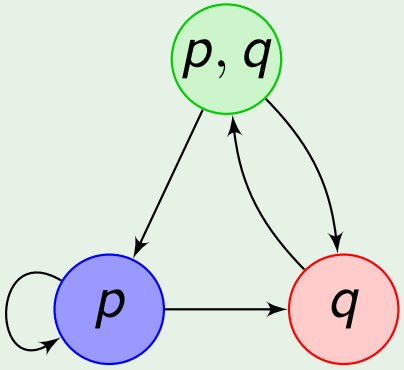

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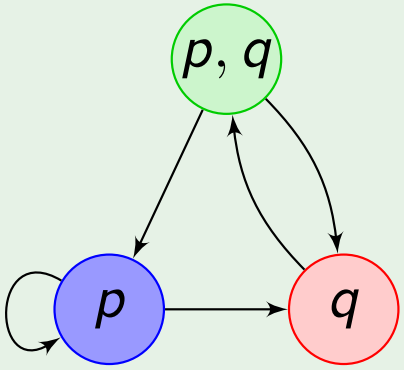

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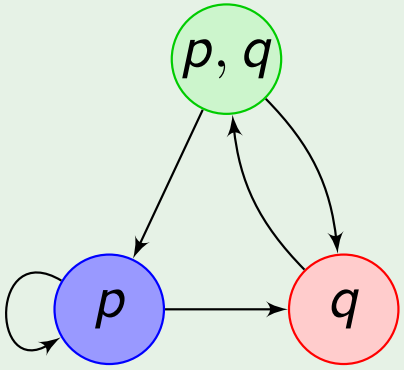

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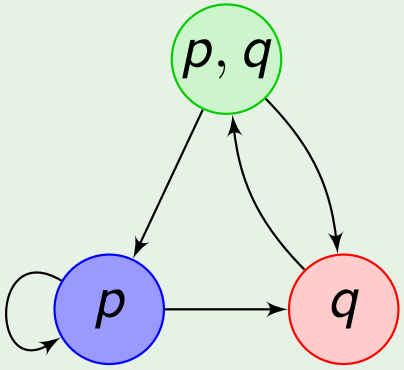

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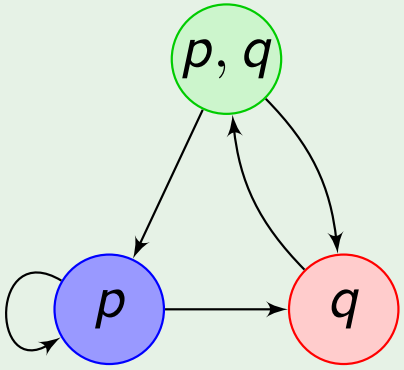

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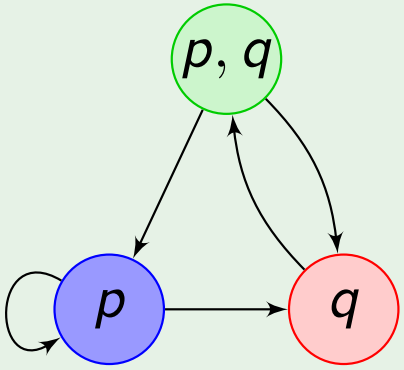

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Lemma

$$\mathcal{M}, \bigcirc \not\models \varphi \iff \neg\varphi \wedge \bigcirc \wedge \Phi_{\mathcal{M}} \text{ is satisfiable.}$$

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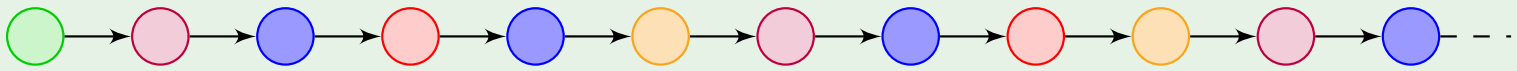

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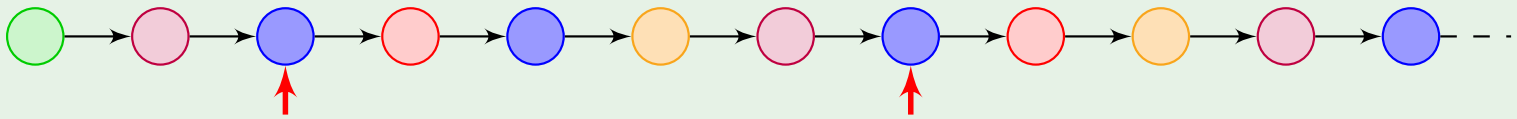

same subformulas hold at both positions;
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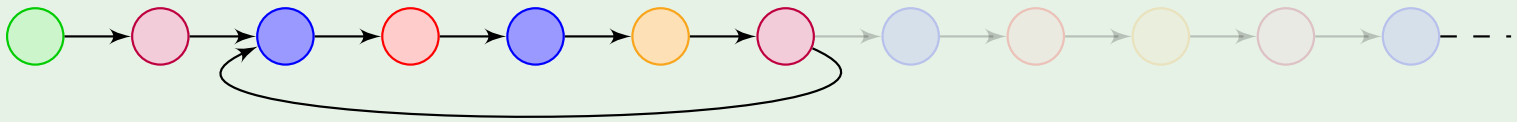

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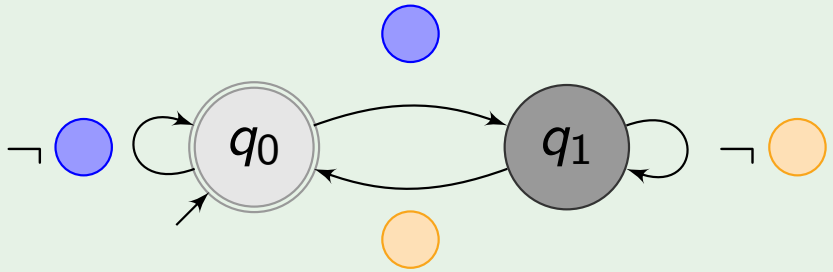

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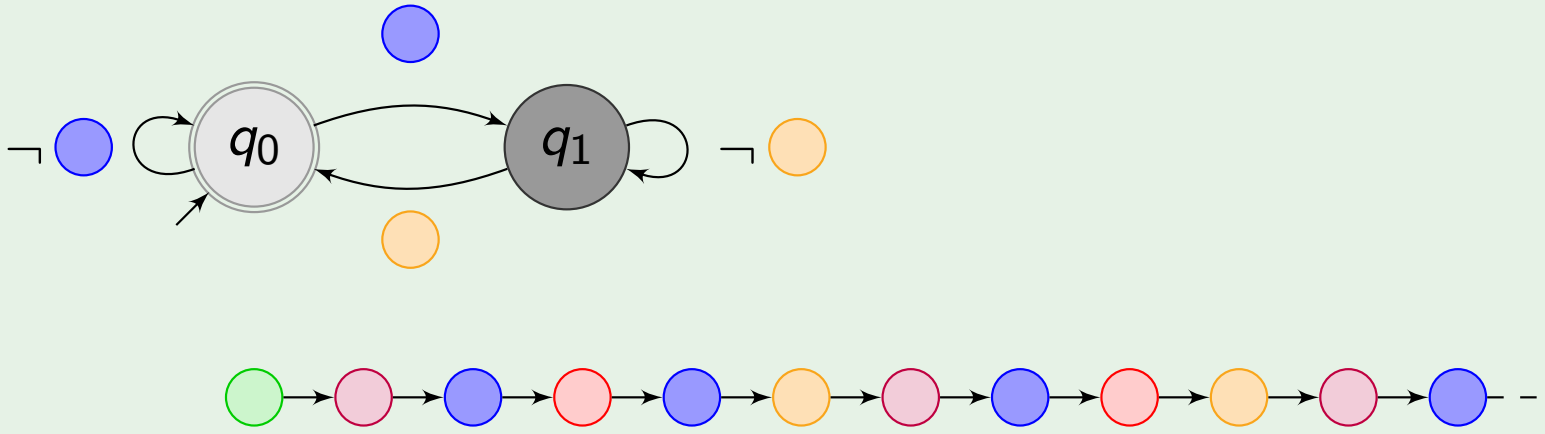

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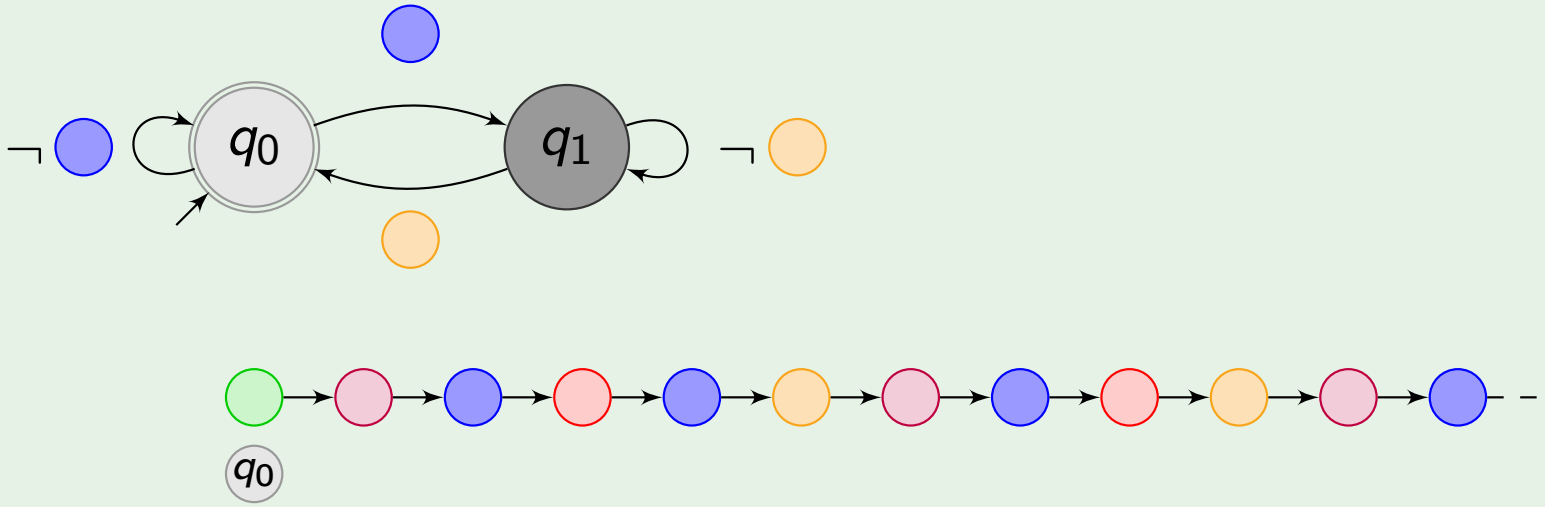

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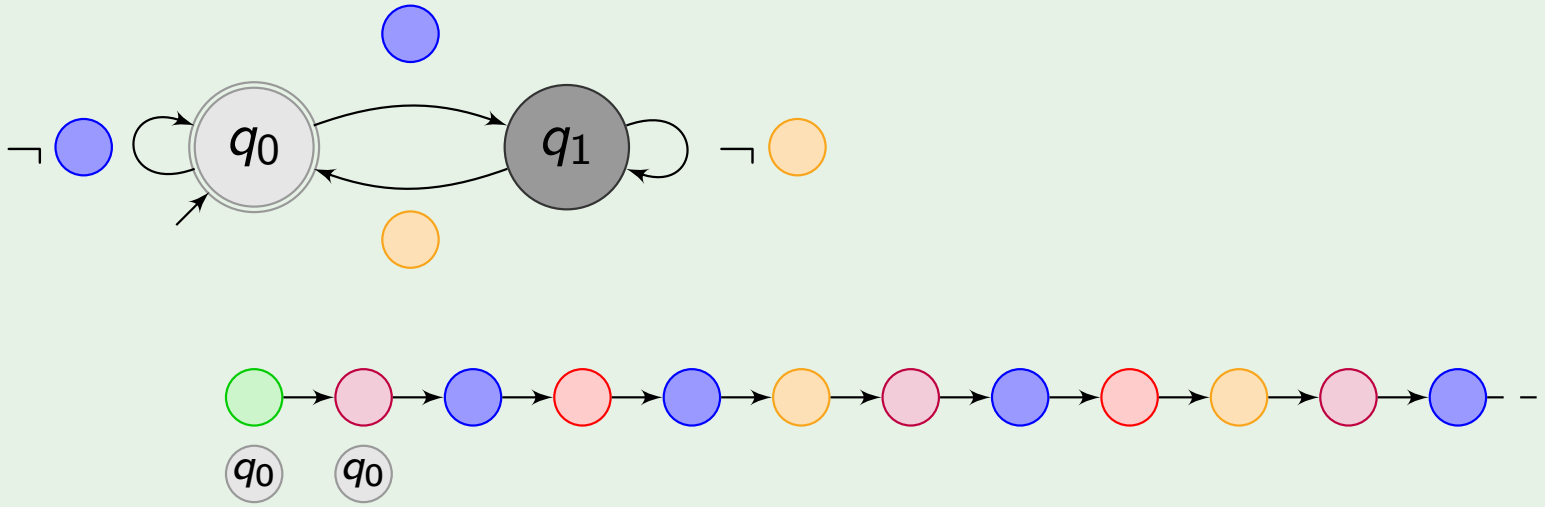

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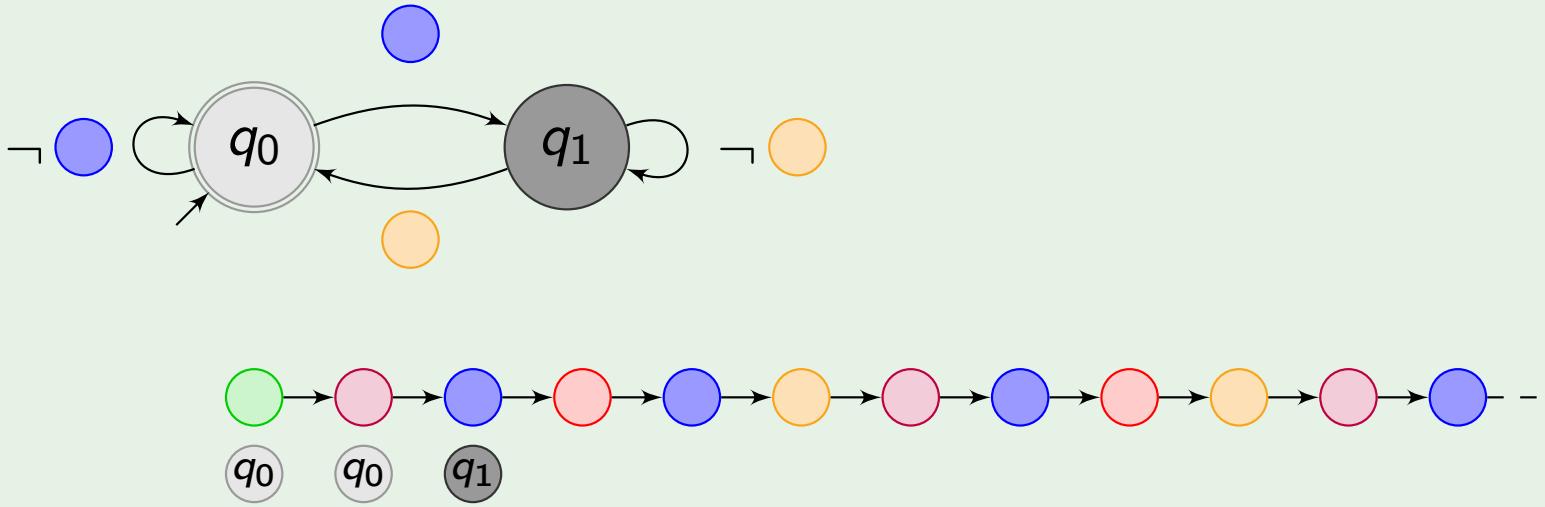

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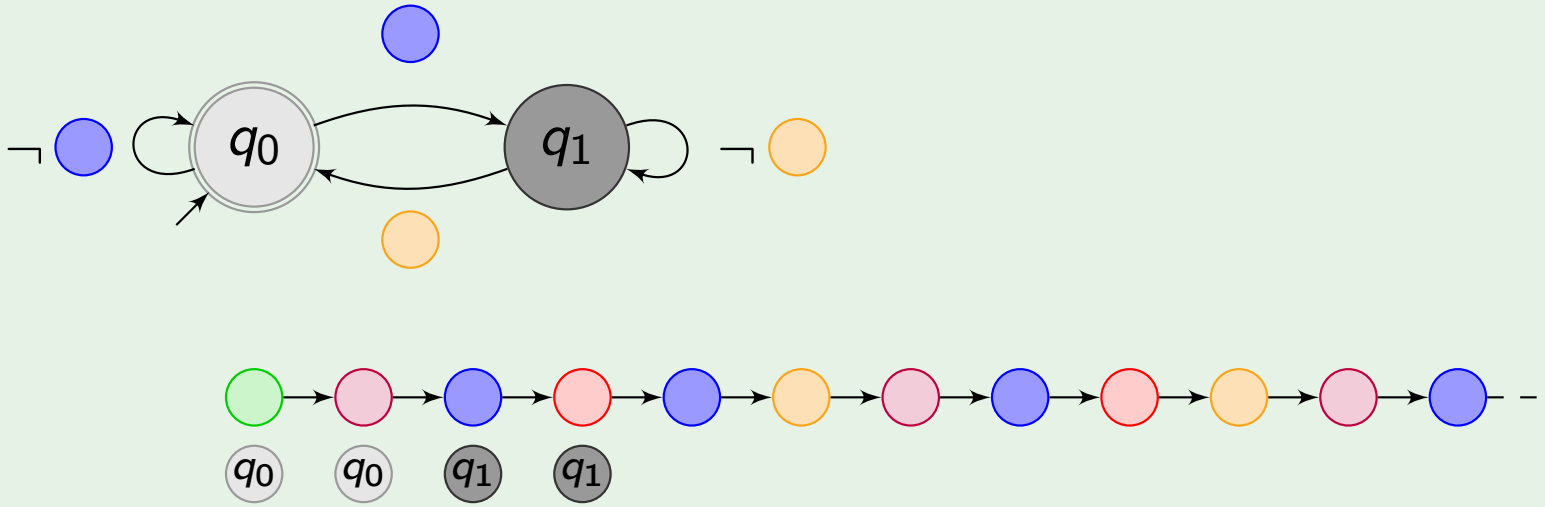

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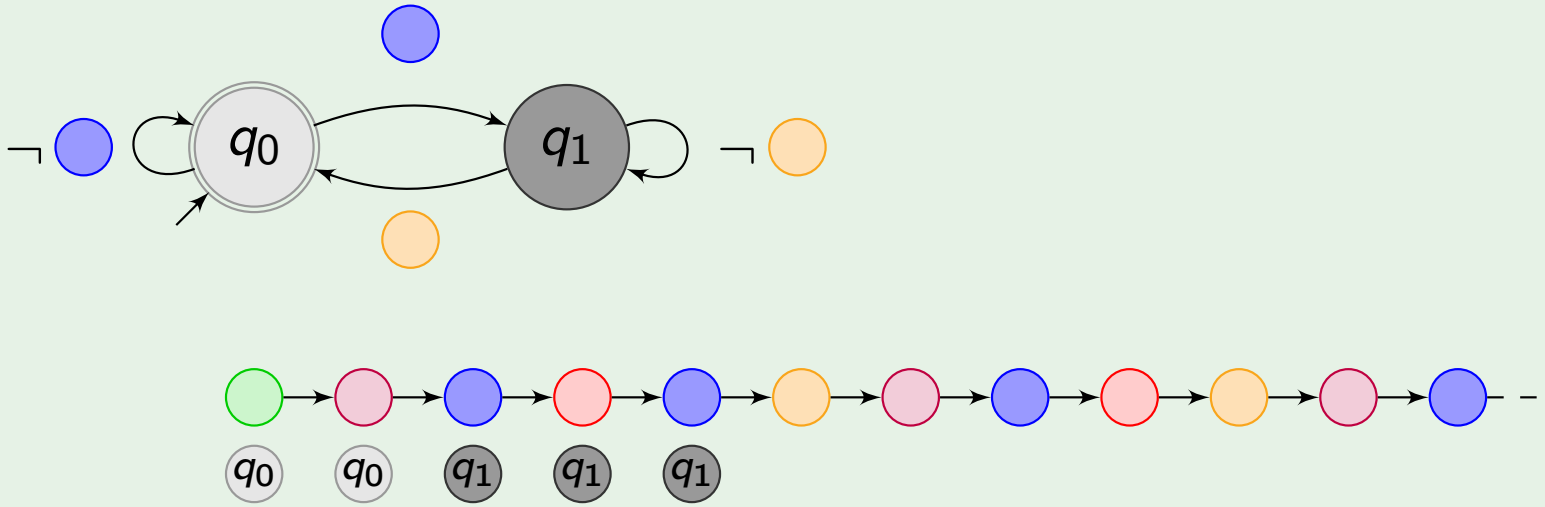

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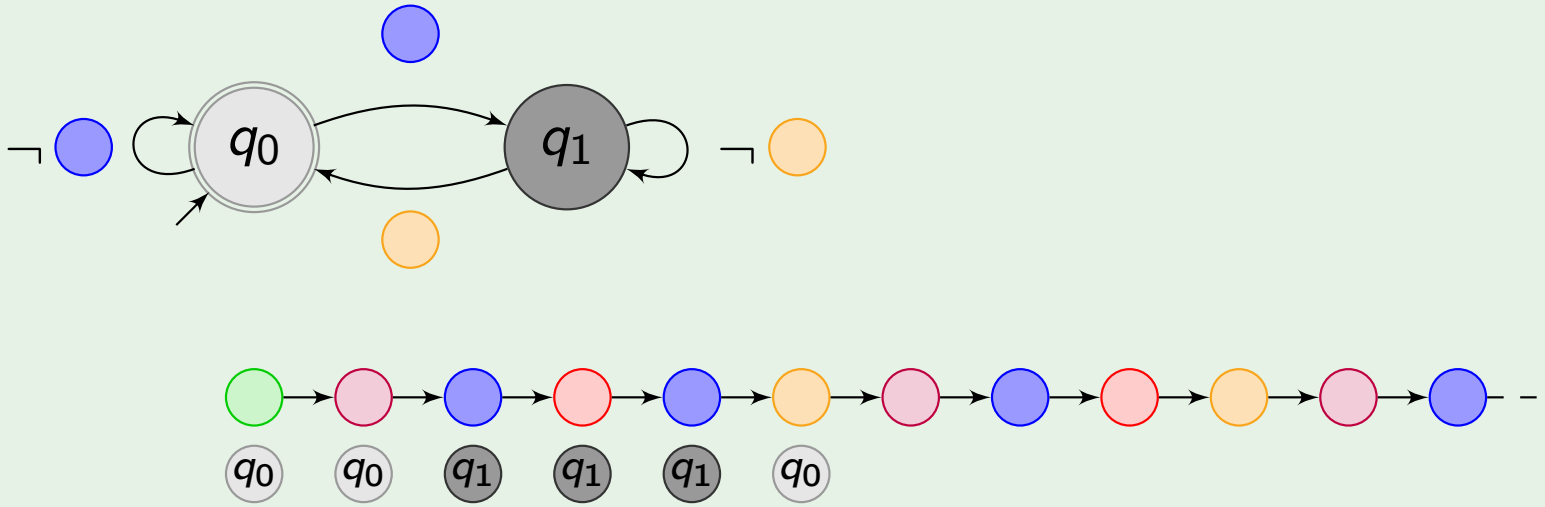

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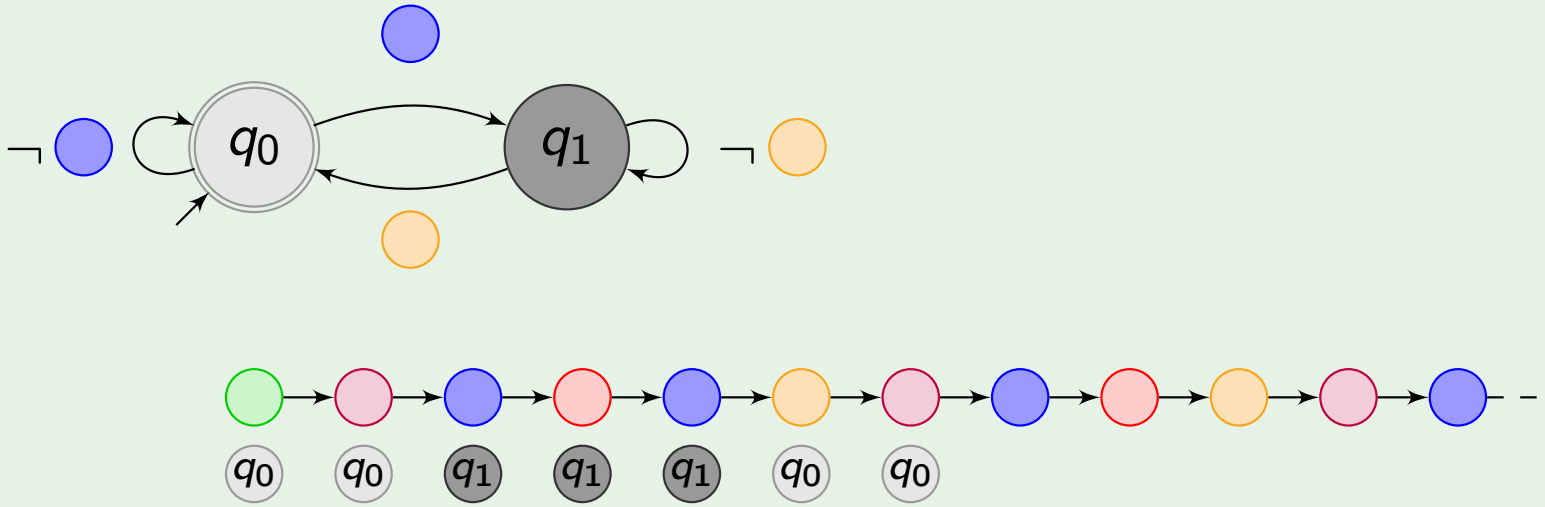

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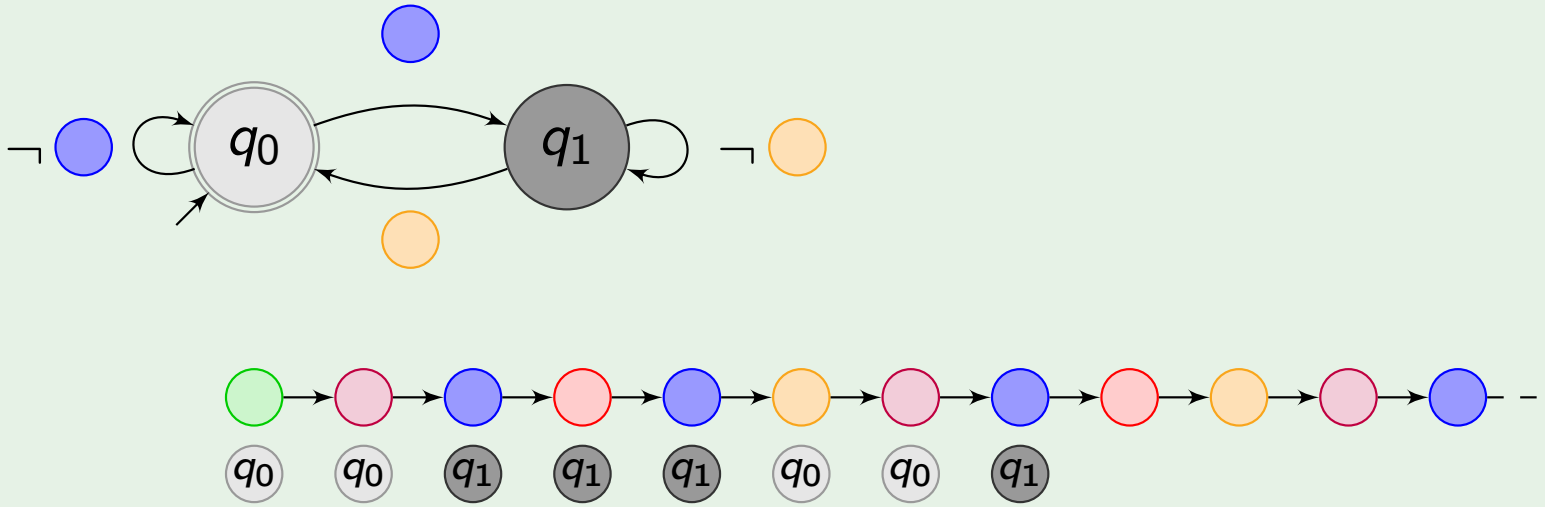

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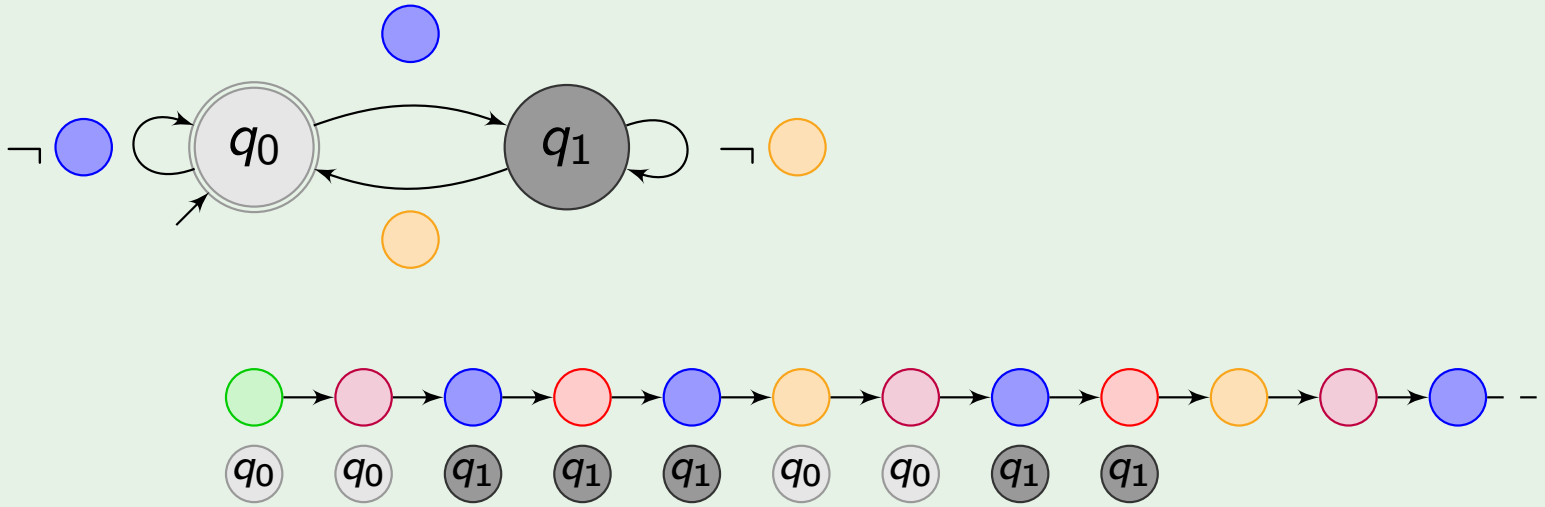

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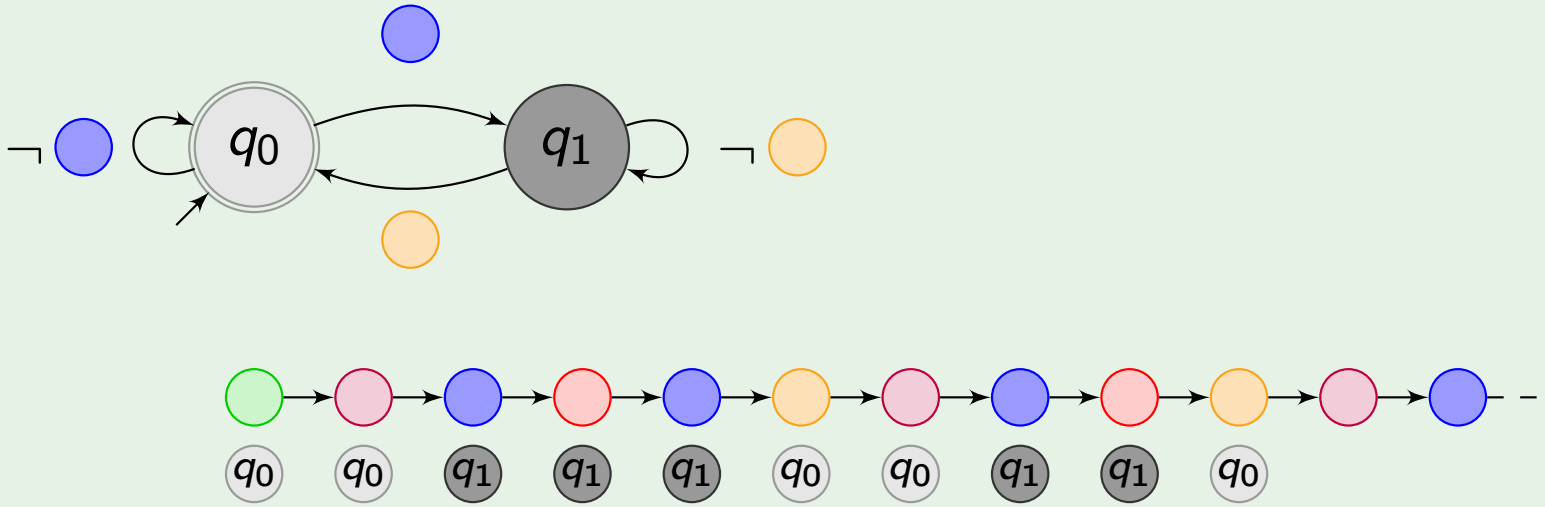

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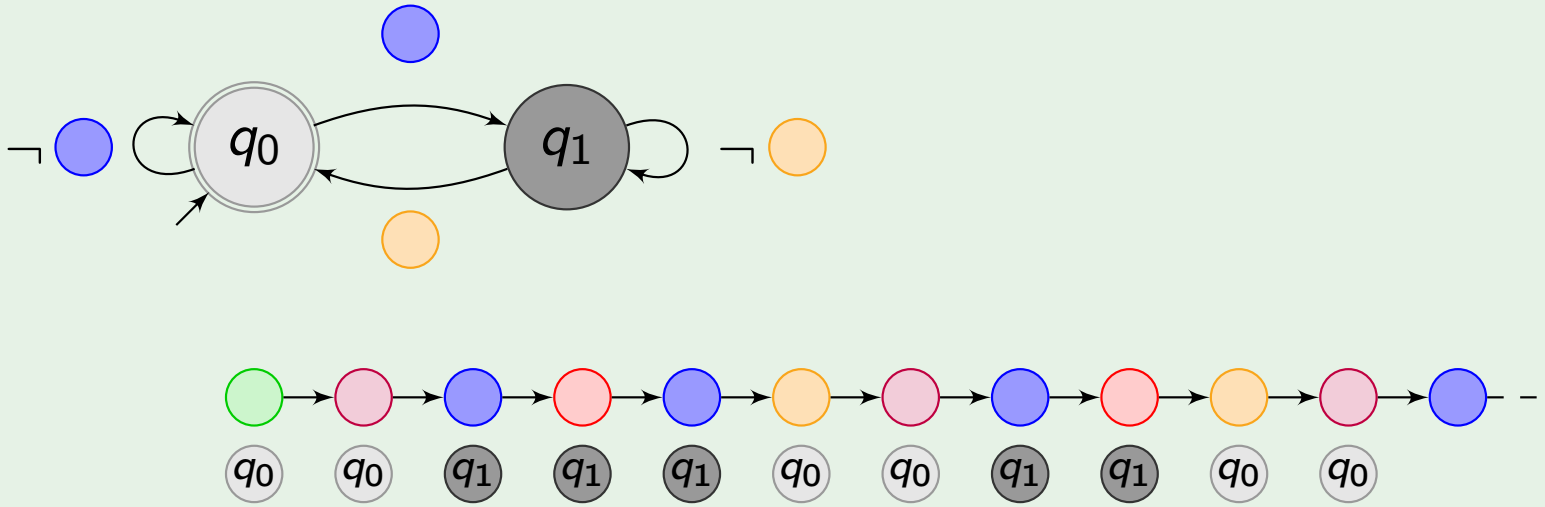

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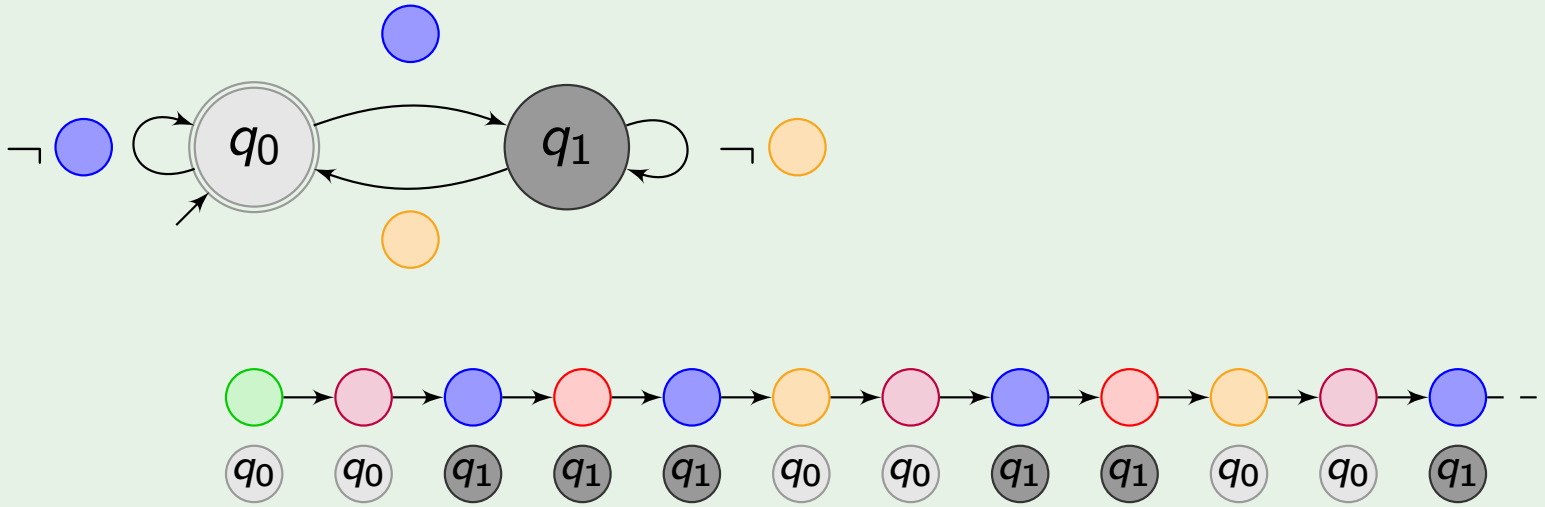

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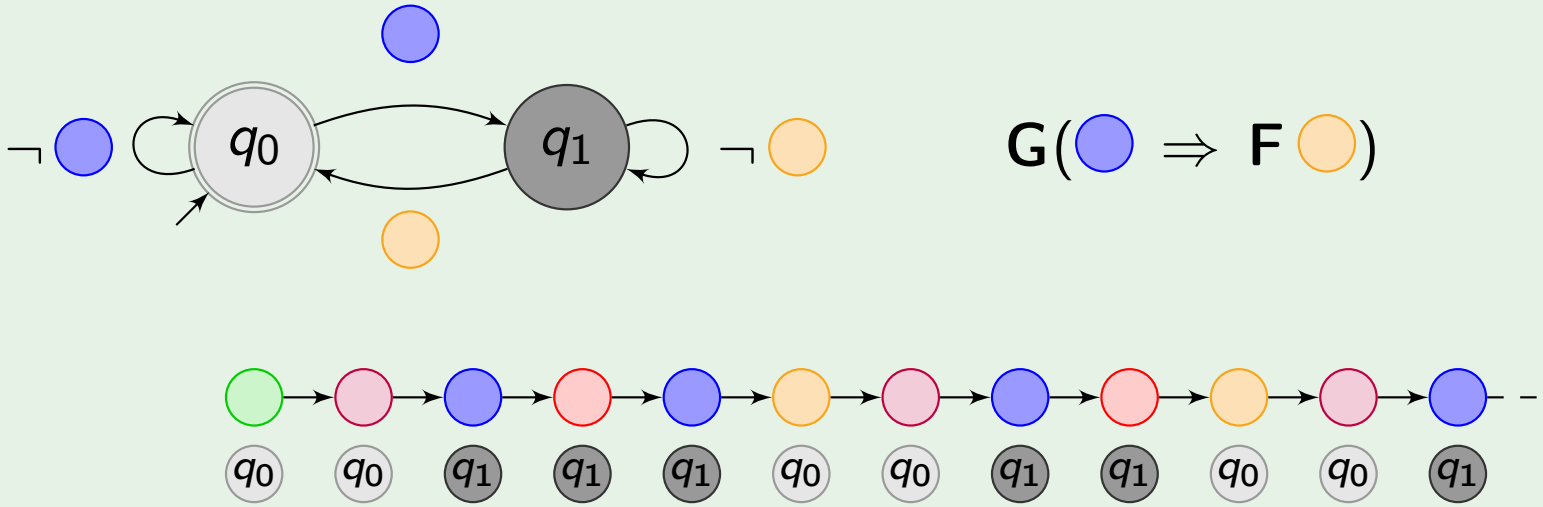

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For any $\varphi \in LTL$, there exists an exponential-size Büchi automaton $\mathcal{A}_\varphi$ such that

$$\forall w. \qquad w \models \varphi \iff w \in Lang(\mathcal{A}_\varphi).$$

Proof (Sketch)

- use a slightly extended notion of subformula:

$$\bigcirc\ \mathbf{U}\ \bigcirc \quad \rightsquigarrow \quad \bigcirc \vee (\bigcirc \wedge \mathbf{X}(\bigcirc\ \mathbf{U}\ \bigcirc)).$$

- states of $\mathcal{A}_\varphi$ = maximal consistent sets of subformulas;
- transition $S \to S'$ whenever for all $\mathbf{X}\,\psi \in S$, it holds $\psi \in S'$;
- acceptance condition: infinitely often $\neg(\bigcirc\ \mathbf{U}\ \bigcirc)$ (generalized Büchi condition)

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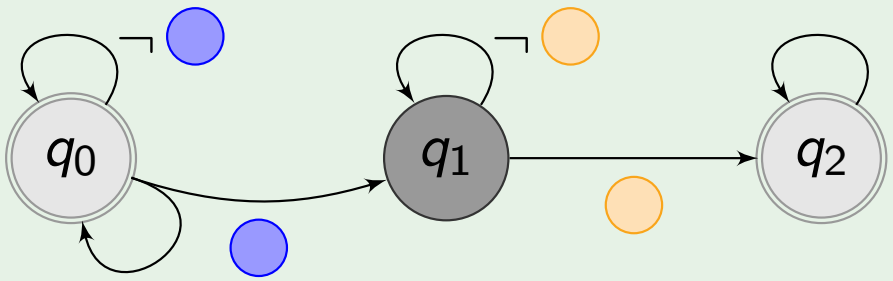

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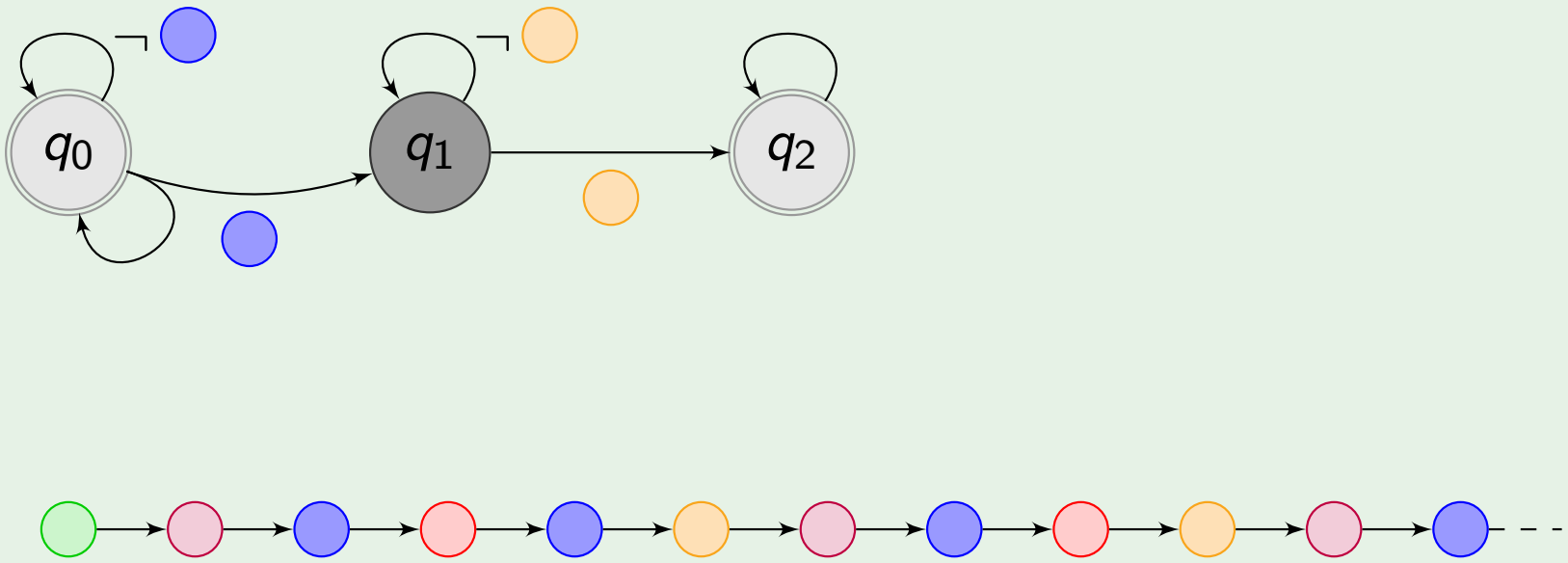

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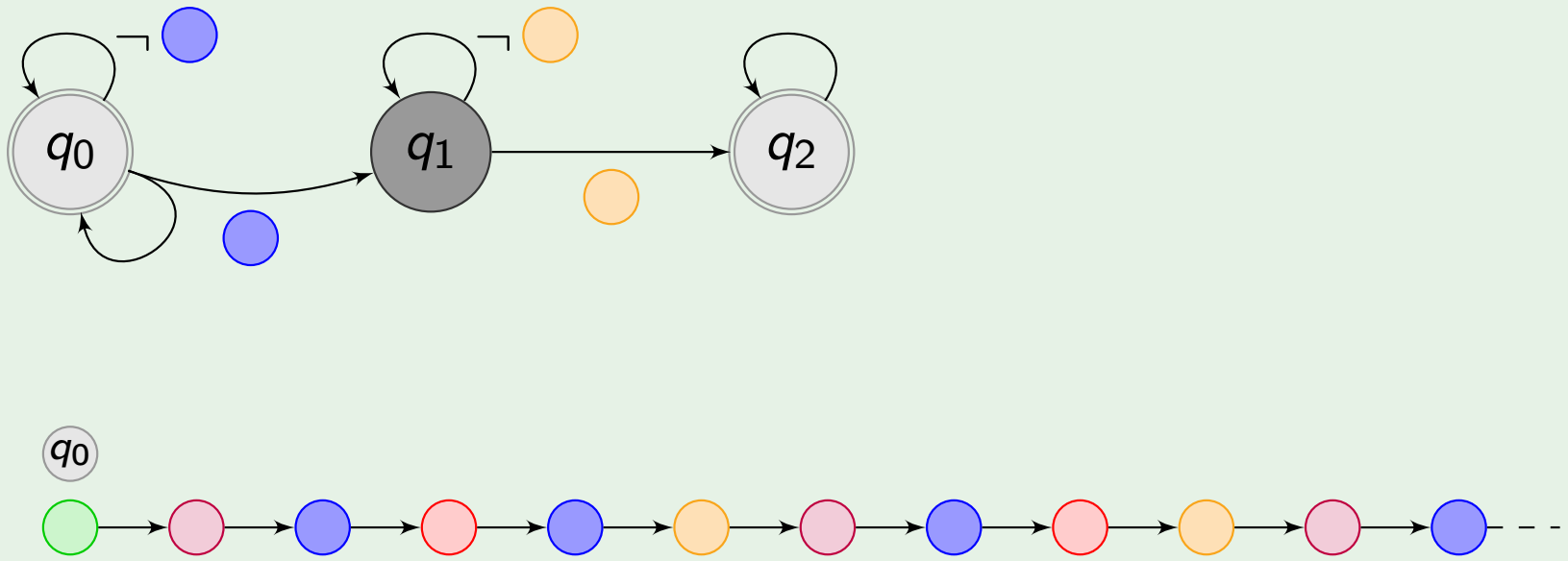

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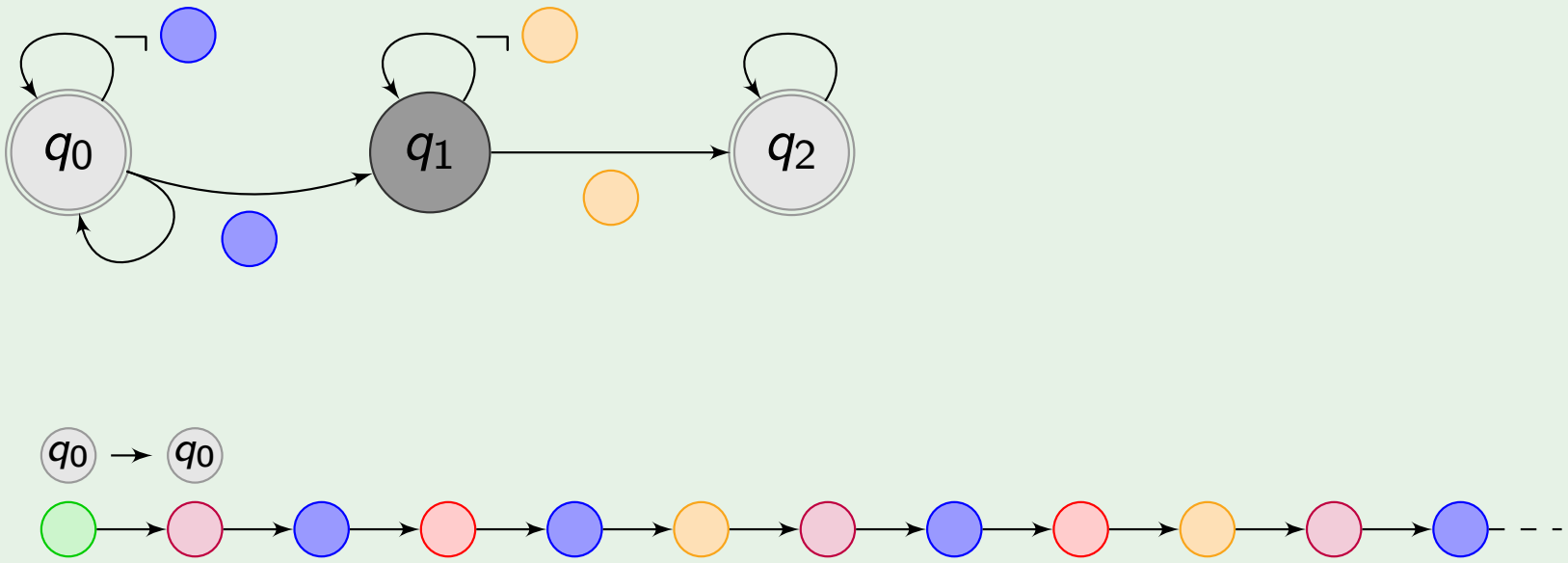

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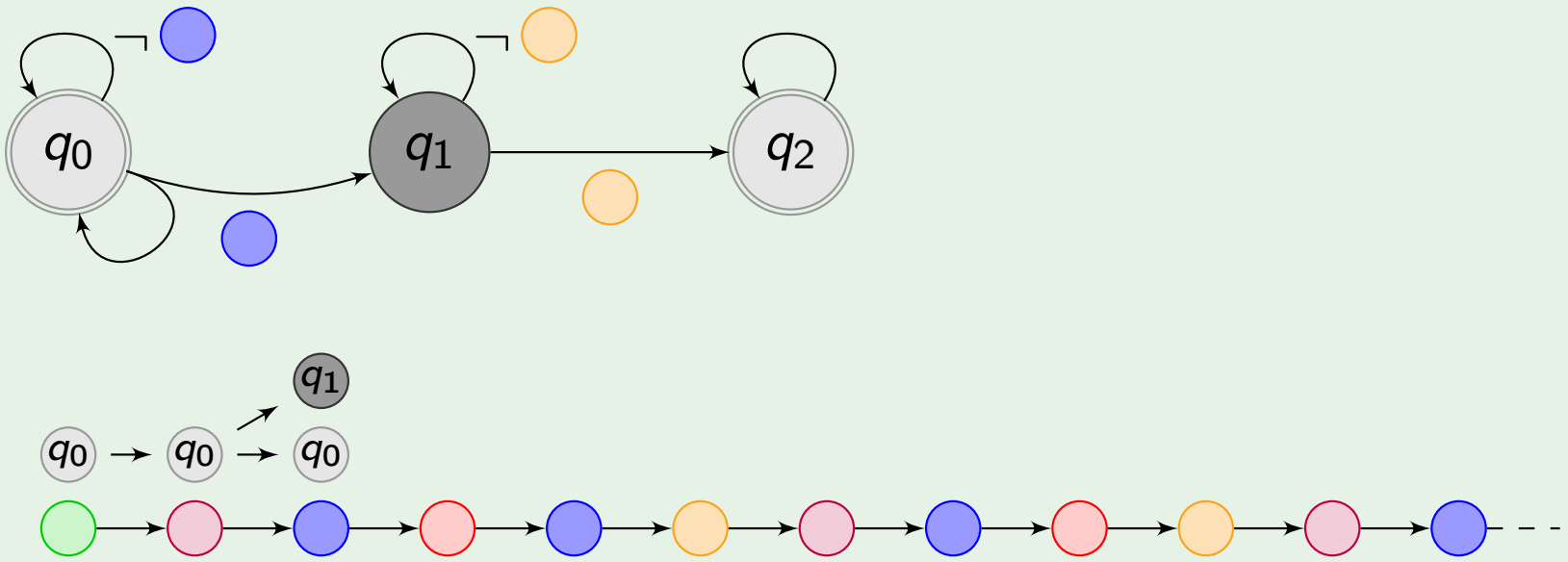

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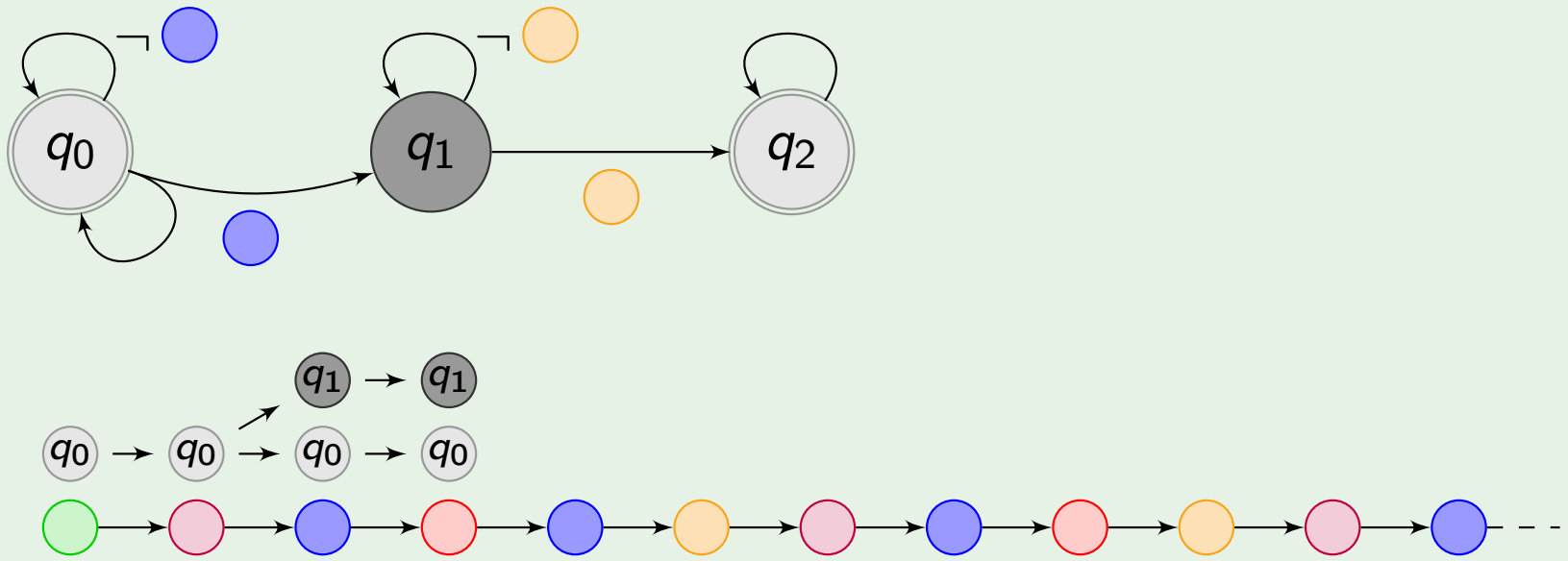

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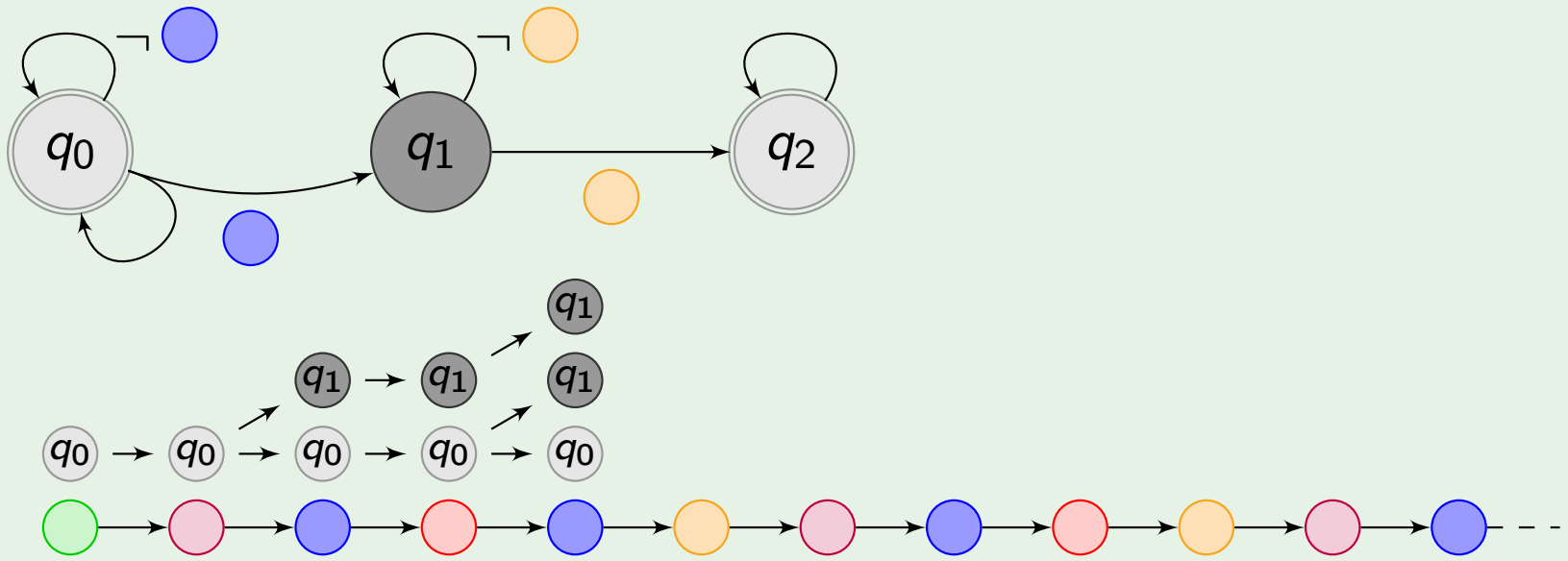

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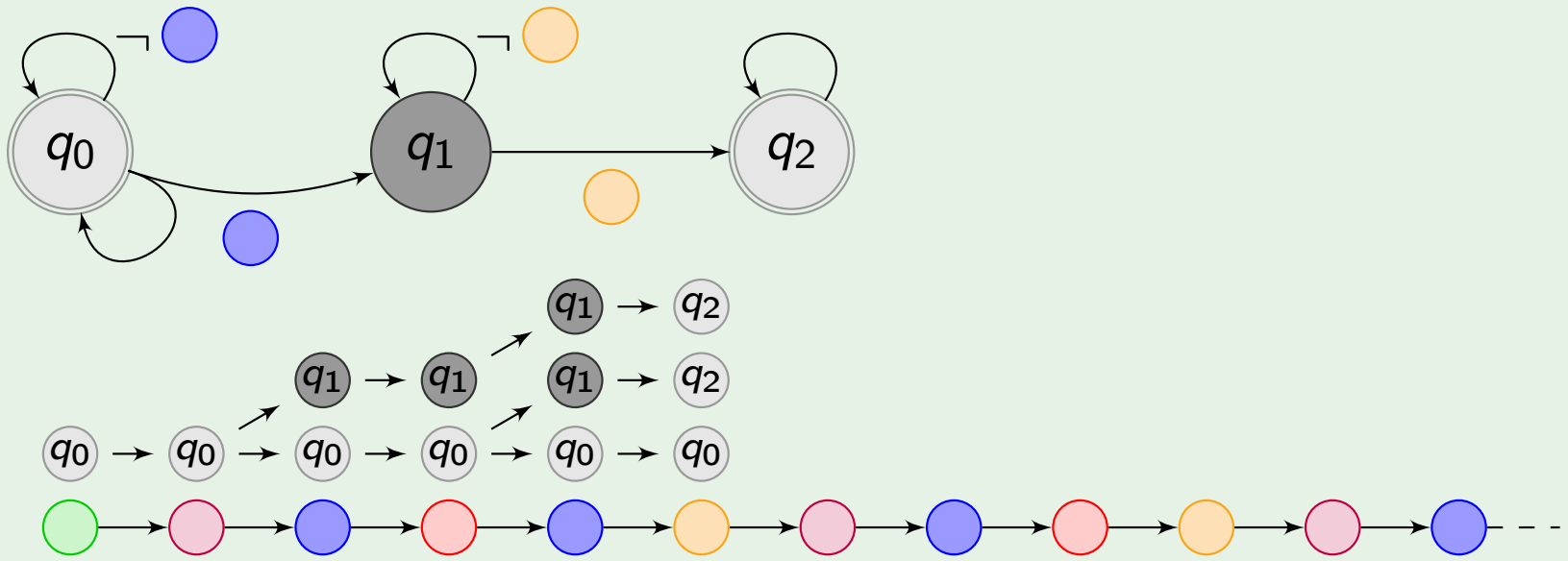

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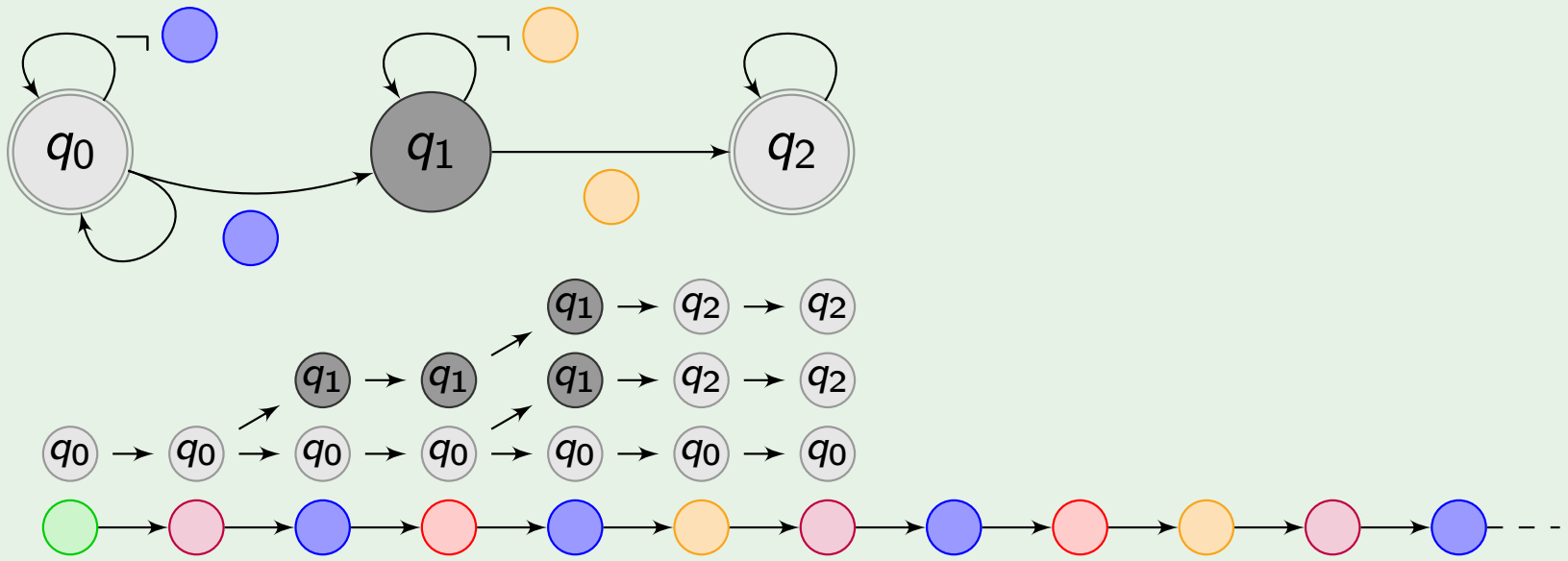

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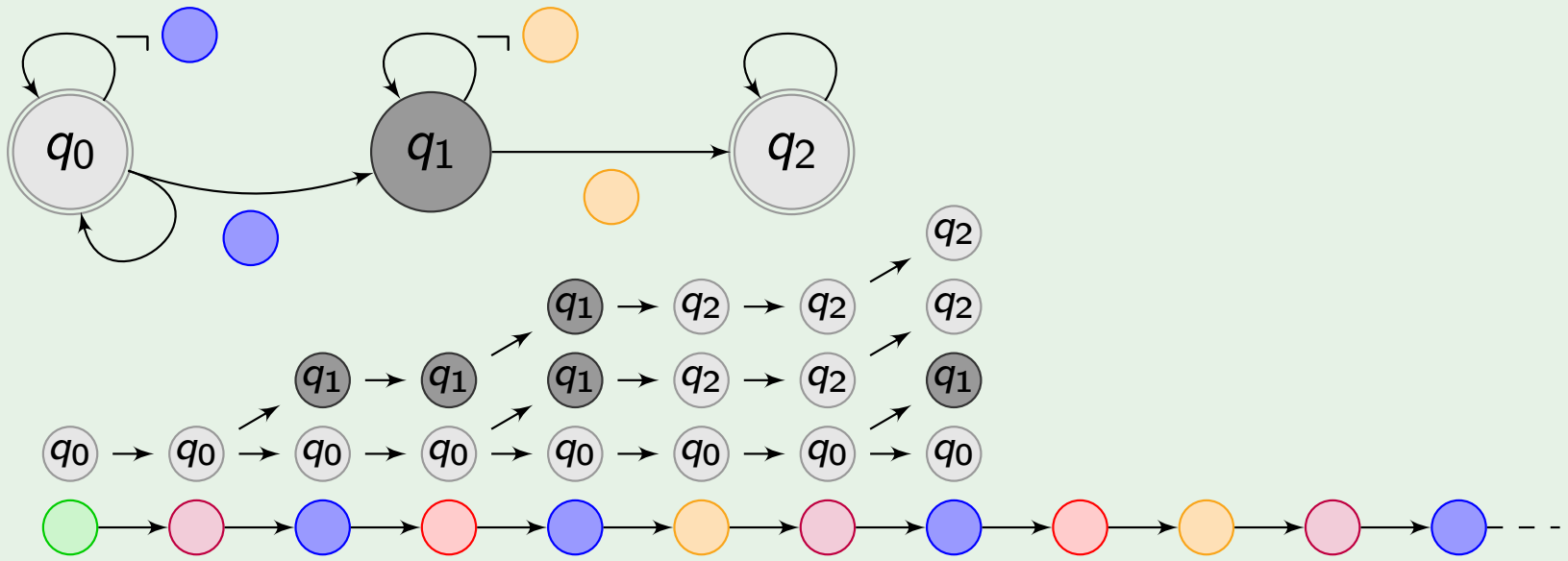

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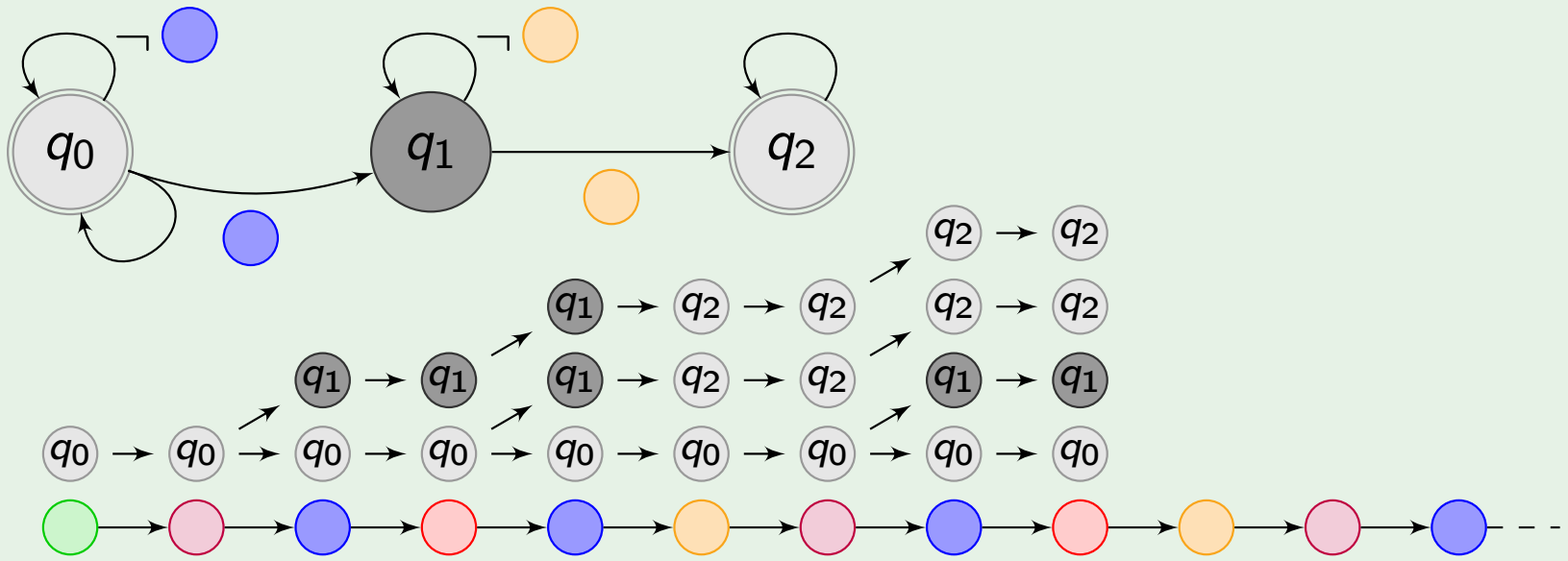

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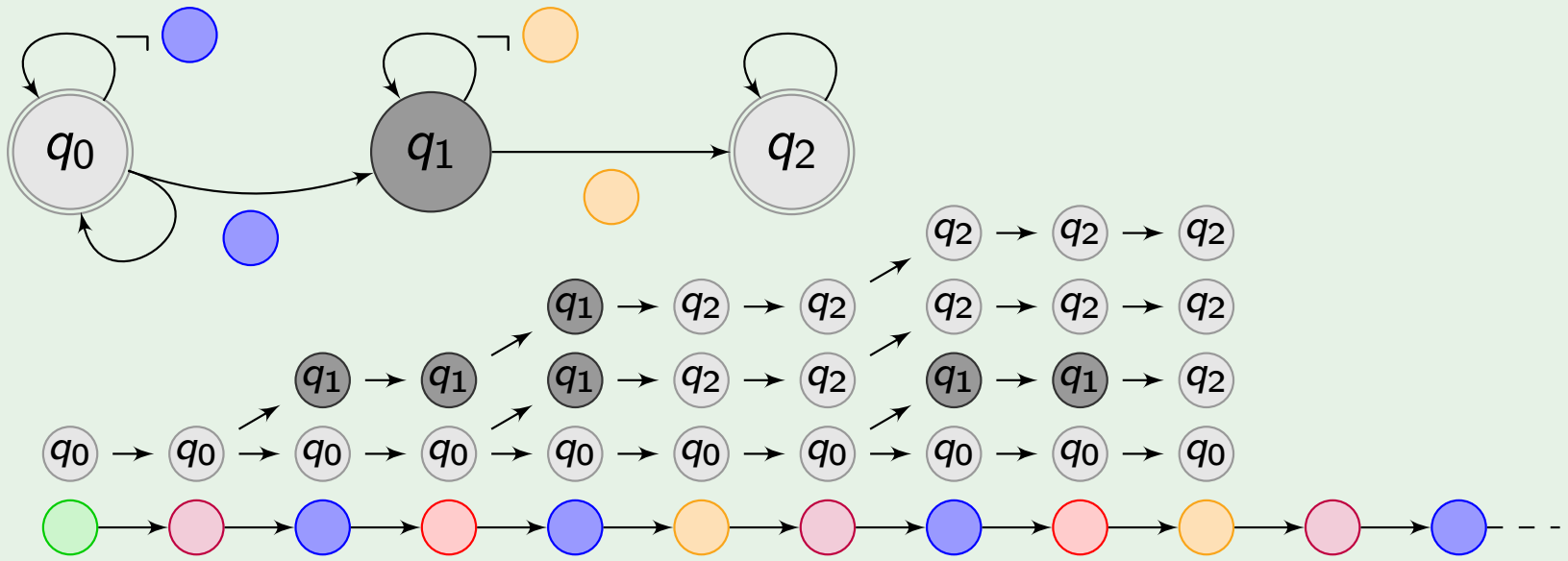

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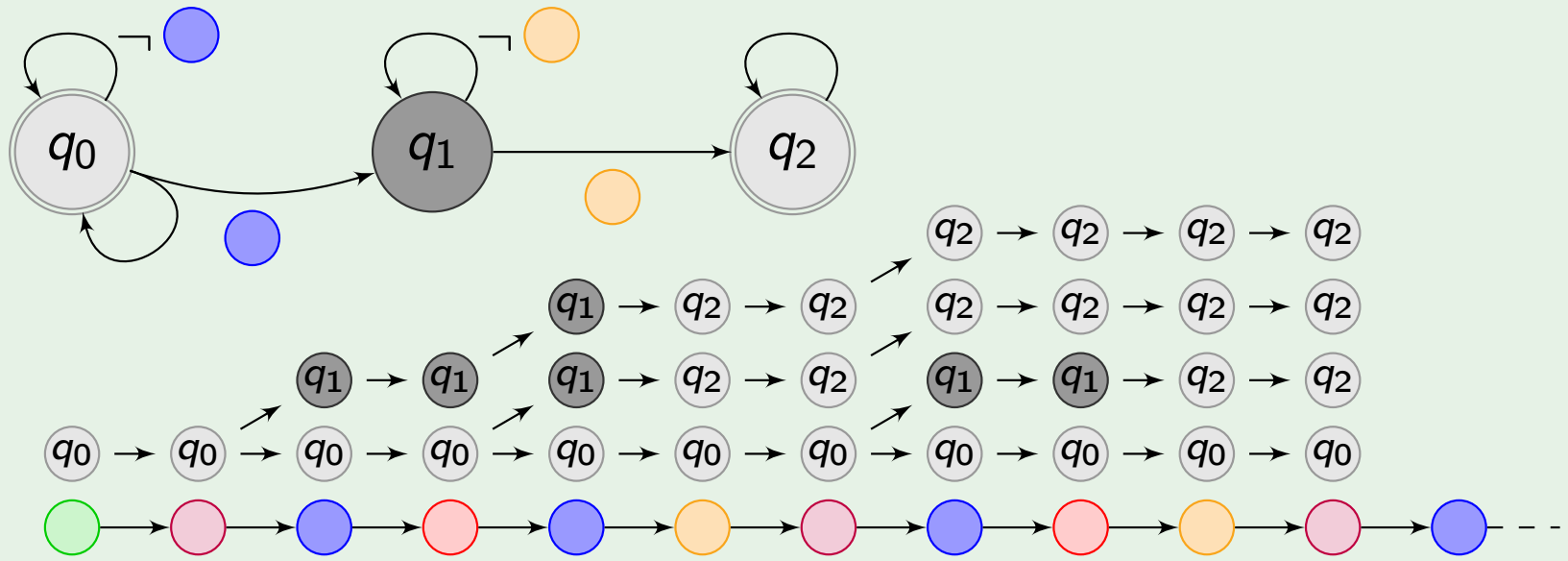

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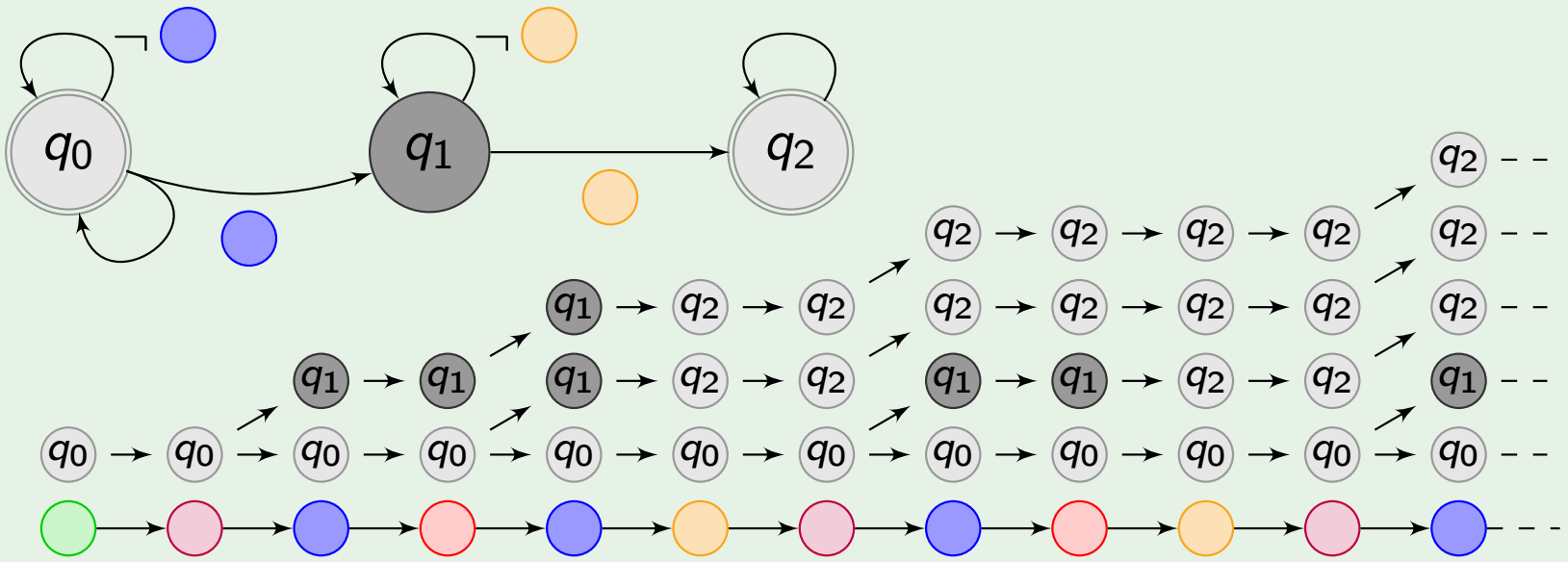

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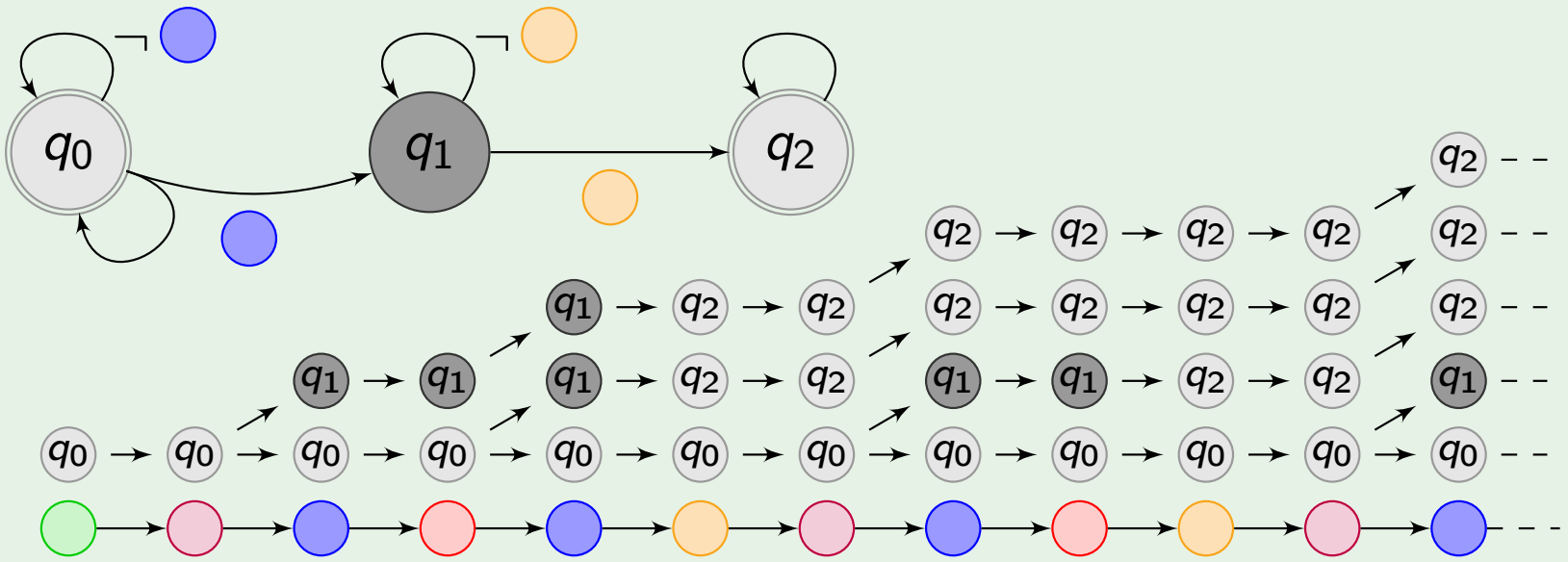

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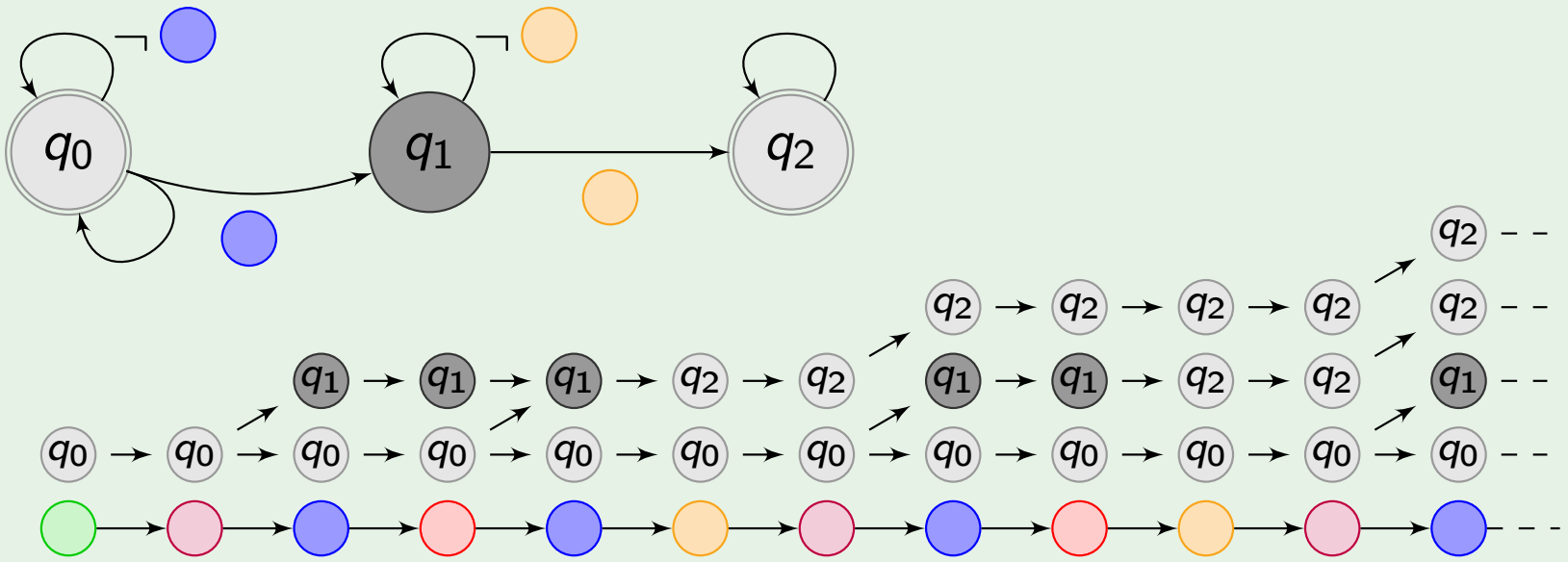

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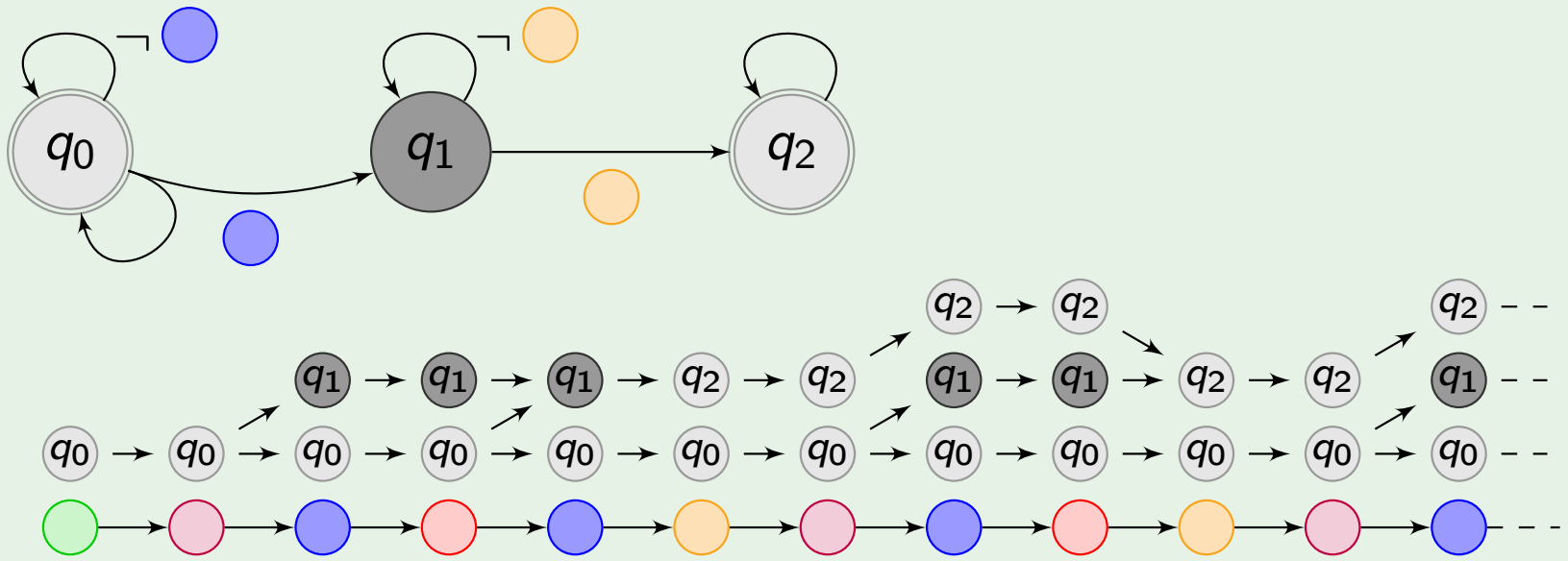

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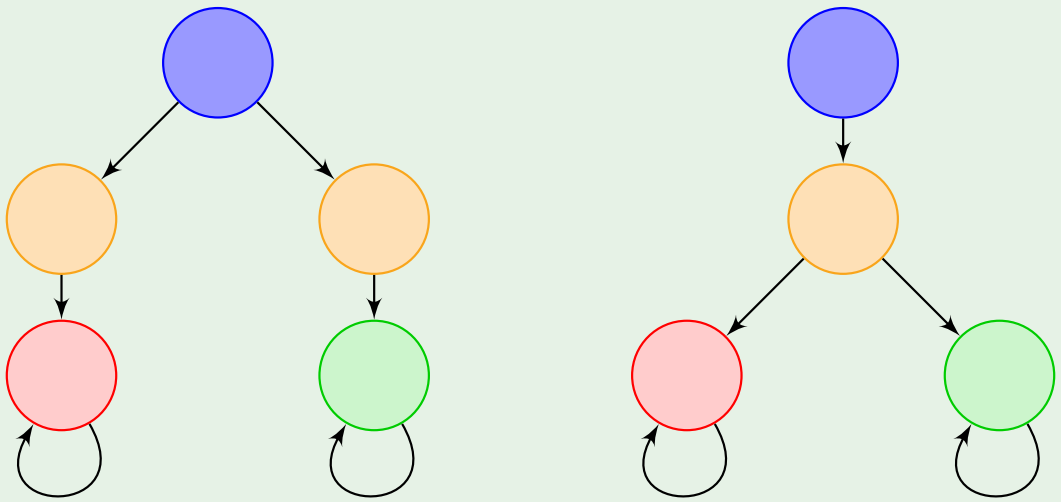

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Lemma

If any finite trace of $\mathcal{M}$ can be generated by $\mathcal{N}$, then

$$\mathit{InfTraces}(\mathcal{M}) \subseteq \mathit{InfTraces}(\mathcal{N})$$

(for finitely branching Kripke structures $\mathcal{M}$ and $\mathcal{N}$).

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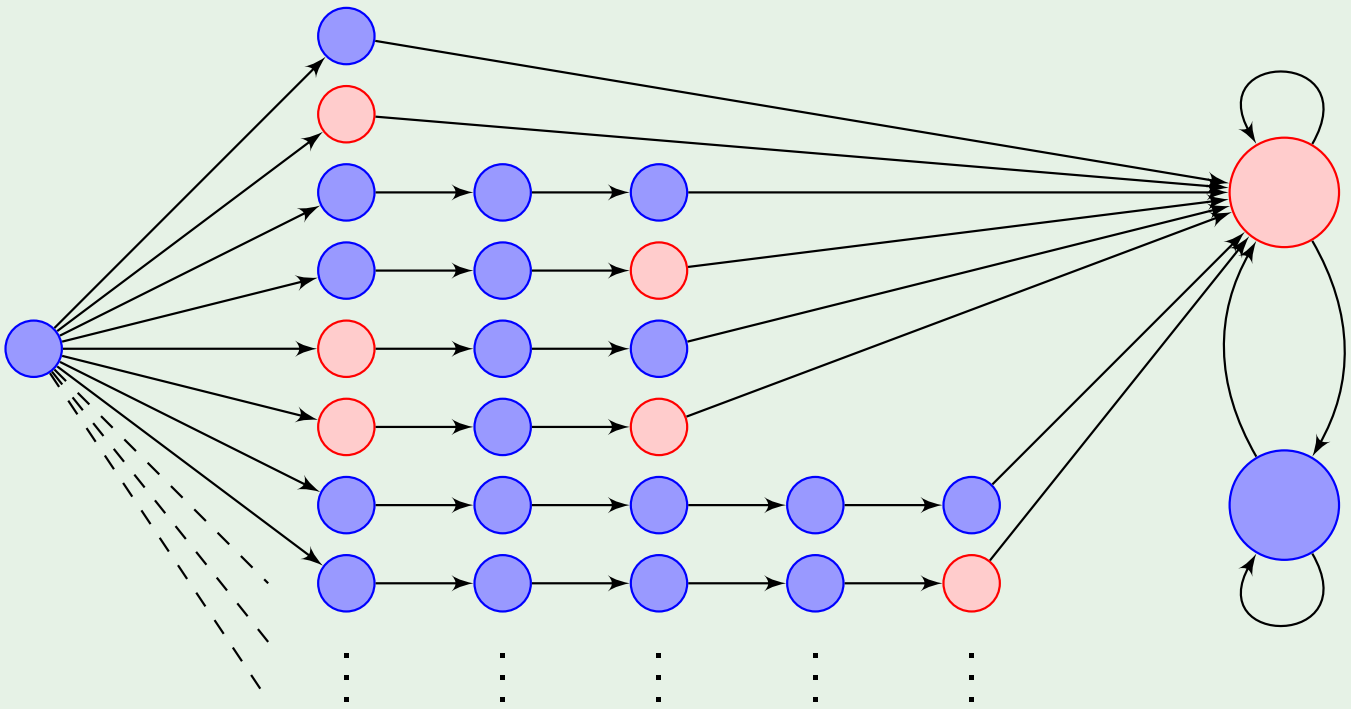
Counter-example



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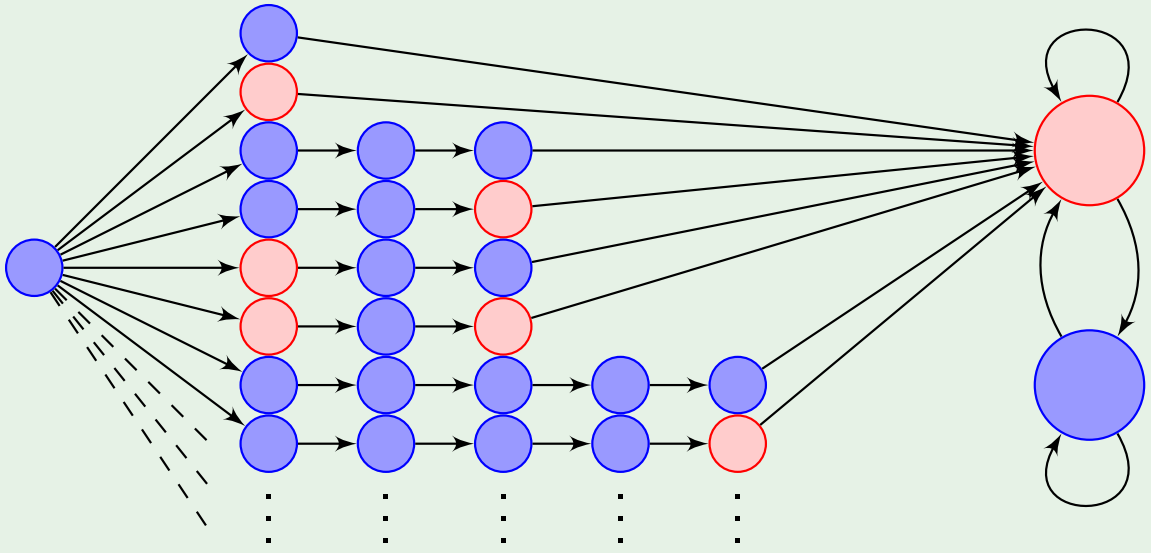

$$w \in \mathsf{Traces}(\mathcal{M}) \iff \exists i > 0.\ w(2i) =$$

Computation-Tree Logic (CTL*)

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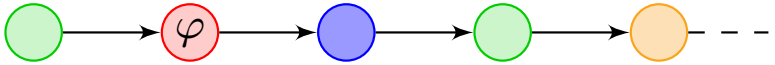
- **atomic propositions:** ●, ●, ...
- **boolean combinators:** $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- **temporal modalities:**

$\mathbf{X}\,\varphi$ 

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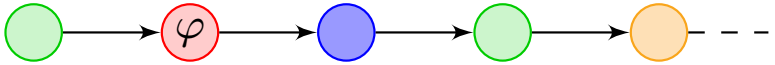
$\mathbf{X}\,\varphi$  "next φ"

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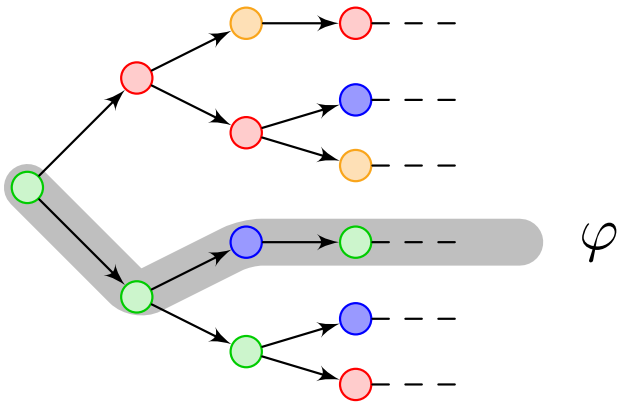
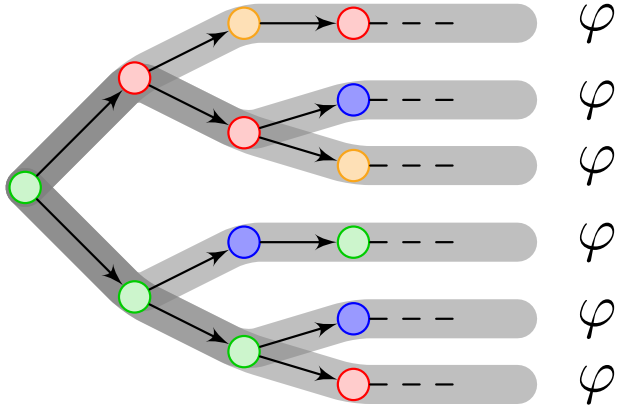


$\mathbf{A}\varphi$



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Expressiveness of CTL

Theorem ([EH86])

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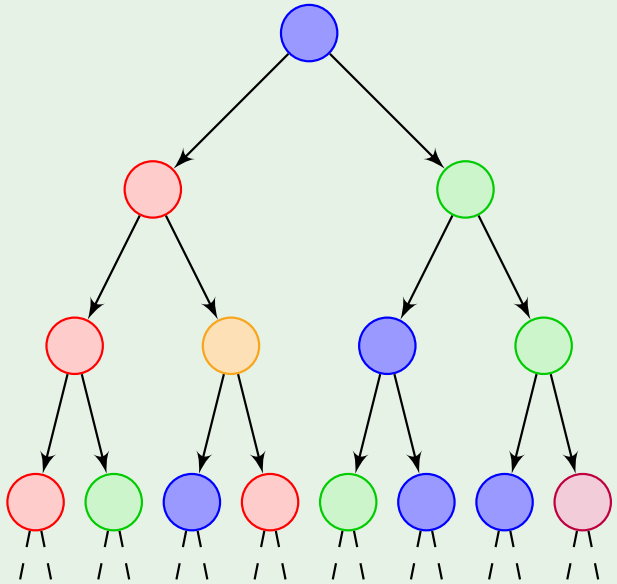

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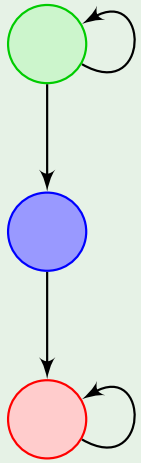

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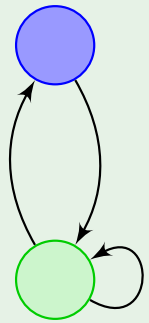

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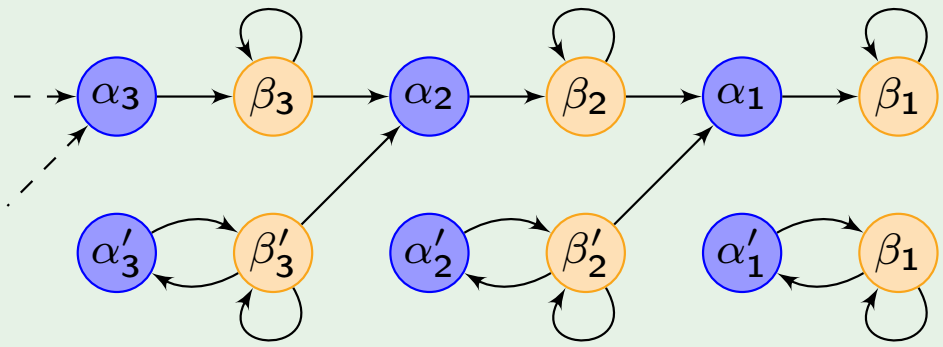

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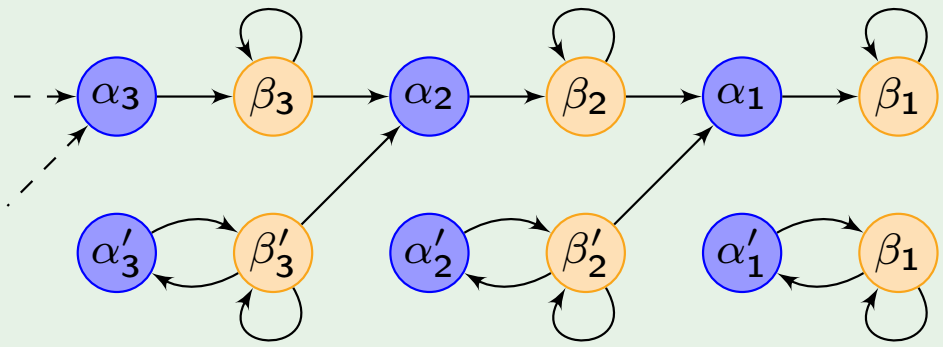

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Lemma

α_i and α'_i satisfy the same CTL formulas of length at most i.

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Modalities $\mathbf{E}\,\mathbf{X}$, $\mathbf{E}\;\mathbf{U}$ *and* $\mathbf{E}\,\mathbf{G}$ *are sufficient to express any CTL formula.*

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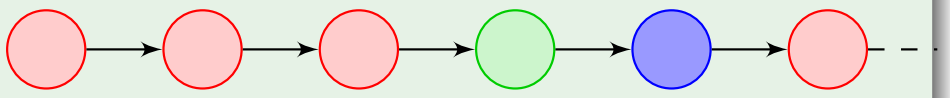

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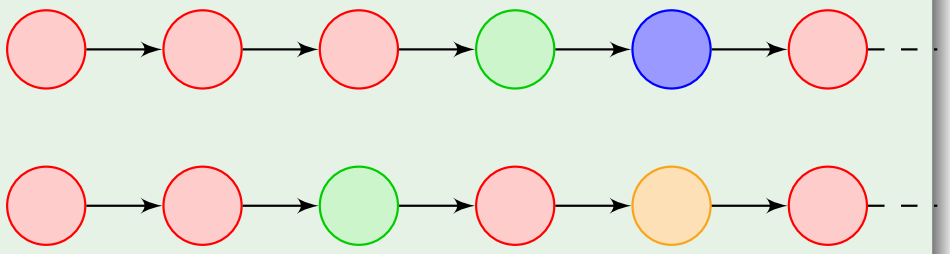

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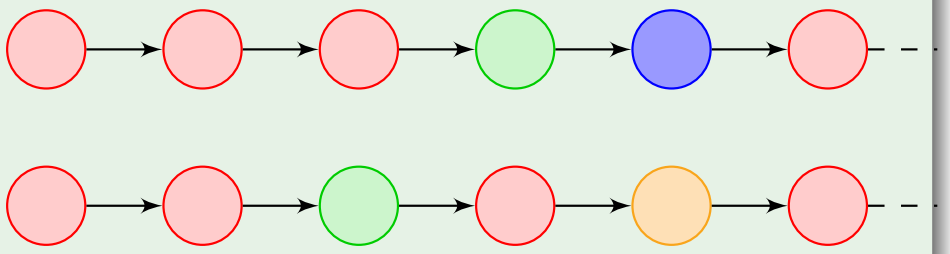

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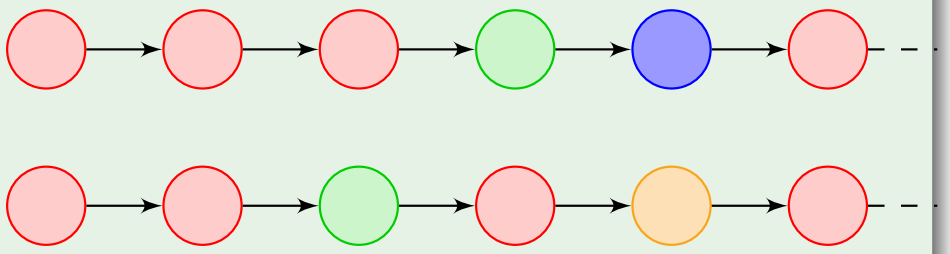

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Theorem ([CE81,QS82])

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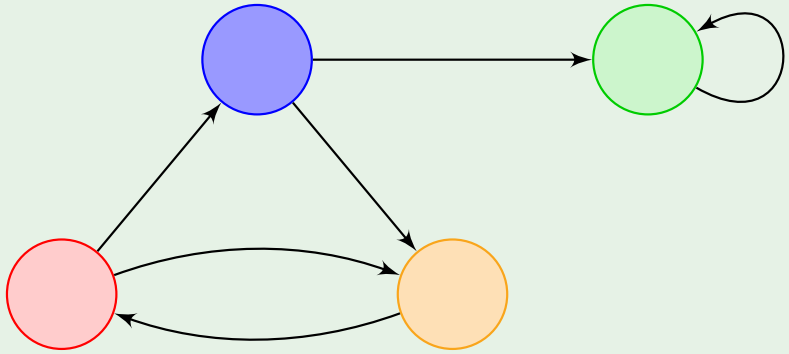

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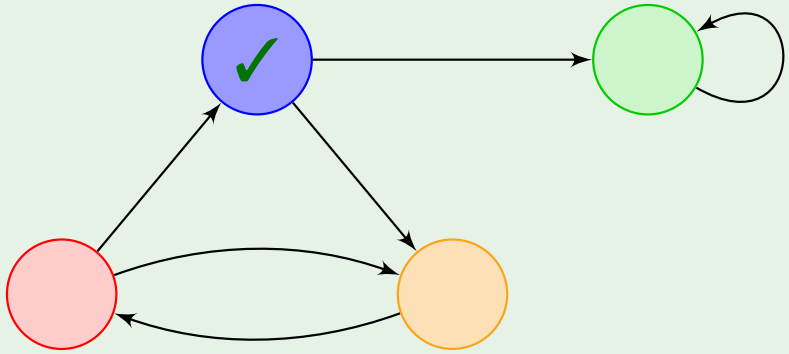

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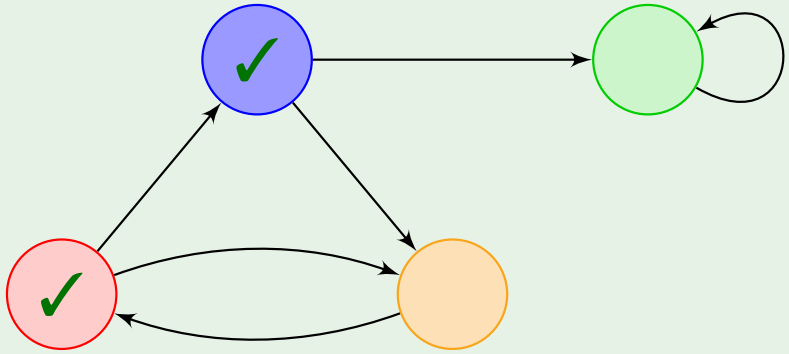

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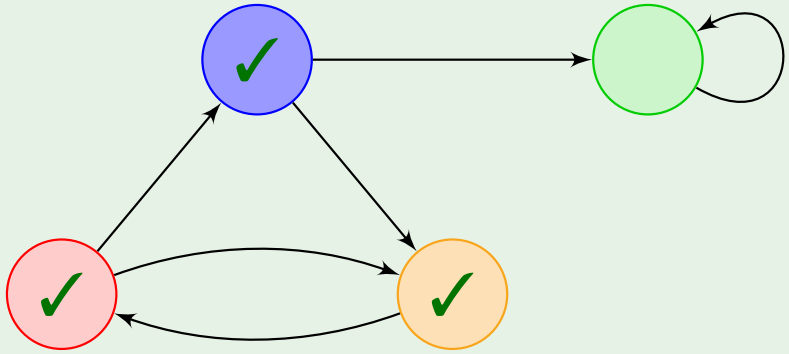

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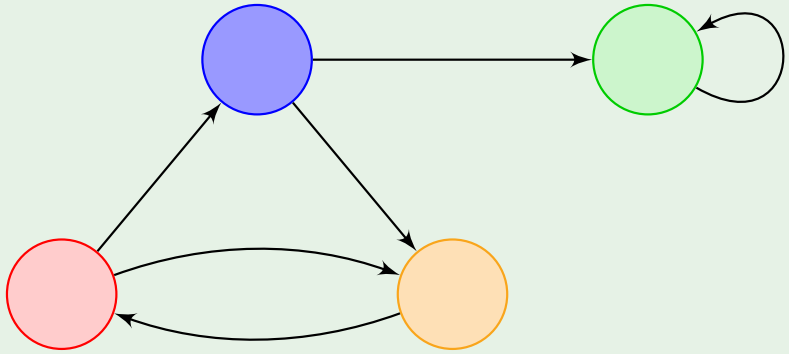

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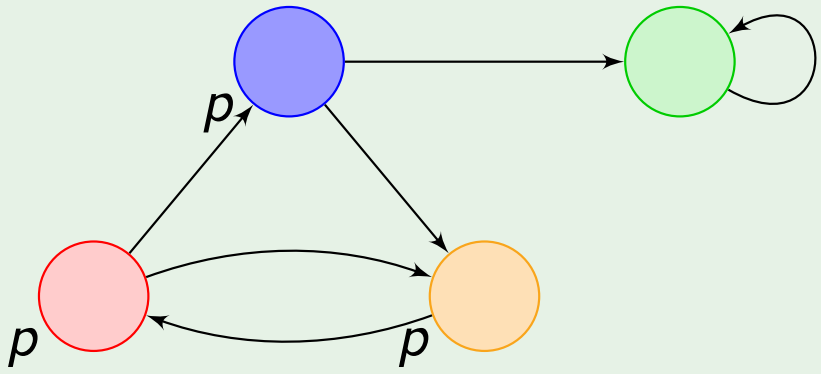

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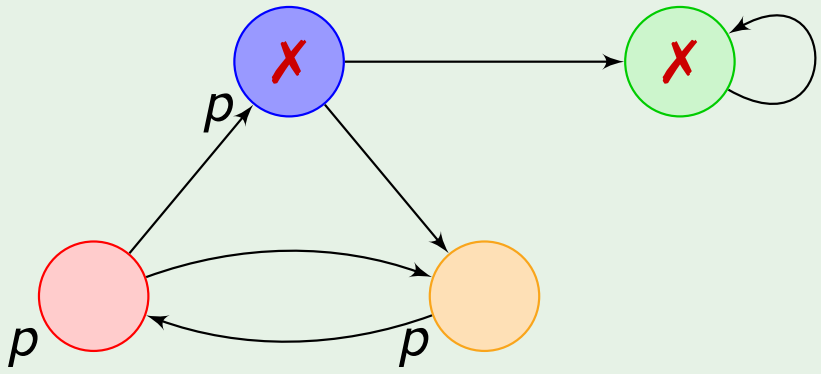

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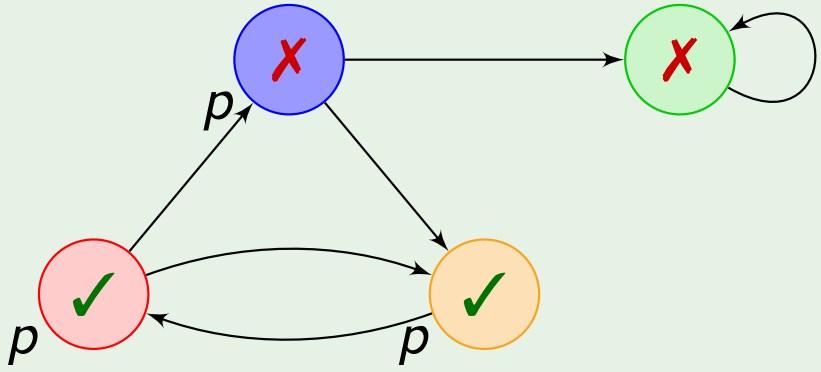

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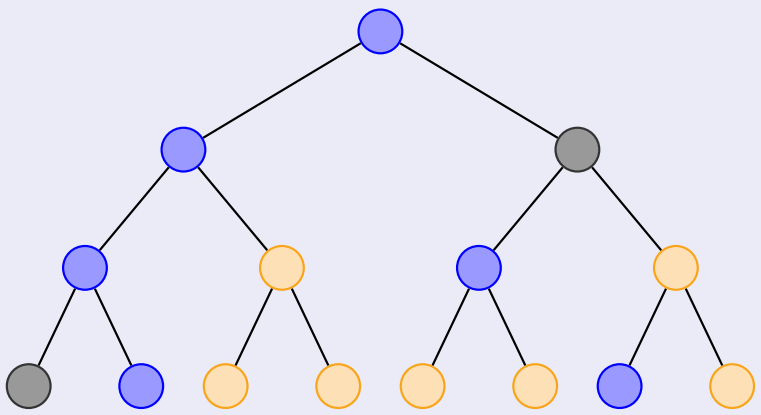

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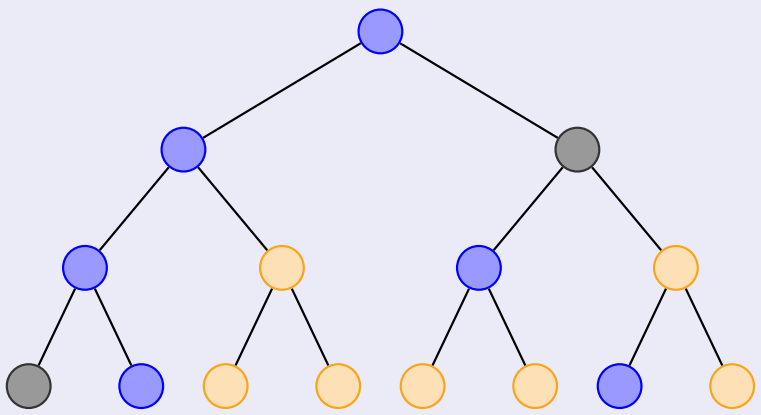

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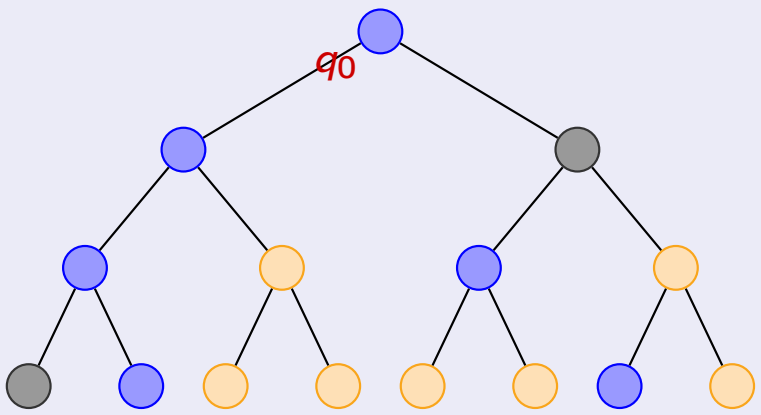

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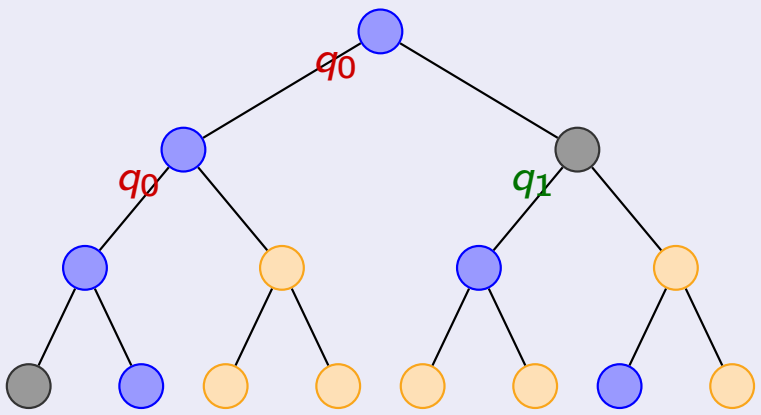

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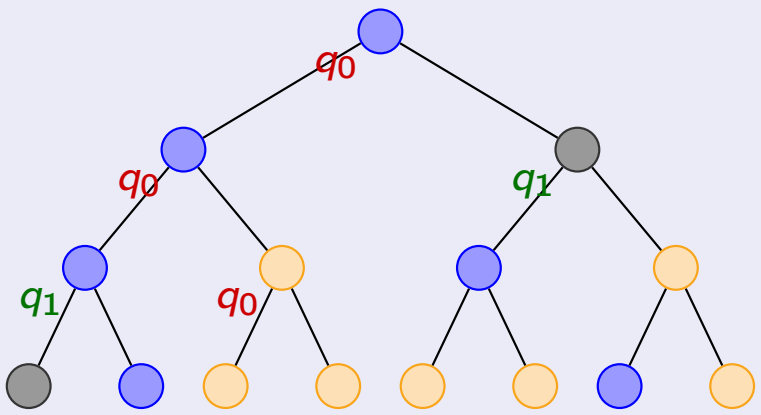

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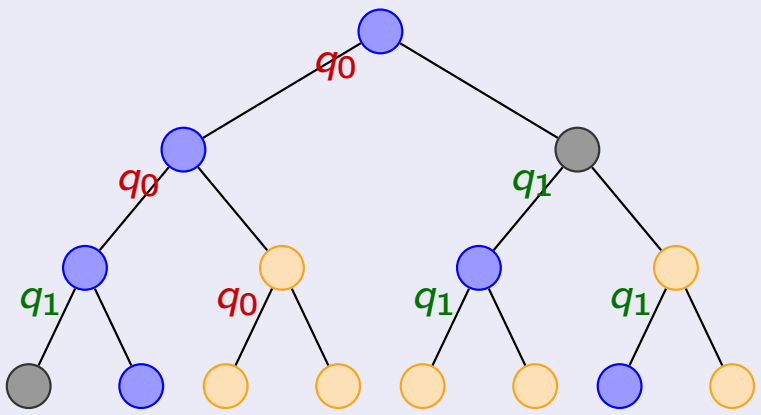

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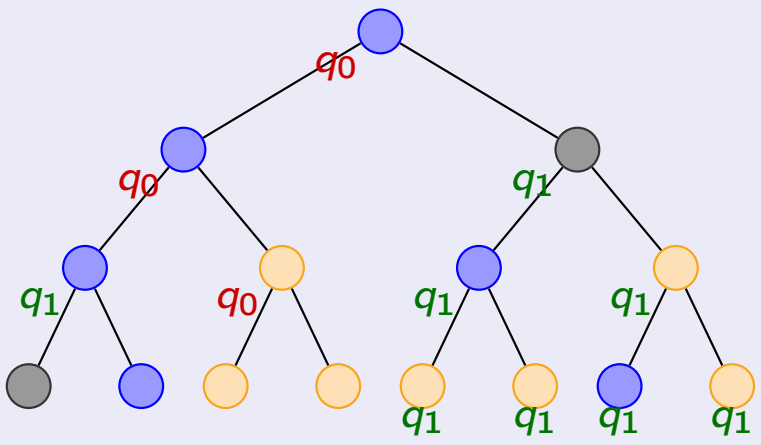

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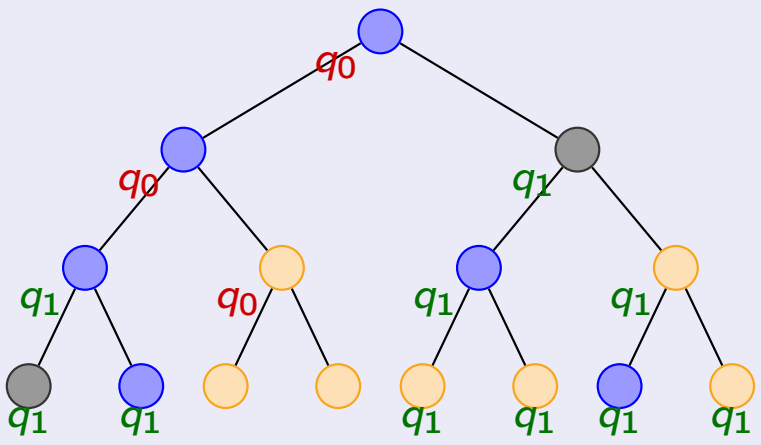

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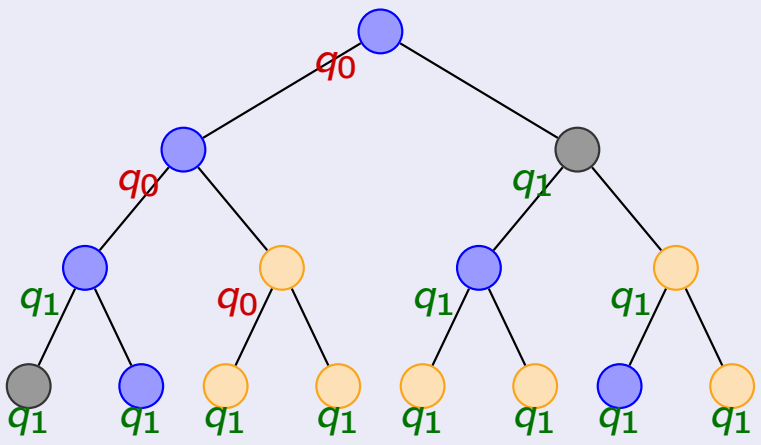

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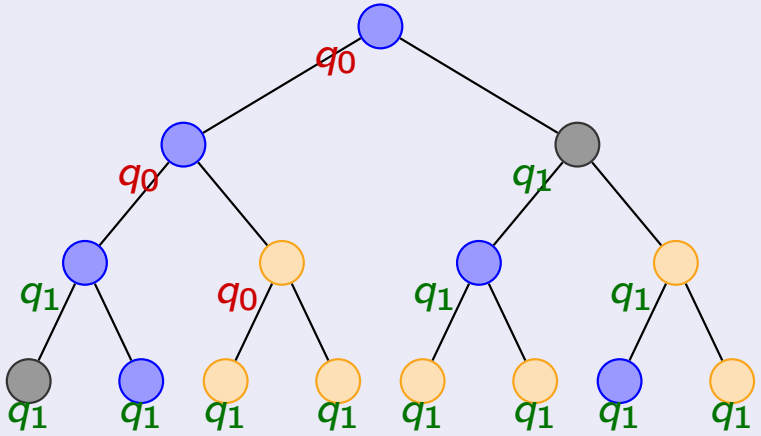

This automaton corresponds to **E** ● **U** ●

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Verifying CTL properties – automata-theoretic approach

Alternating tree automata

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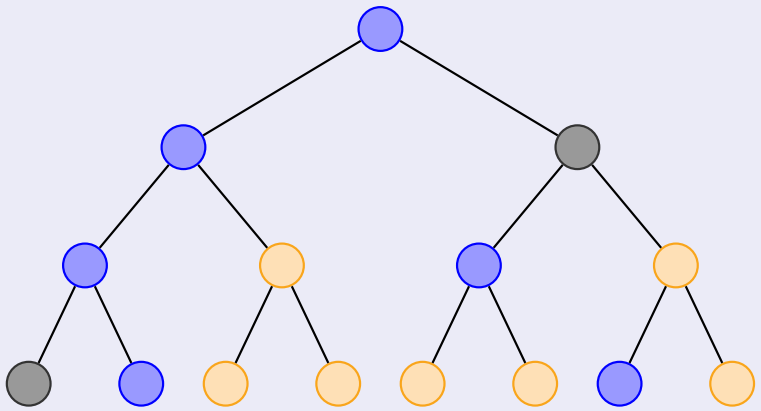

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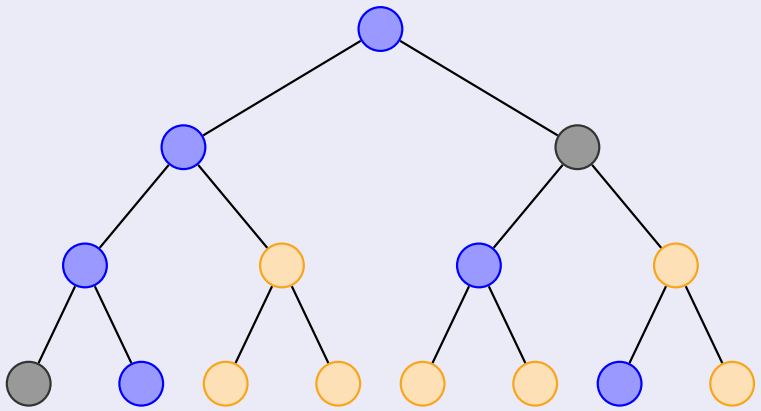

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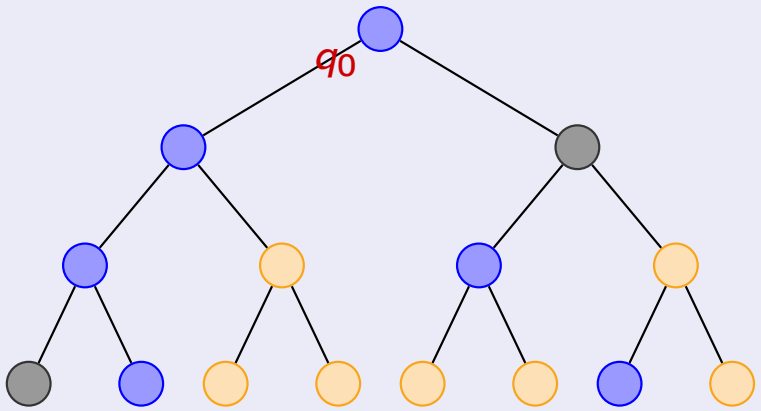

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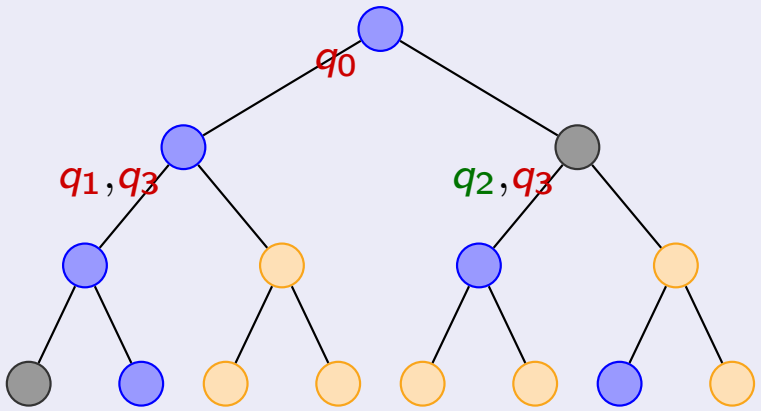

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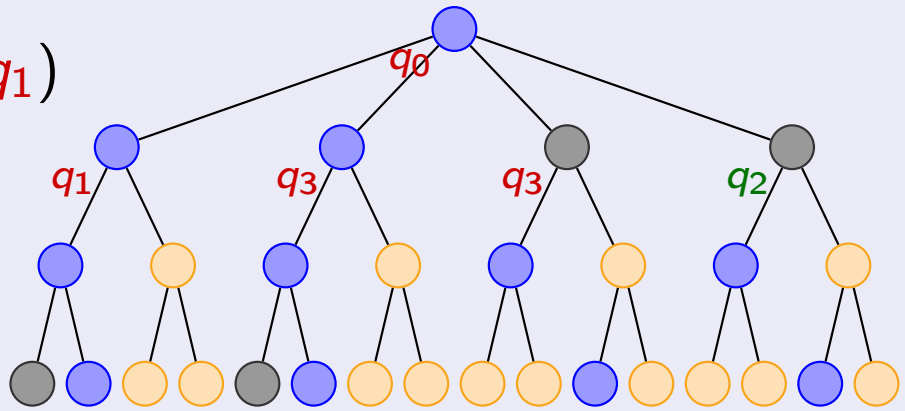

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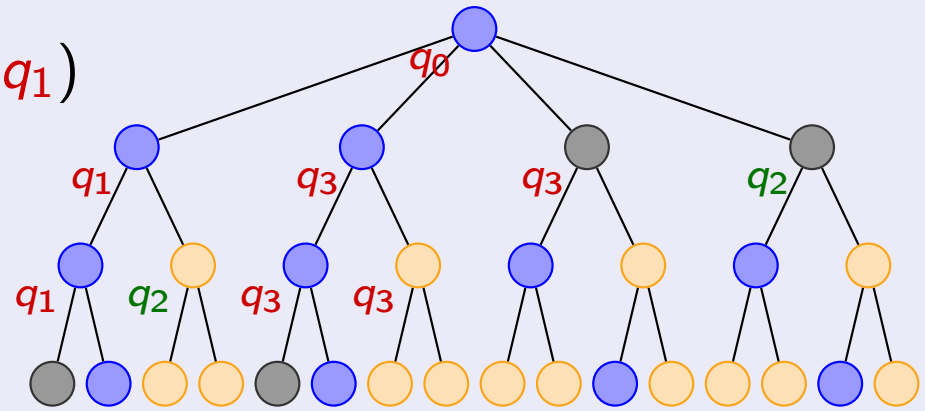

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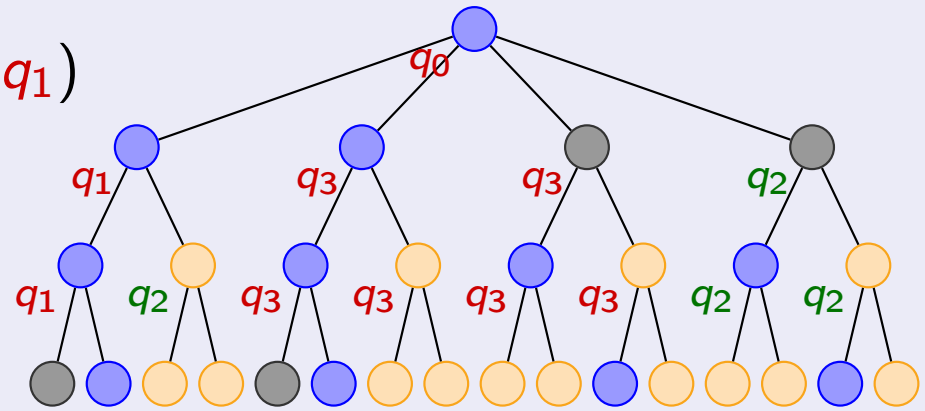

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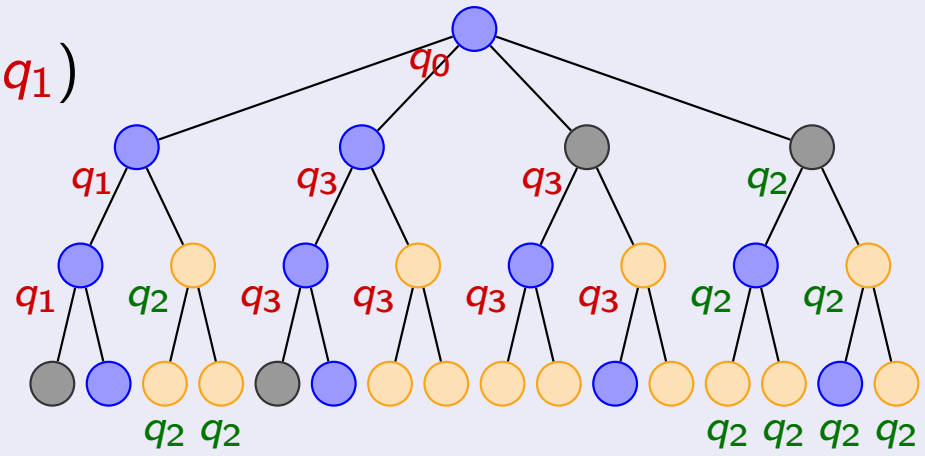

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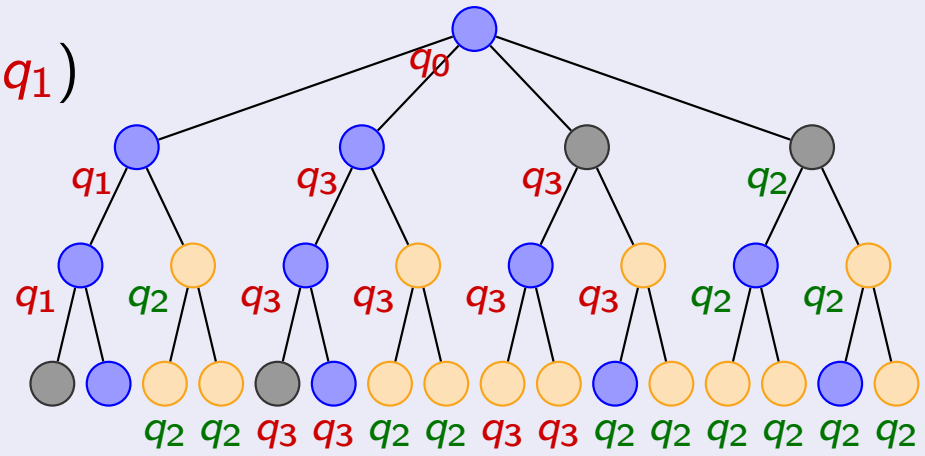

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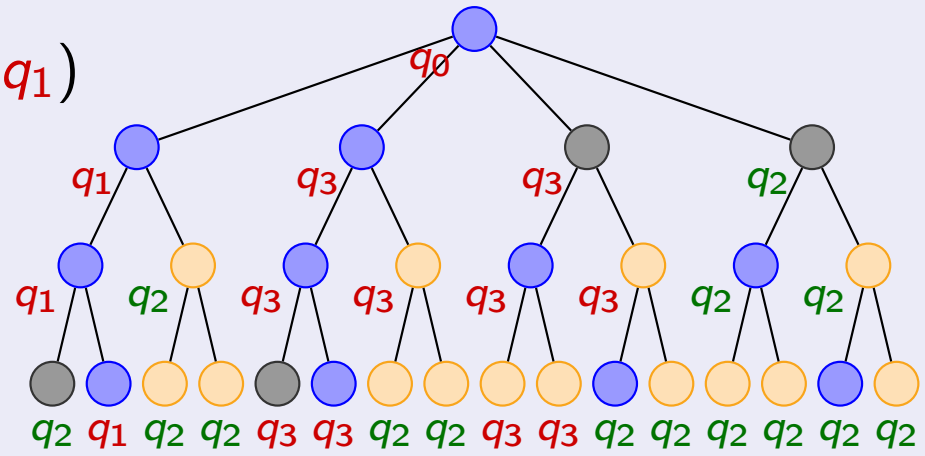

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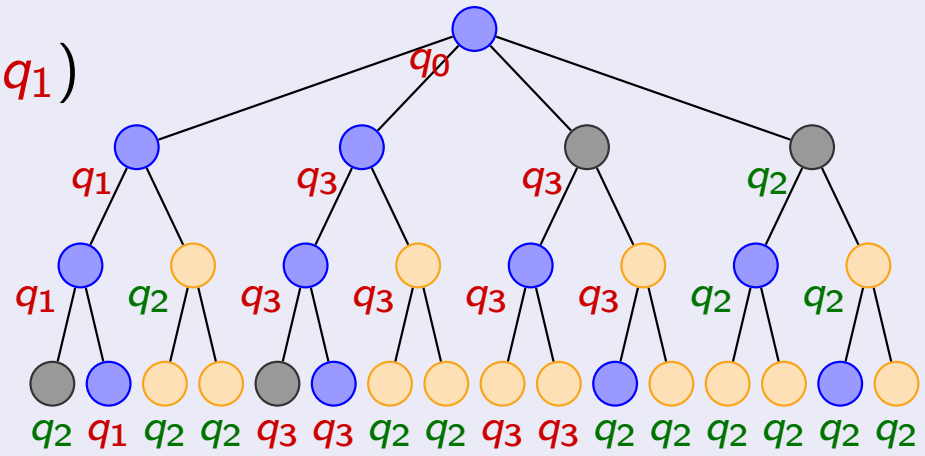

This automaton corresponds to $\mathbf{E}\,\mathbf{G}\,\bullet \wedge \mathbf{A}\,\mathbf{F}\,\bullet$

Detailled explanations during François' lectures.

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Behavioural equivalence for CTL

When are two Kripke structures indistinguishable by CTL?

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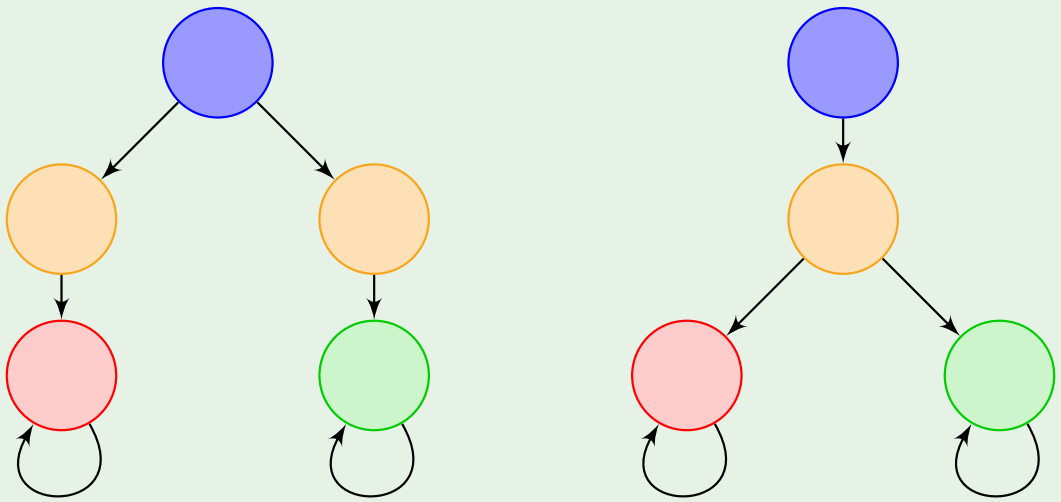
Example



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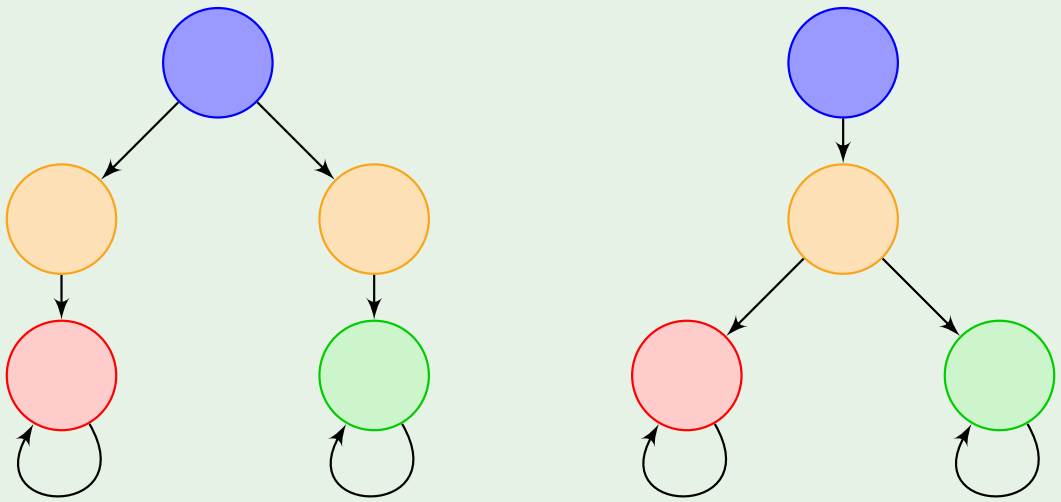

$\mathbf{EX}(\mathbf{EX}\,\bigcirc \wedge \mathbf{EX}\,\bigcirc)$

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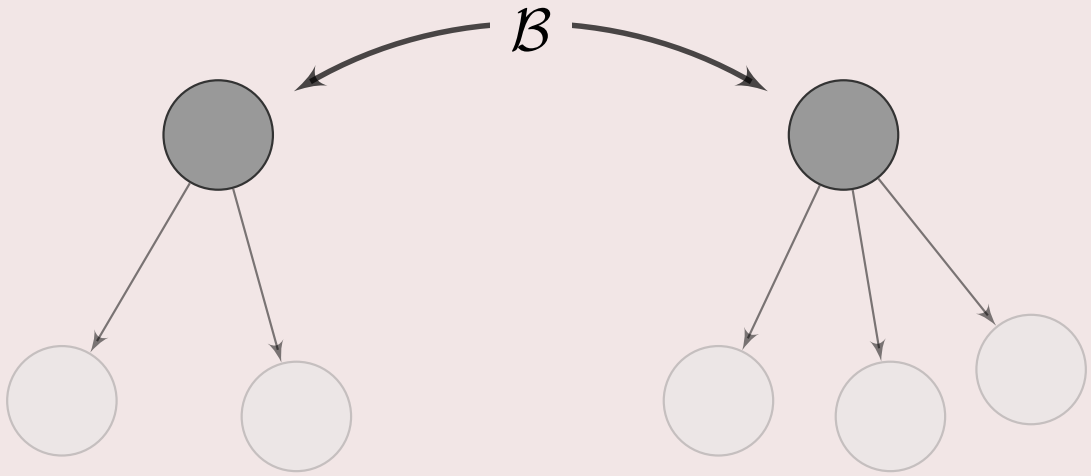

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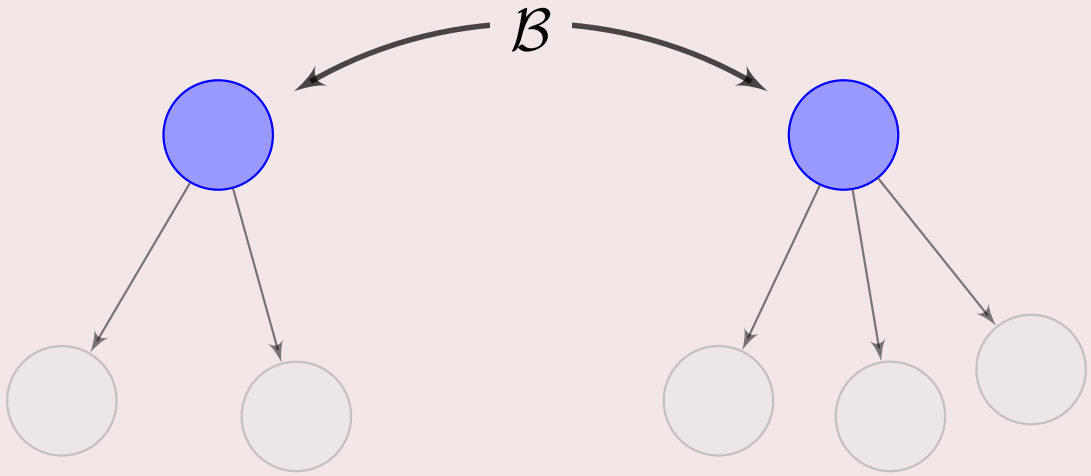

- same label;

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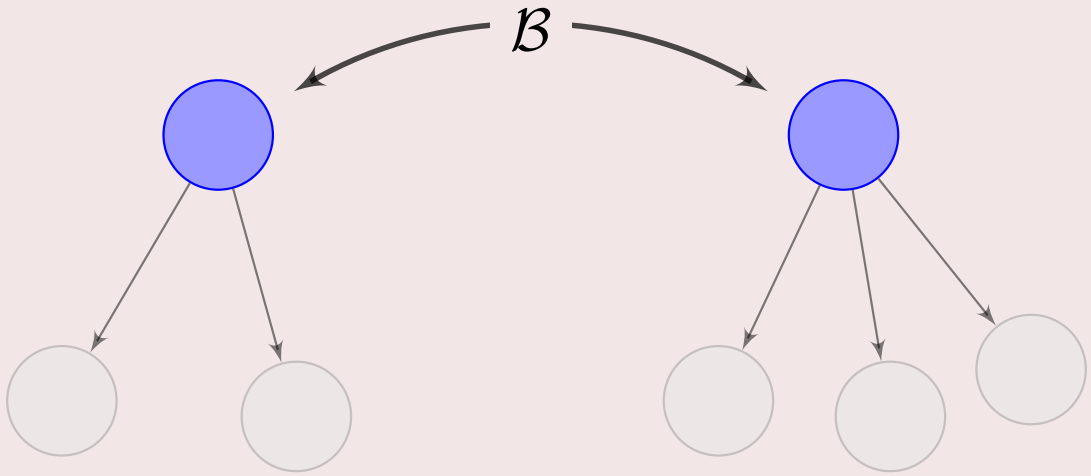

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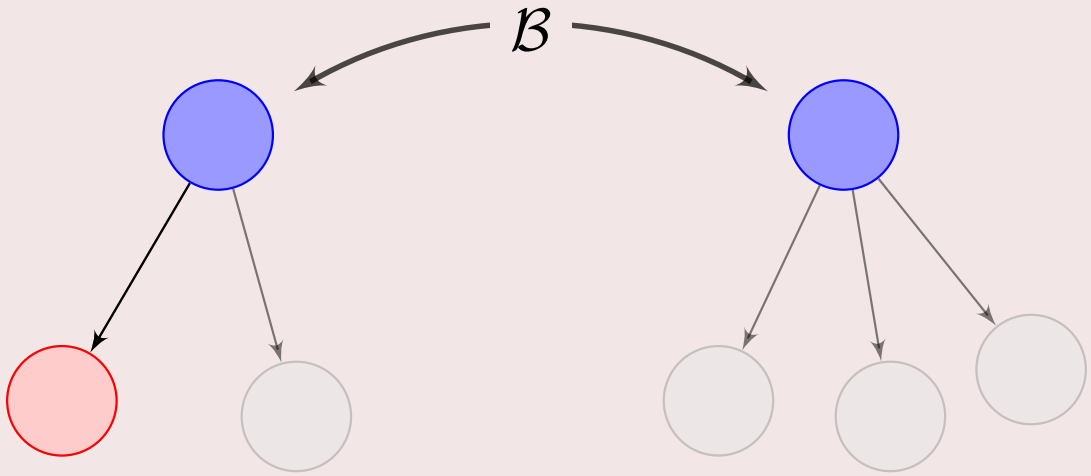

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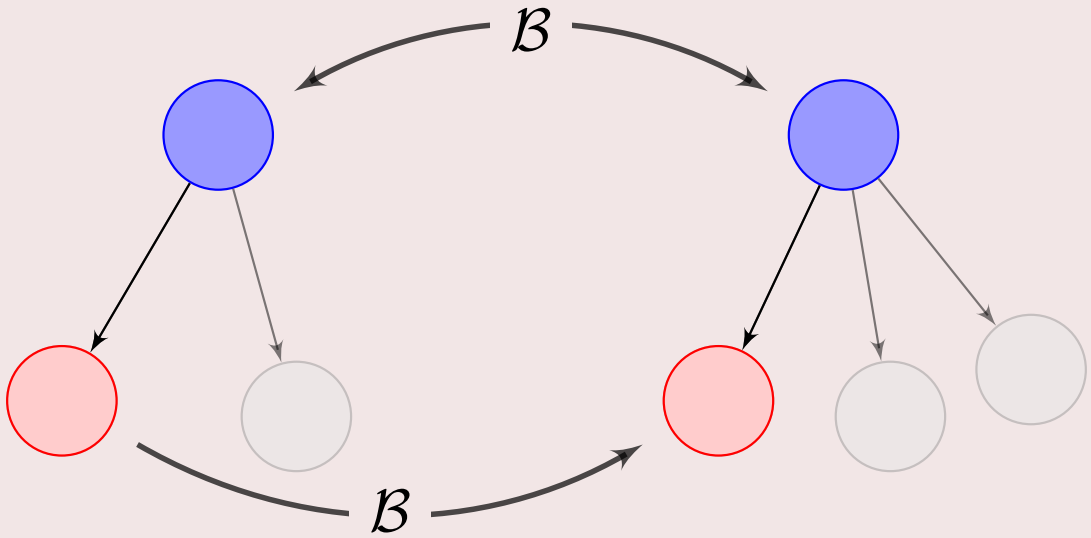

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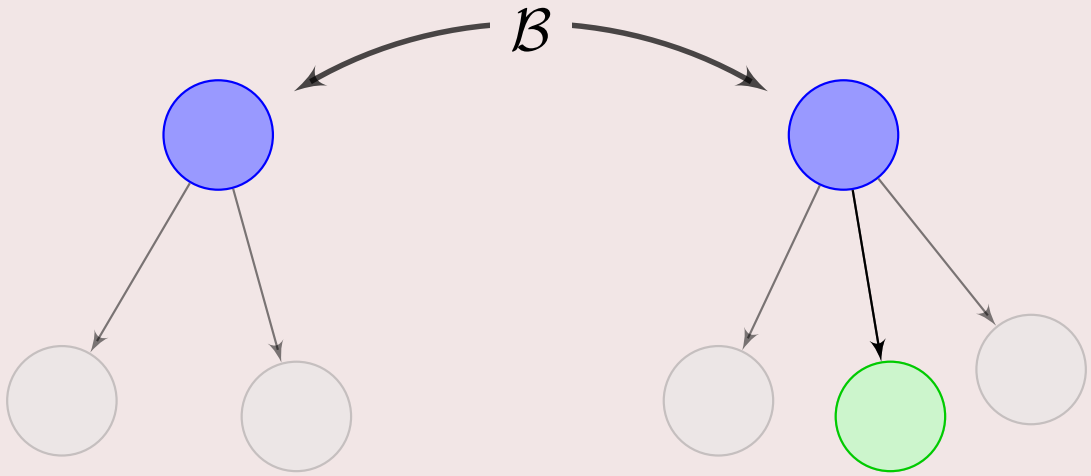

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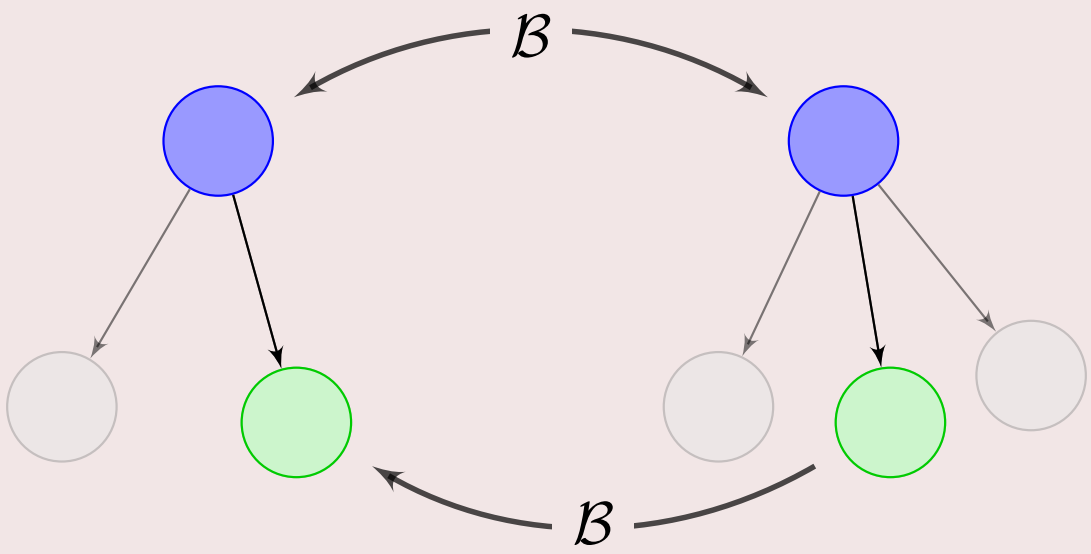

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Theorem ([BCG88])

(The initial states of) two finitely branching Kripke structures satisfy the same CTL formulas if, and only if, they are bisimilar.

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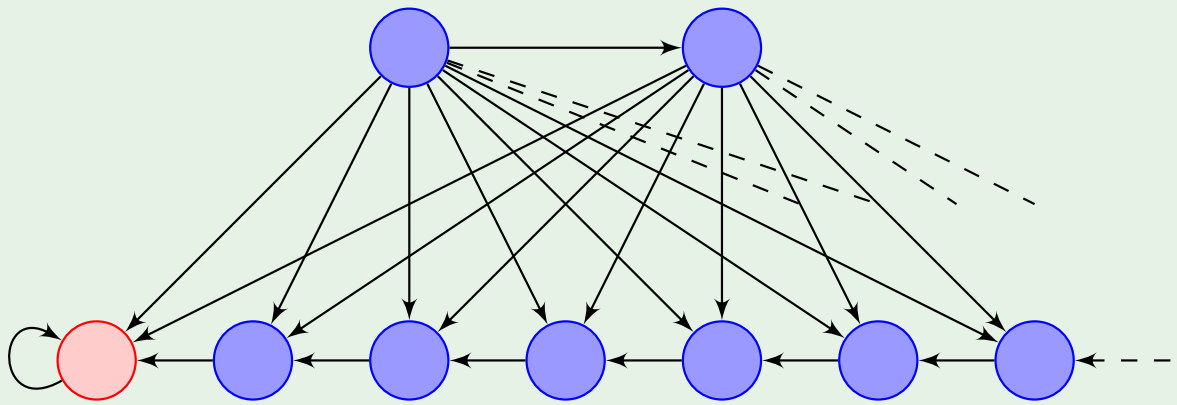
Counter-example



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Computation-Tree Logic (CTL*)

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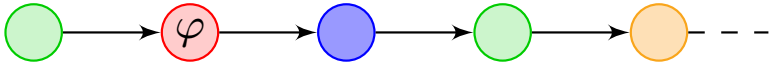
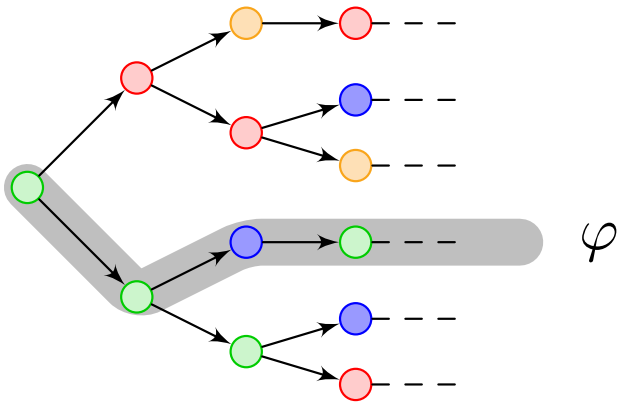
- **atomic propositions:** ◯, ◯, ...
- **boolean combinators:** $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- **temporal modalities:**

$\mathbf{X}\,\varphi$  "next φ"

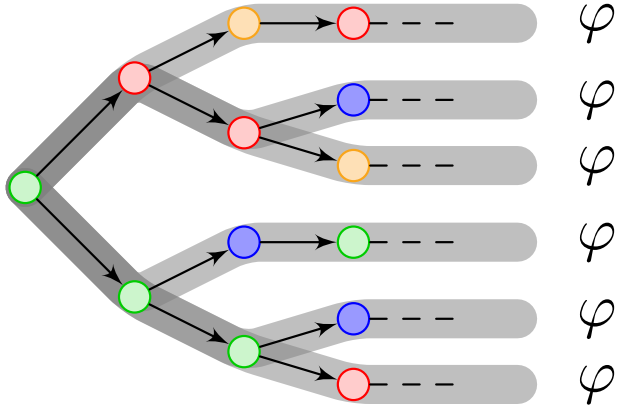
$\varphi\ \mathbf{U}\ \psi$  "φ until ψ"

- **path quantifiers:**

$\mathbf{E}\varphi$



$\mathbf{A}\varphi$



Computation-Tree Logic (CTL*)

$\mathbf{A}\,\mathbf{G}(\mathsf{req}_2 \Rightarrow \mathbf{A}(\mathbf{G}\,\mathbf{F}\,\mathsf{serv}_i \Rightarrow \mathbf{F}\,\mathsf{serv}_2))$ *any request is eventually served along any fair execution*

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Expressiveness

- CTL* subsumes LTL and CTL;
- two states are bisimilar iff they satisfy the same CTL* formulas.

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Theorem ([ES84])

CTL model checking is* PSPACE*-complete.*

[ES84] Emerson, Sistla. Deciding full branching-time logic. STOC, 1984.

Outline of the presentation

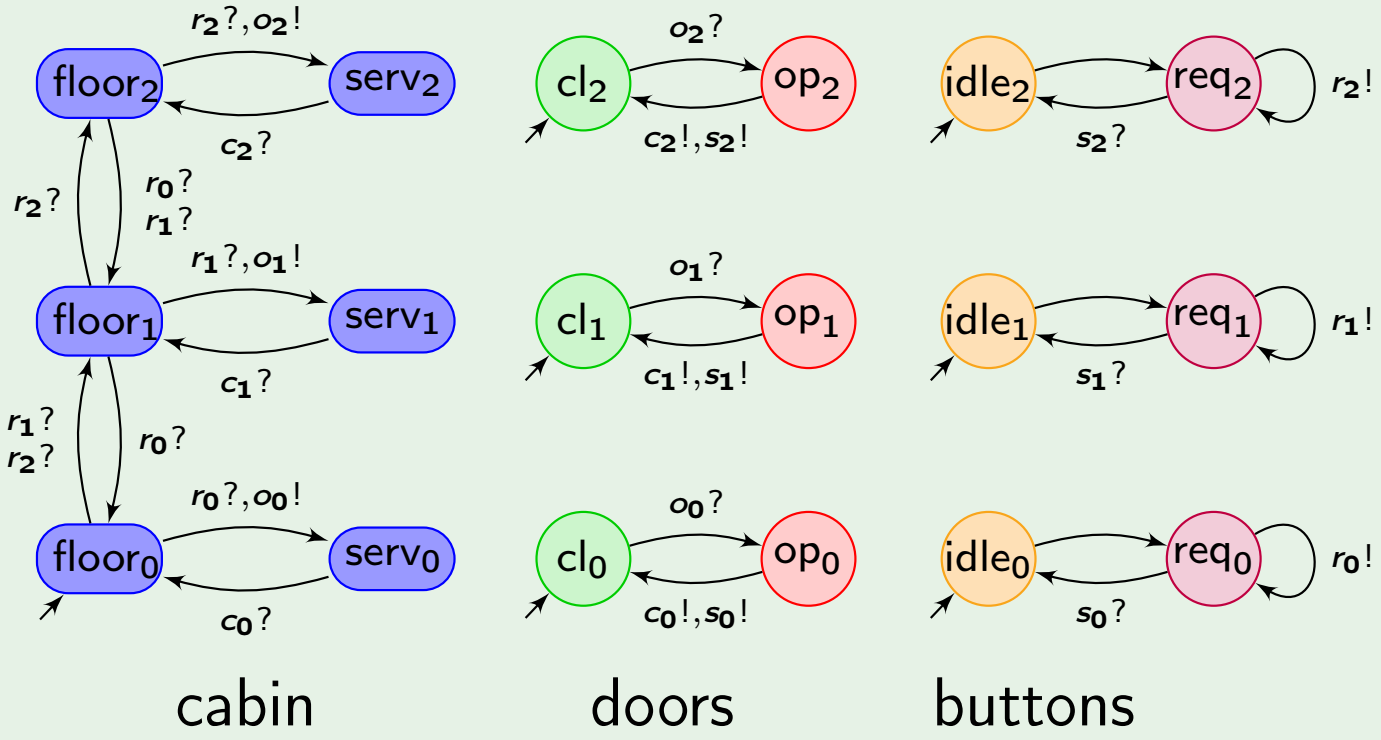
1. Introduction

2. Basics of temporal logics
   - convenient formalism for expressing properties of reactive systems
   - efficient verification algorithms for finite-state models

3. Temporal logics for multi-agent systems
   - reasoning about interacting components in complex systems
   - temporal logics for expressing controllability properties

Reasoning about open systems

Example (An elevator)



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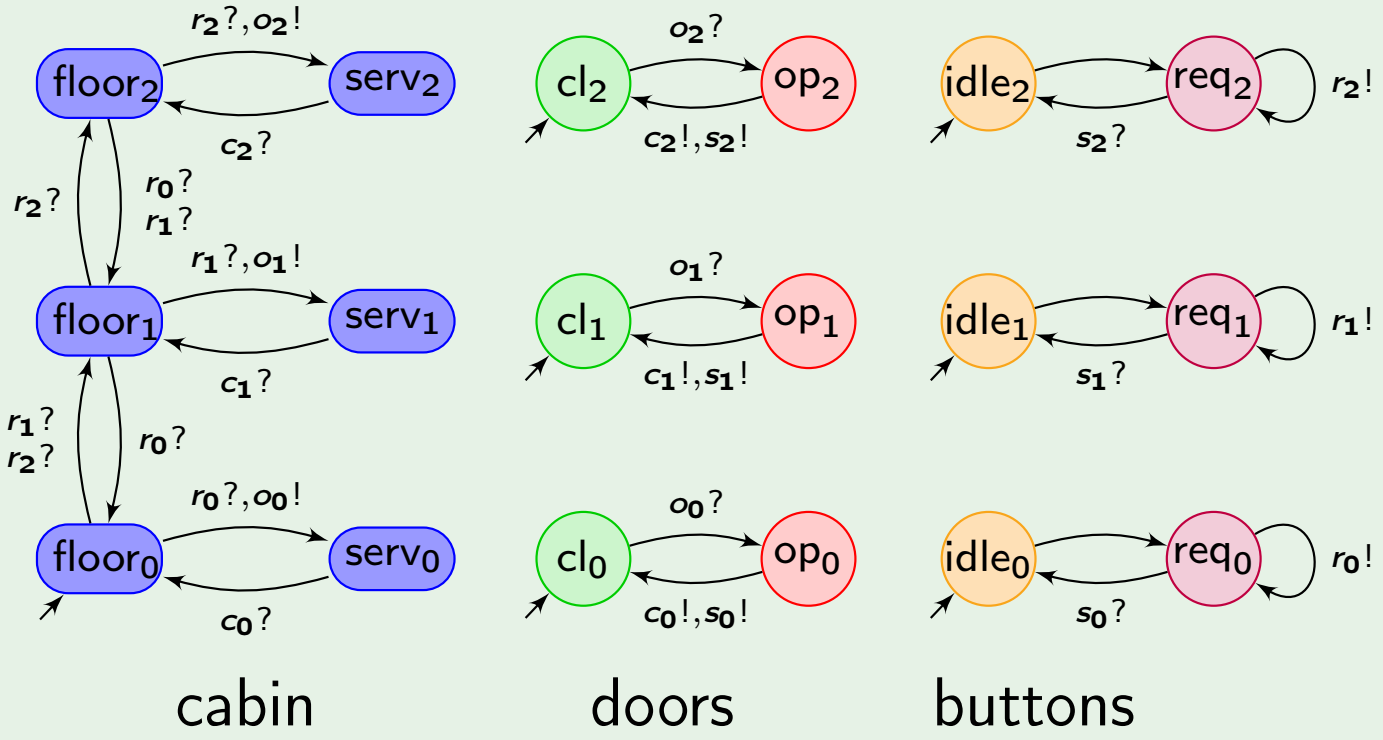

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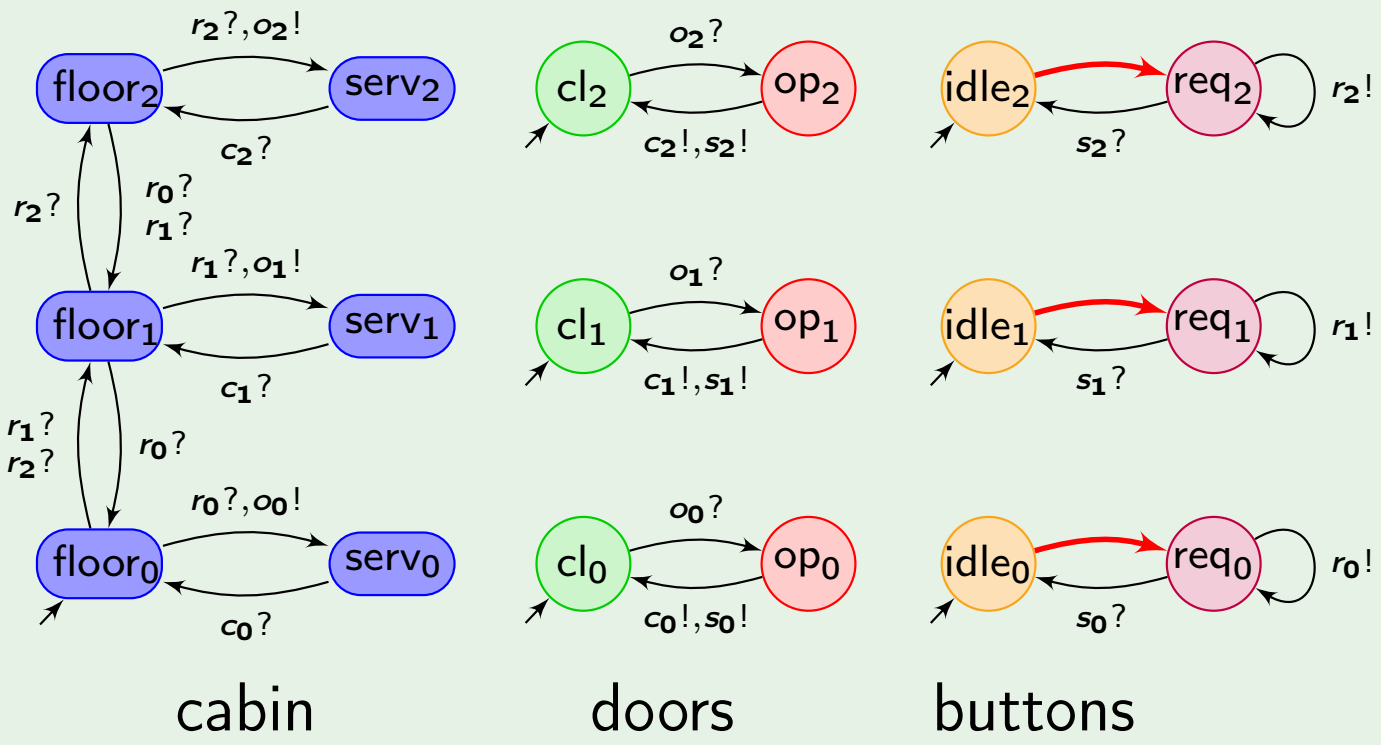

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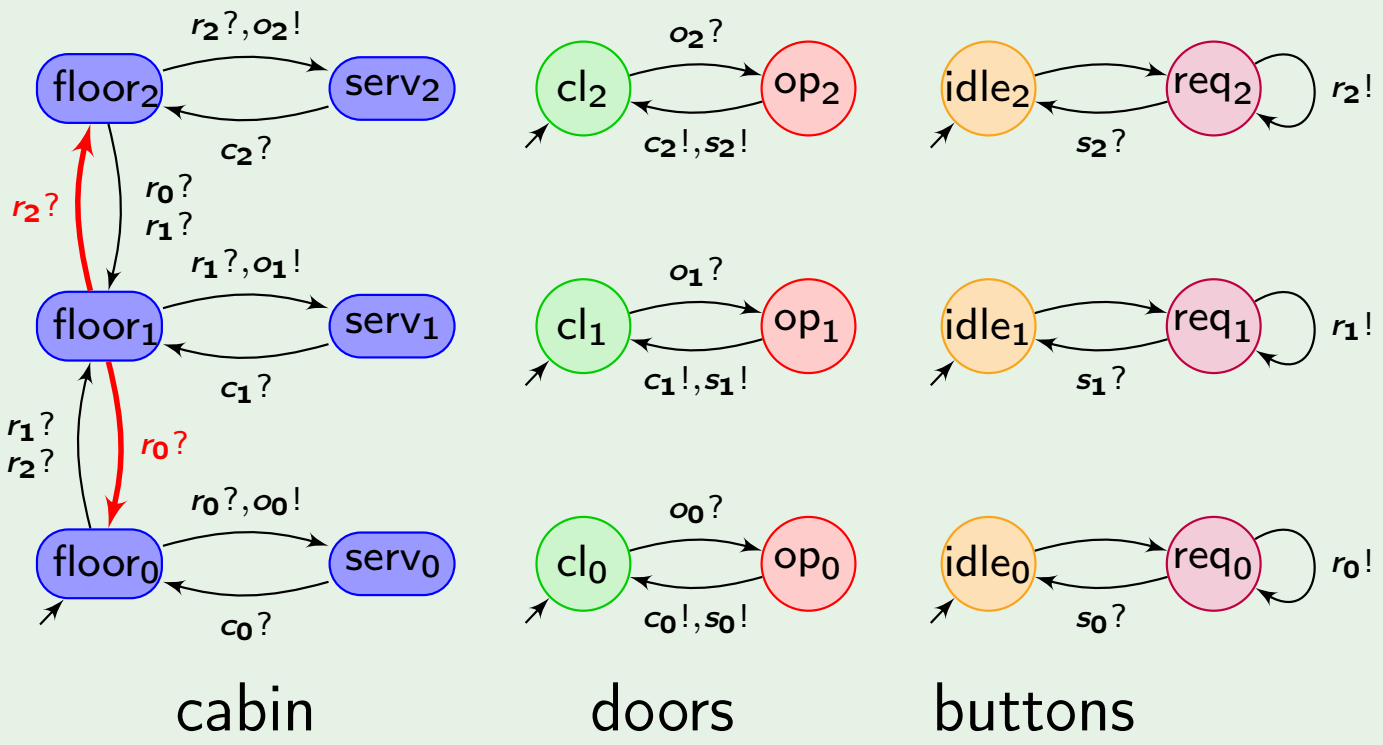

Non-determinism due to

- interactions with users;
- underspecified controller for the lift.

Reasoning about open systems

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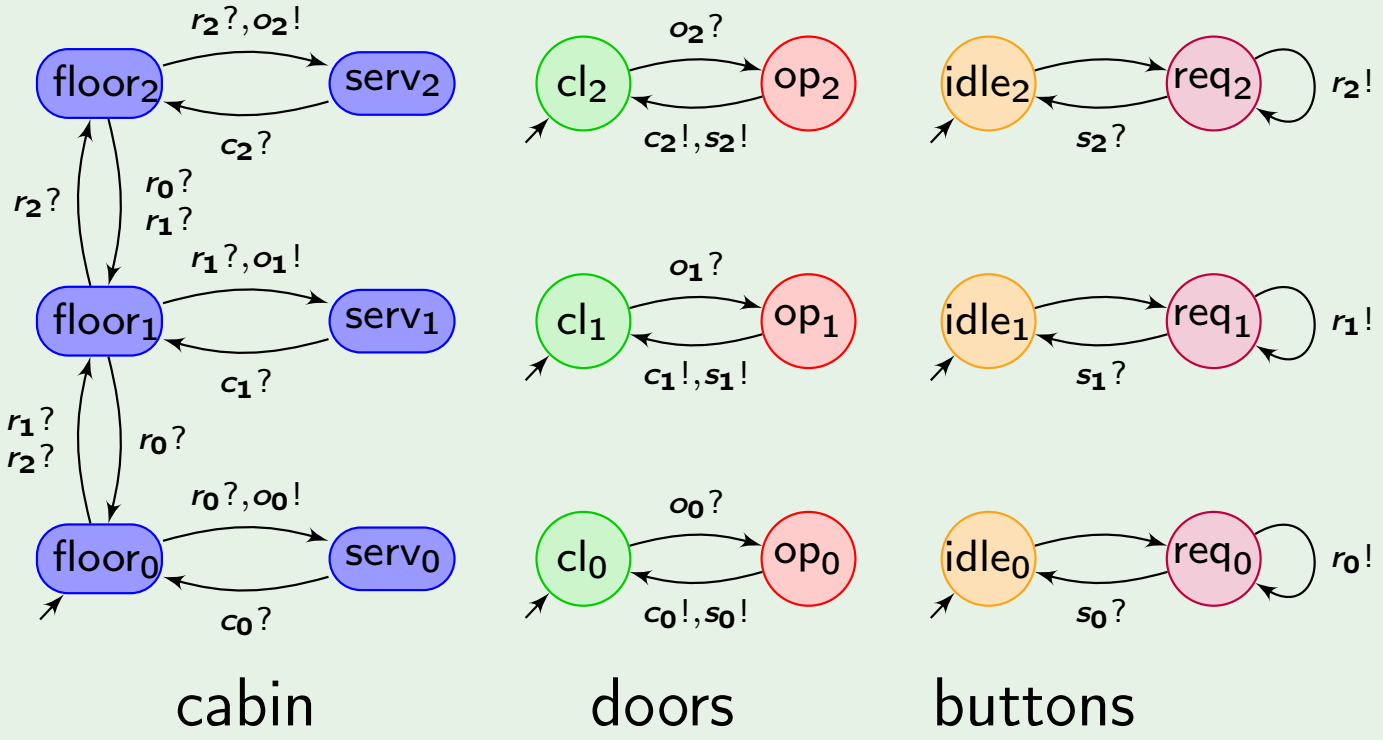

Controllability properties:

does there exist a controller under which the system satisfies some property, whatever the users do?

Reasoning about open systems

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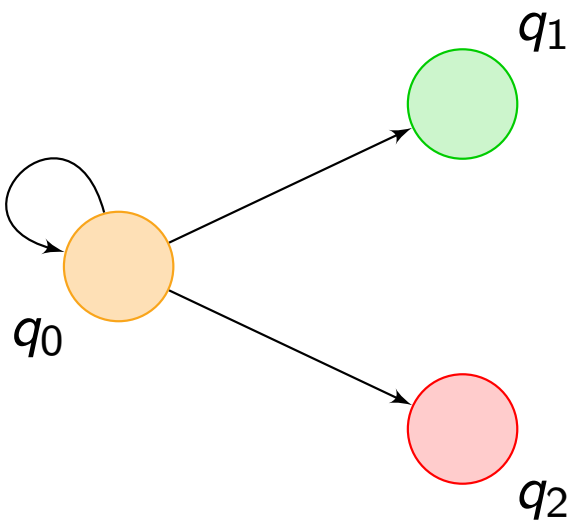
Games on graphs

A concurrent game structure is made of

- a Kripke structure



Reasoning about open systems

Games on graphs

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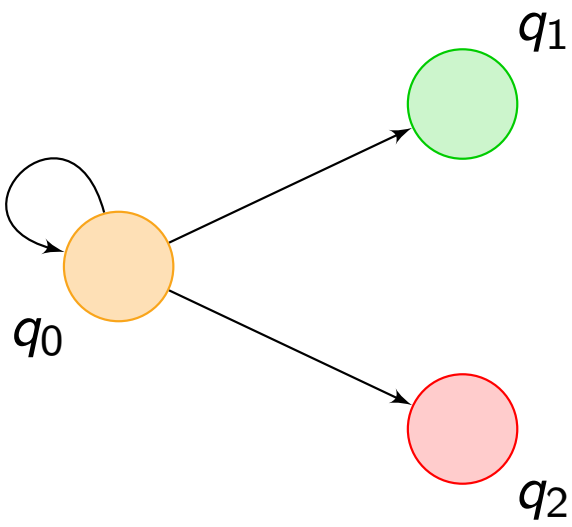

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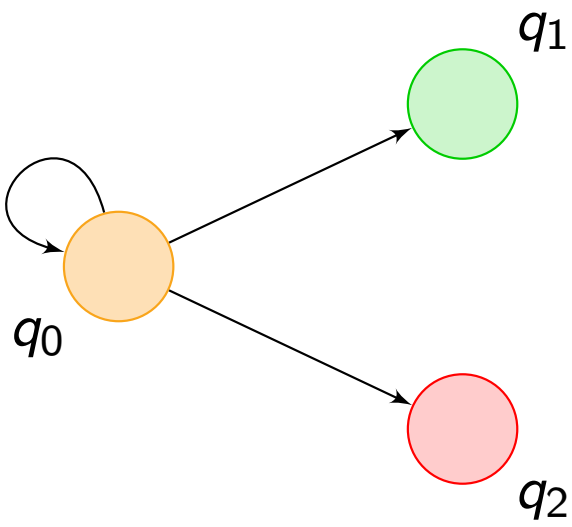

		player 1		
				
player 2		q_0	q_2	q_1
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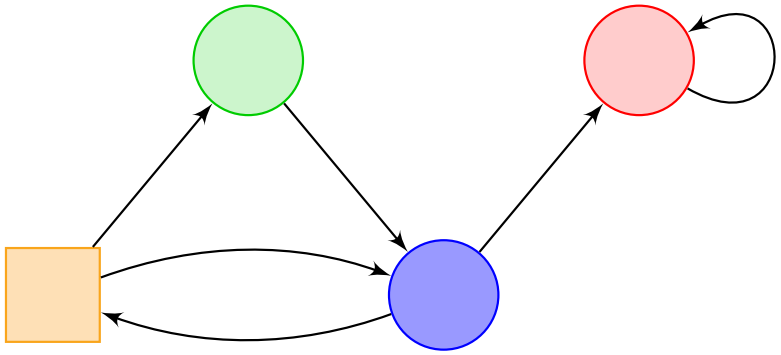
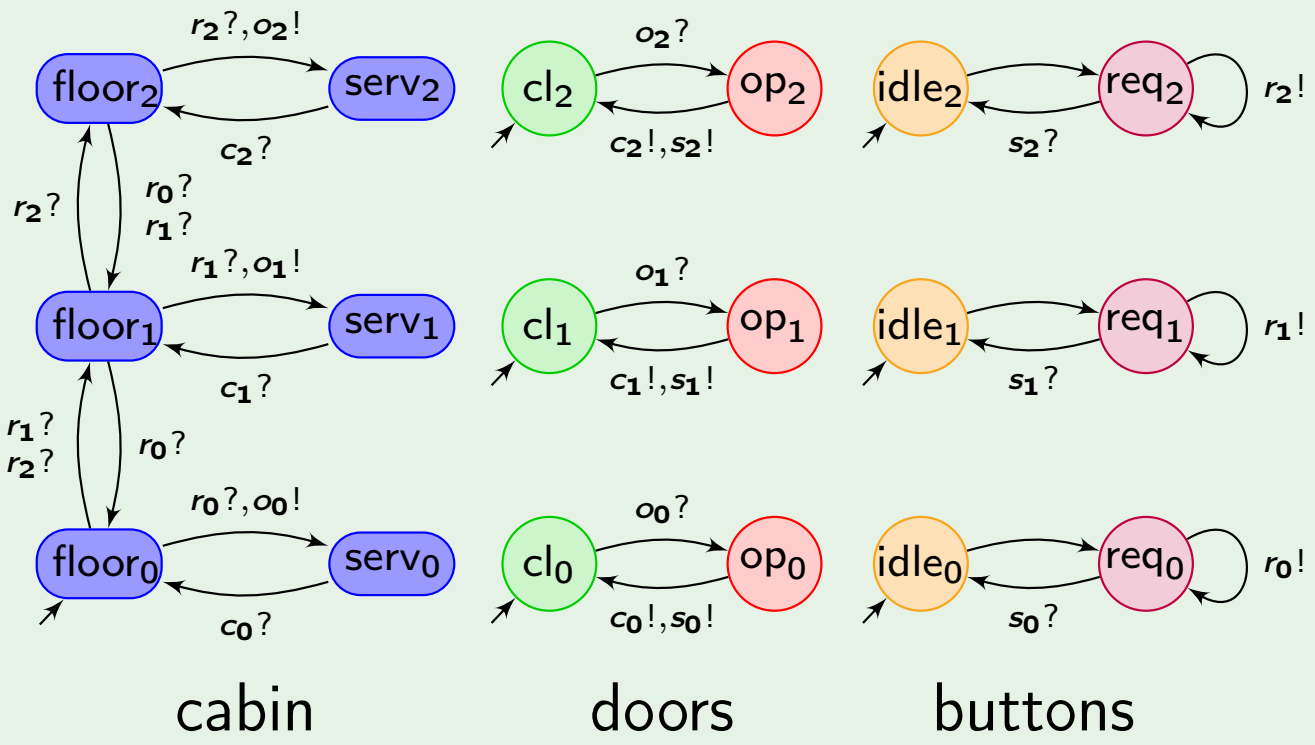
Turn-based games

A turn-based game structure is a game where each state is controlled by a single player.



Reasoning about open systems

Example (An elevator)



Controllability properties:

does there exist a controller under which the system satisfies some property, whatever the users do?

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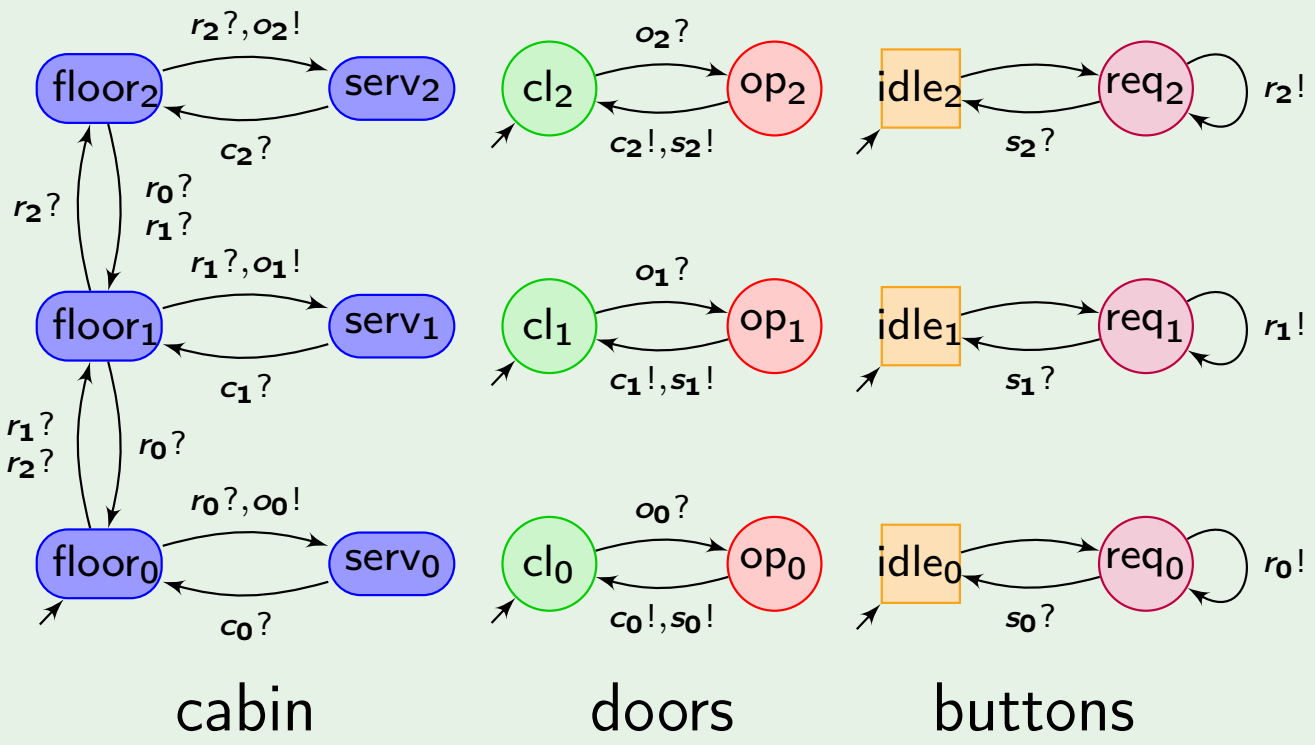

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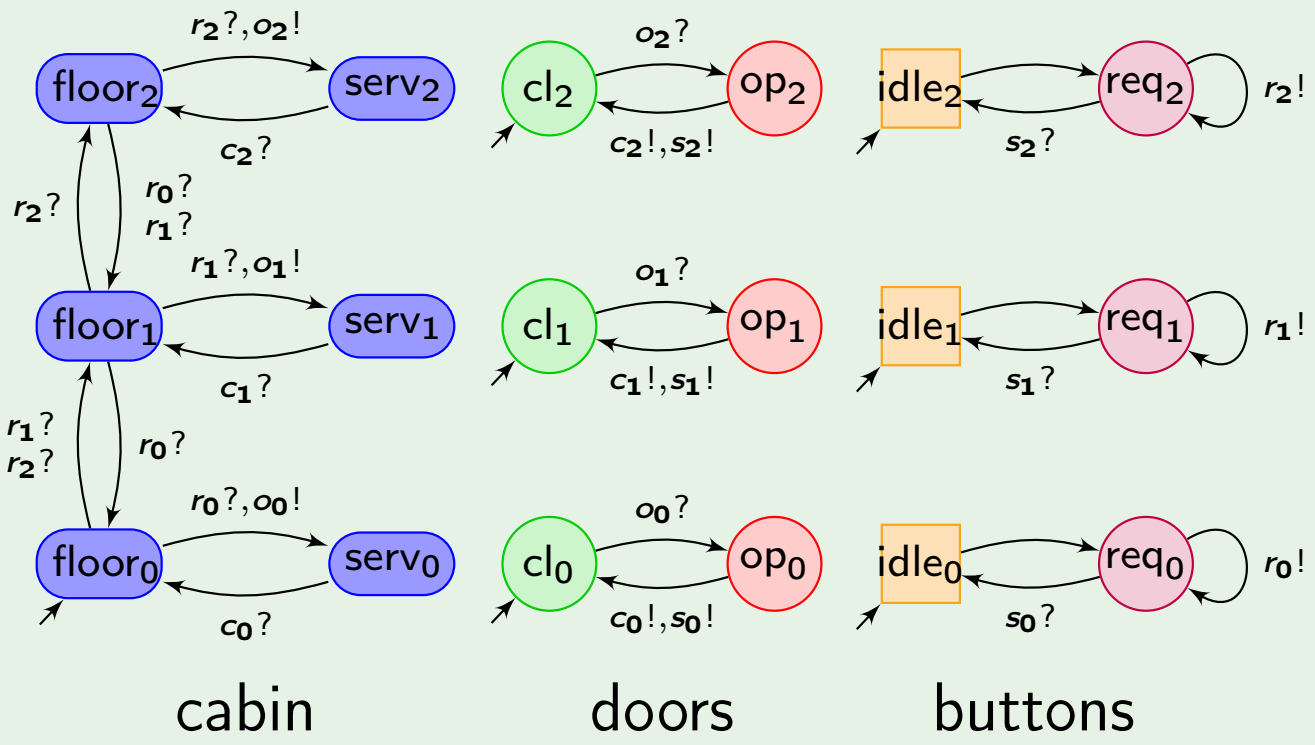

Controllability properties:

is there a strategy for the circle player to enforce some property against any strategy of the square player?

Reasoning about open systems

Strategies

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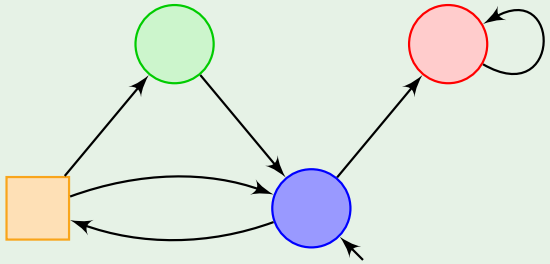

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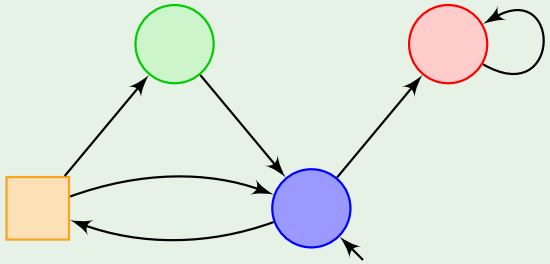
Strategy for player □



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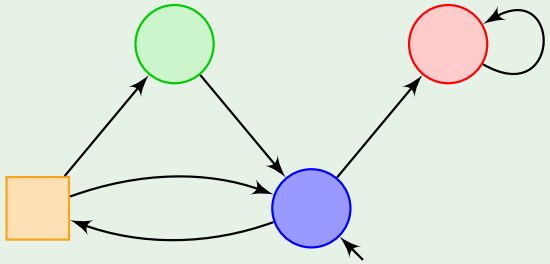

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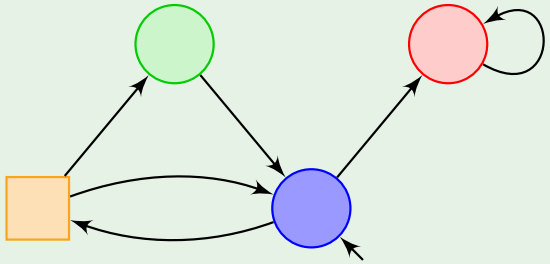

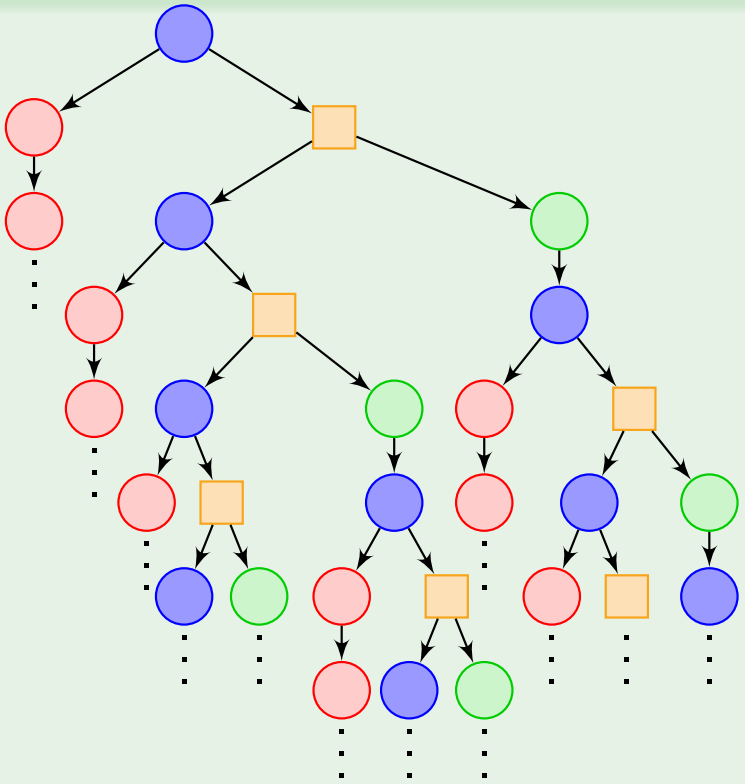

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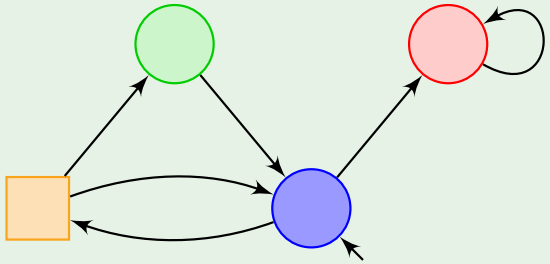

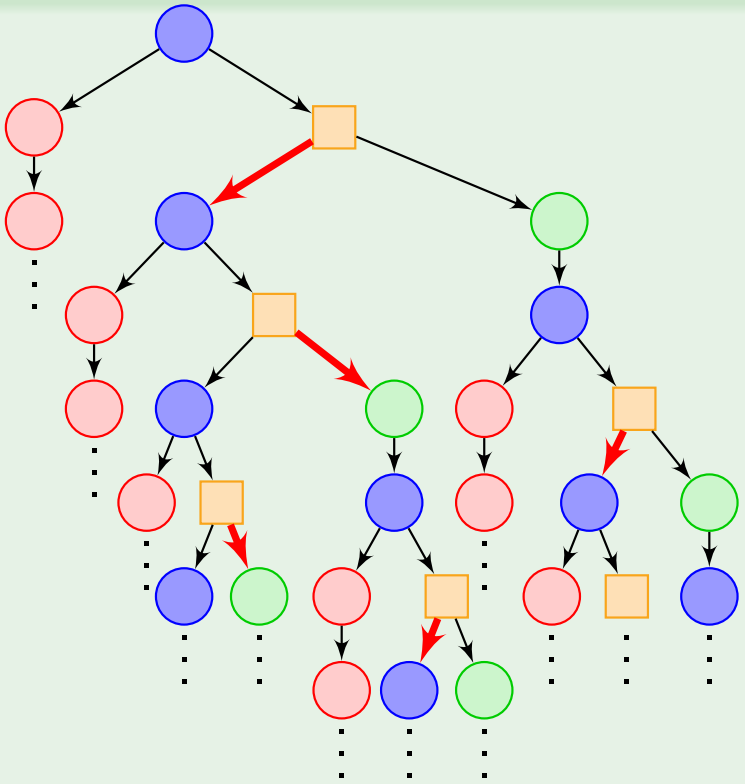

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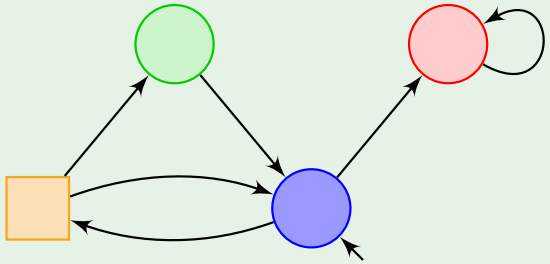

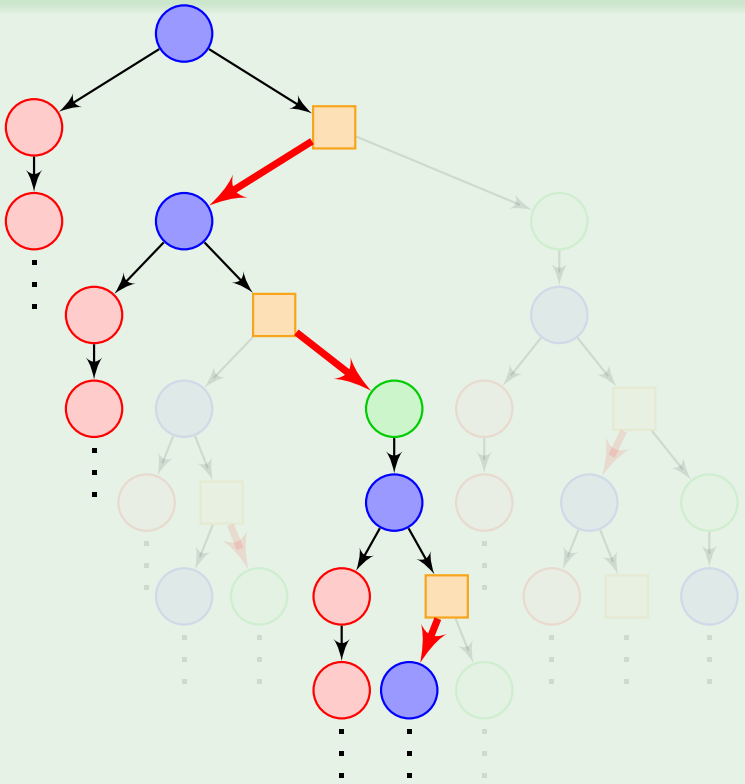

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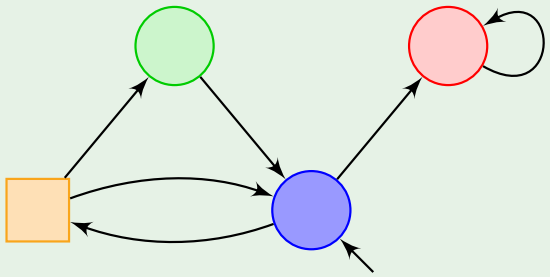

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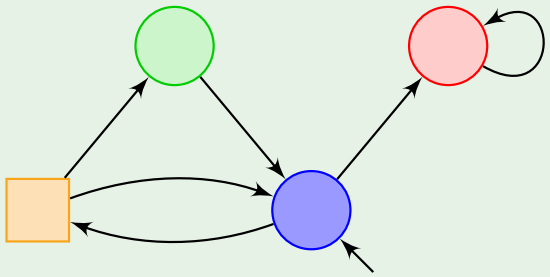

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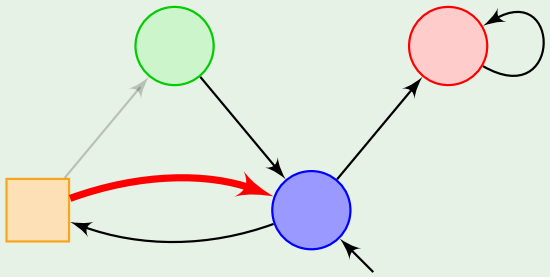

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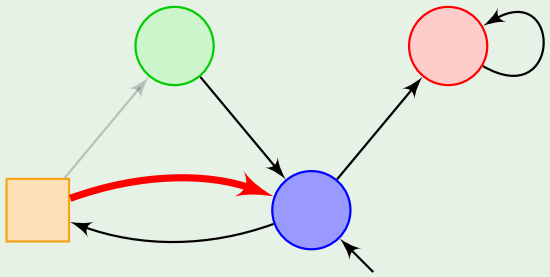

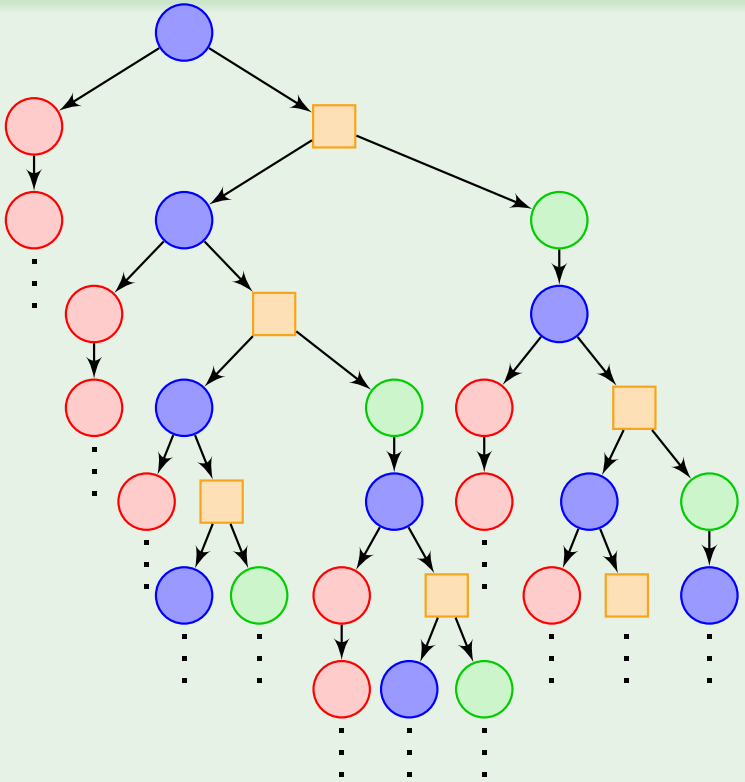

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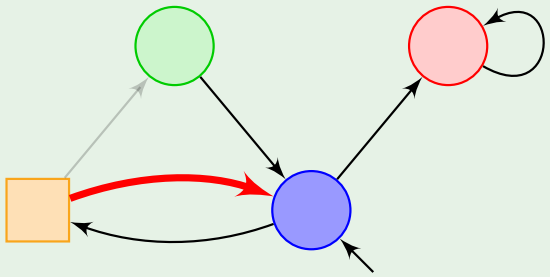

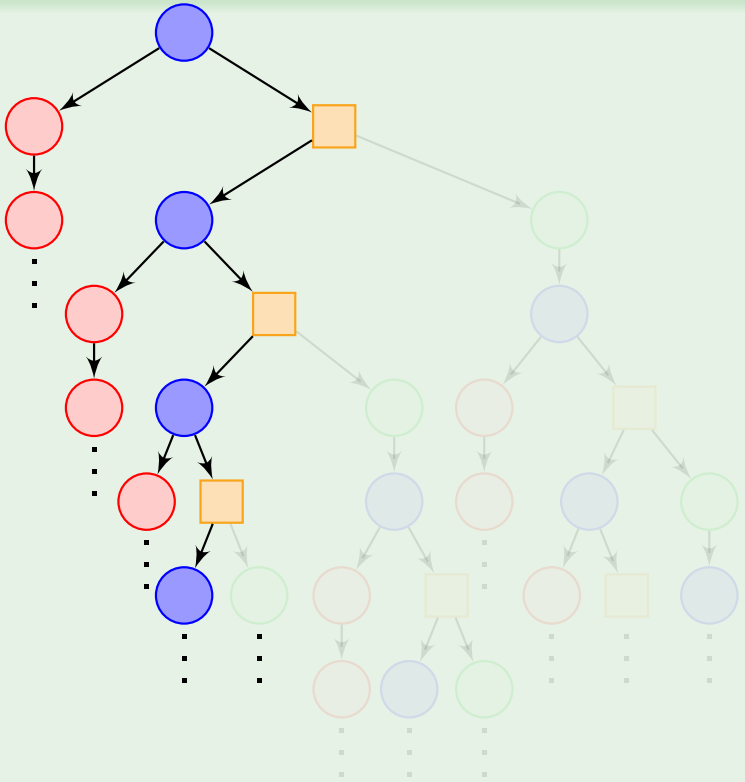

Temporal logics for games: ATL [AHK02]

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ.

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$\mathbf{A}\, \mathbf{G}(\mathrm{req}_0 \Rightarrow \langle\langle \bigcirc \rangle\rangle\, \mathbf{F}\, \mathrm{serv}_0)$ — *any request can be granted by the controller*

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Expressiveness of ATL

Semantics of $\langle\langle A \rangle\rangle\, \varphi$

Existential quantification (over strategies) implicitly includes a universal quantification (over outcomes):

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Writing $\mathbf{E}\varphi \equiv \langle\langle \mathrm{Agt} \rangle\rangle\, \varphi$ would depend on the set of agents...

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$[\![A]\!]\ \mathbf{F}$ ◯ *for any strategy of A, there is an outcome visiting* ◯

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Theorem

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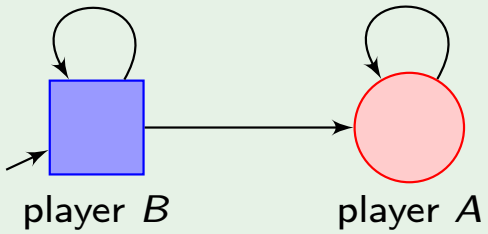

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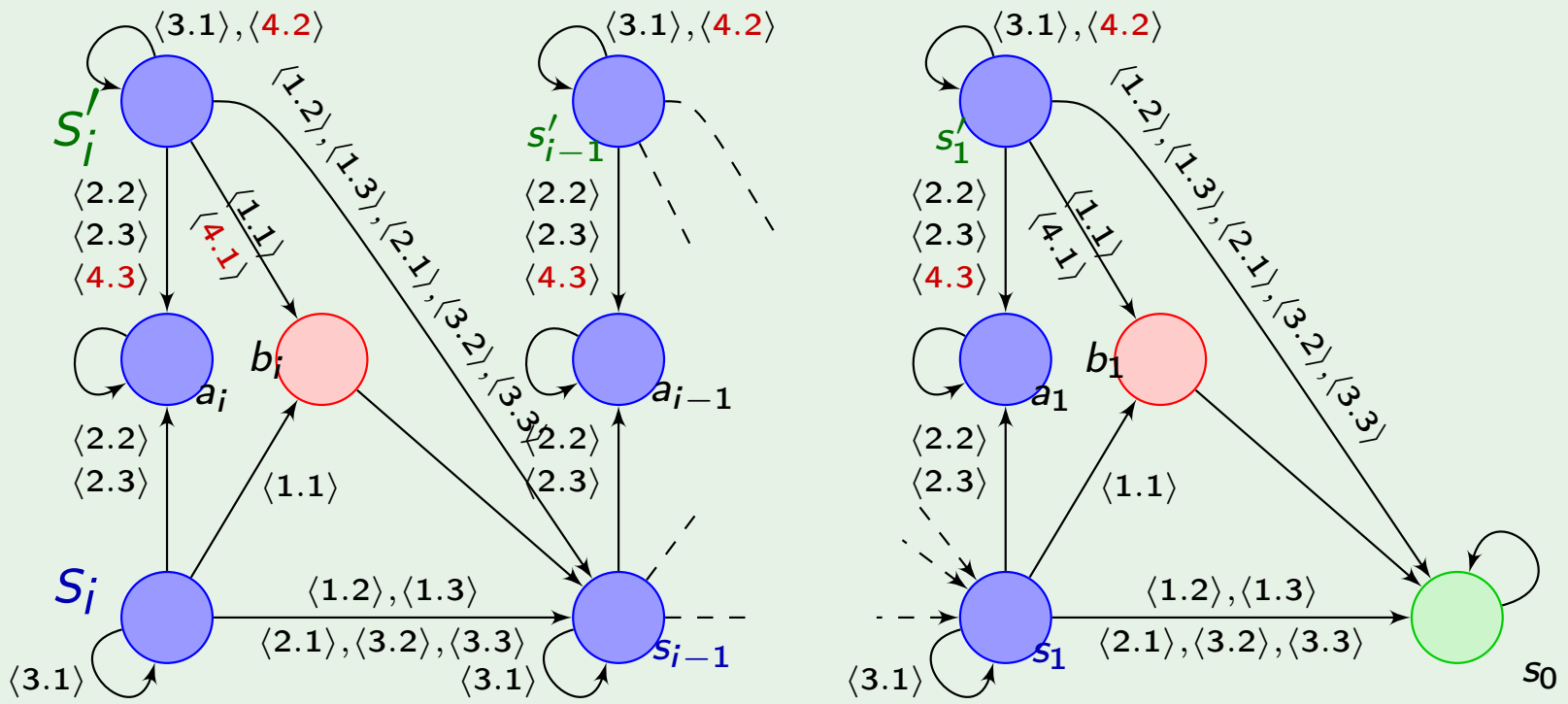

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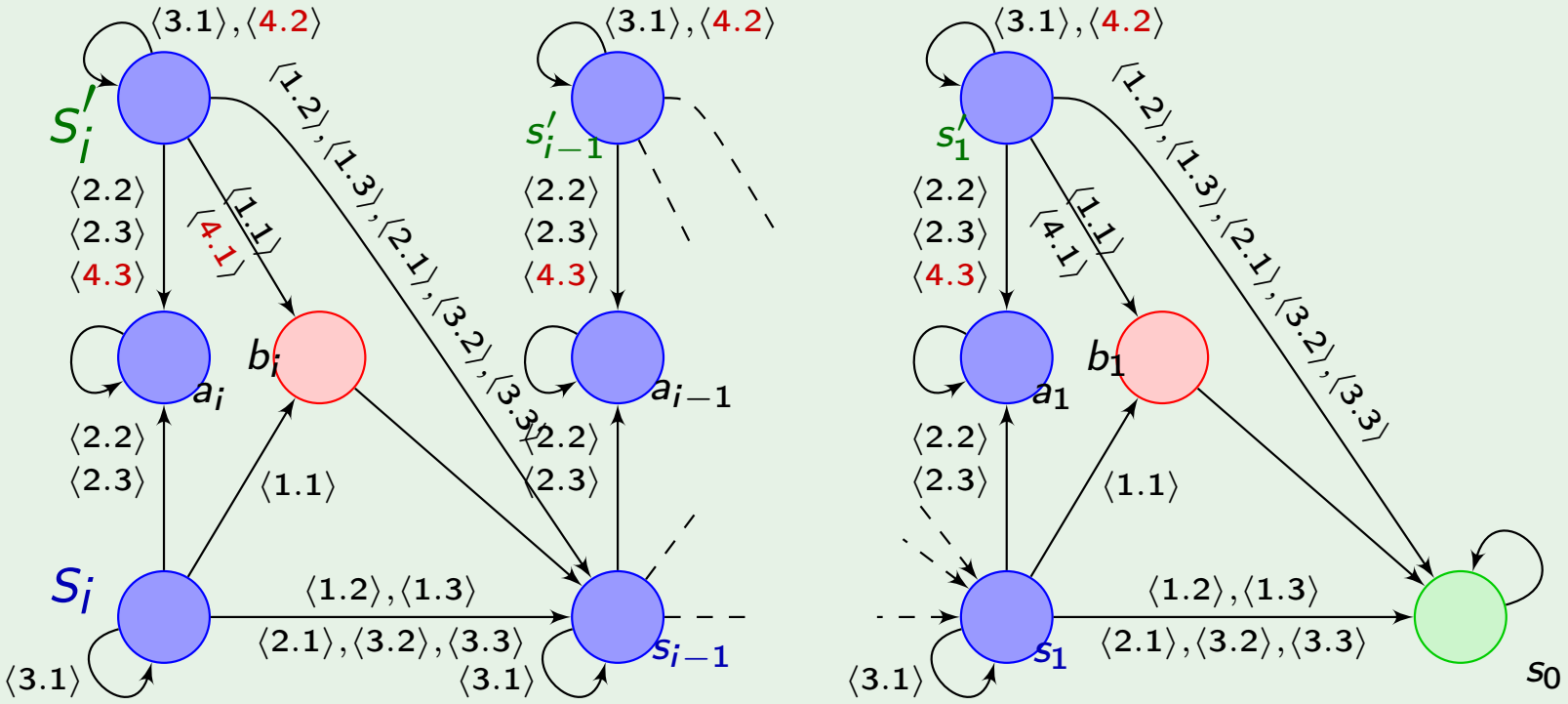

$s_i \not\models \langle\langle A\rangle\rangle$ ● $\mathbf{W}$ ○ but $s_i' \models \langle\langle A\rangle\rangle$ ● $\mathbf{W}$ ○

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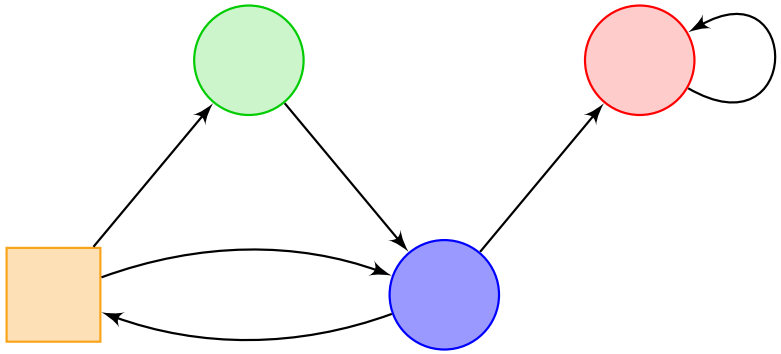
Model checking ATL [AHK02]



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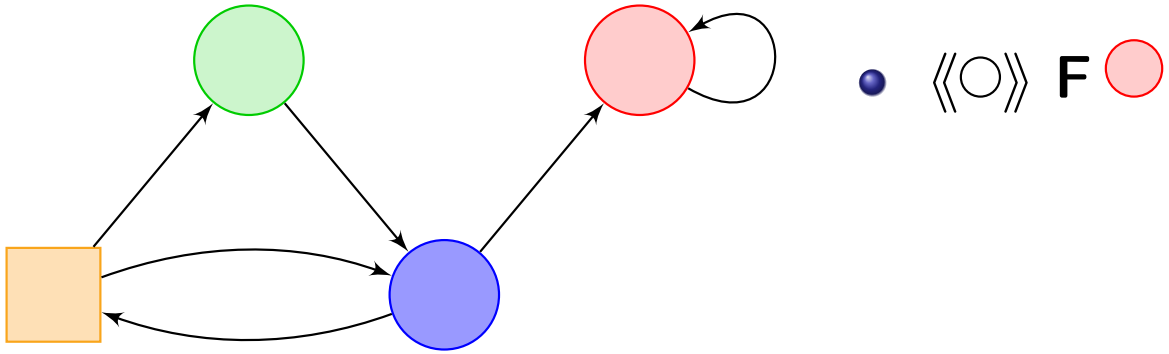

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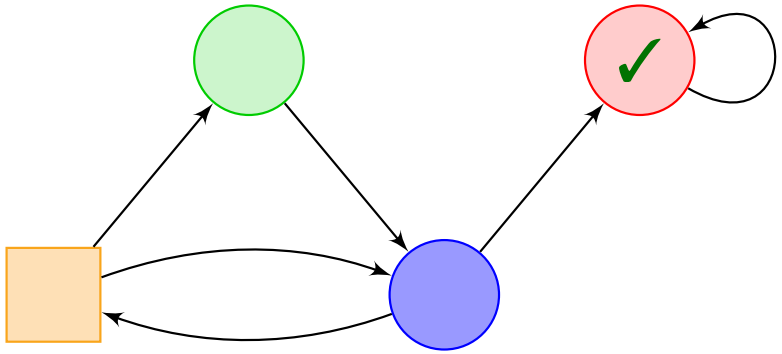
Model checking ATL [AHK02]



- $\langle\langle \bigcirc \rangle\rangle$ **F**

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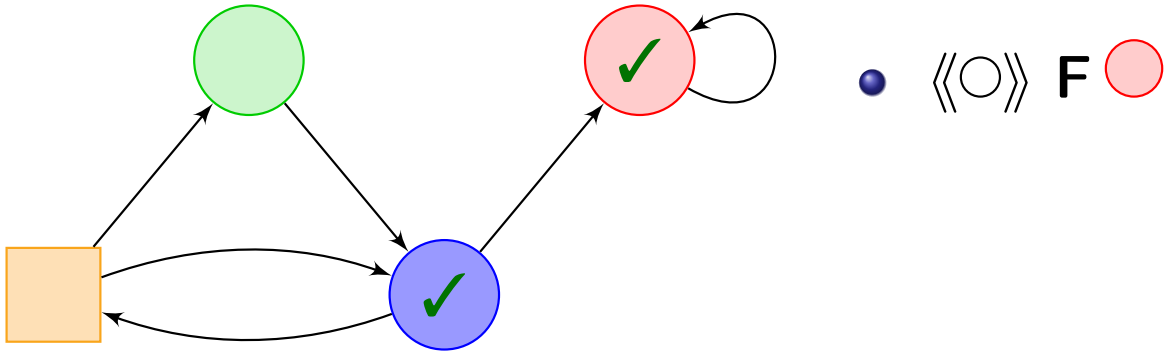

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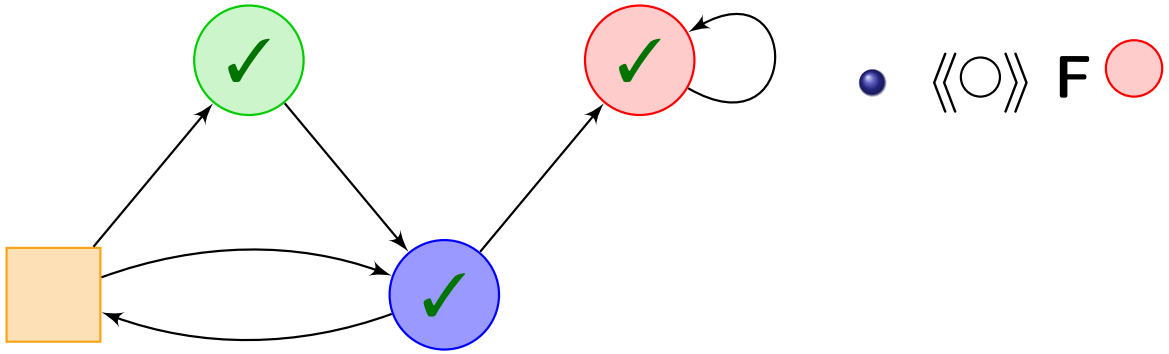

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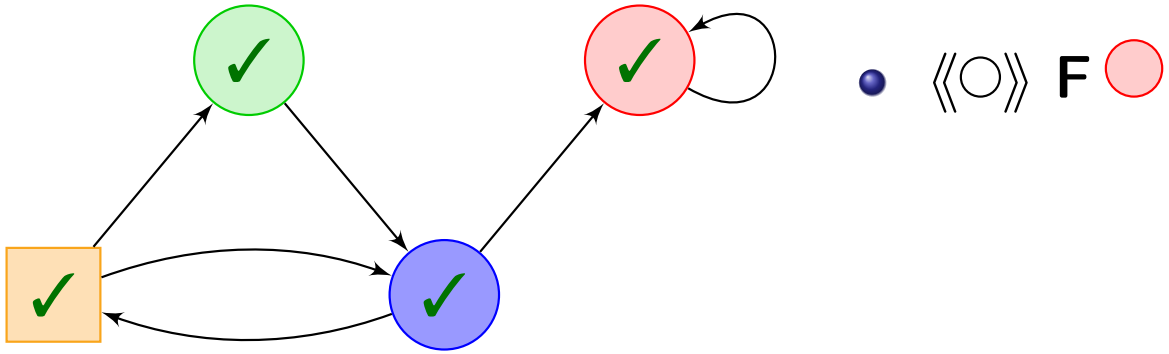

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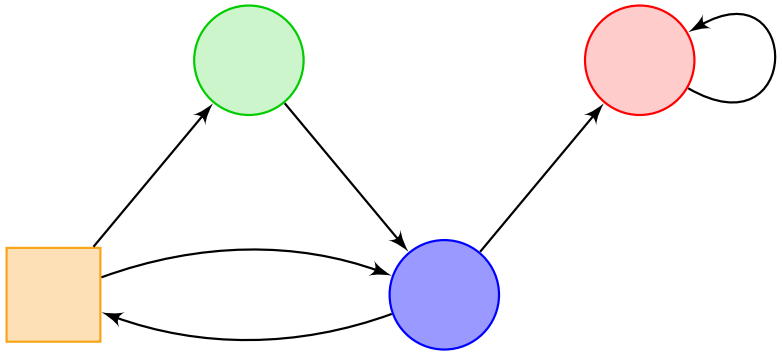

- $\langle\langle \bigcirc \rangle\rangle$ **F** 
- $\langle\langle \Box \rangle\rangle$ **F** 

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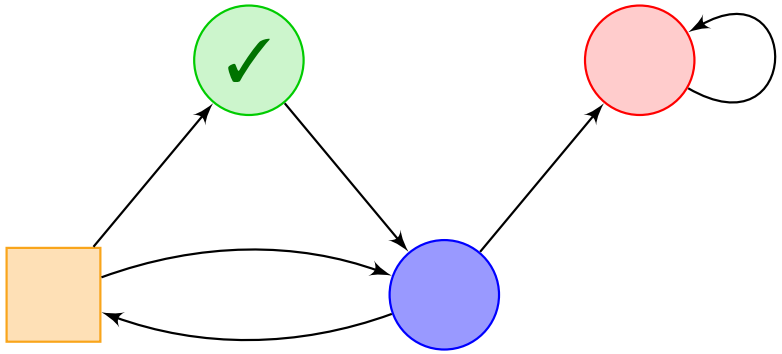

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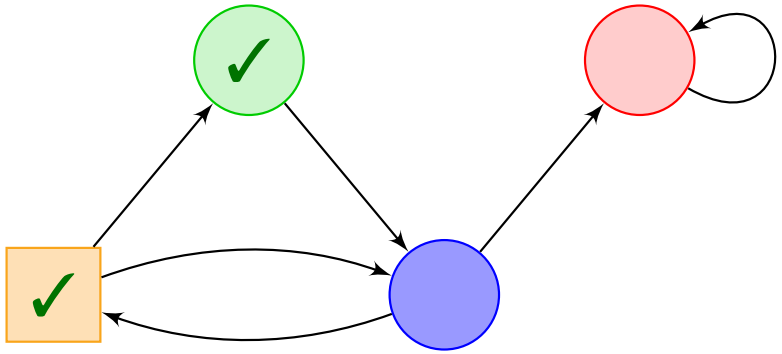

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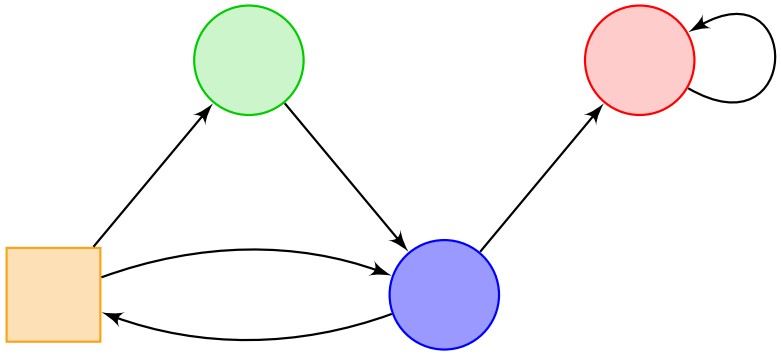

- $\langle\langle\bigcirc\rangle\rangle\ \mathbf{F}$ ●
- $\langle\langle\Box\rangle\rangle\ \mathbf{F}$ ●
- $\langle\langle\bigcirc\rangle\rangle\ \mathbf{G}(\ \langle\langle\Box\rangle\rangle\ \mathbf{F}$ ●$)$

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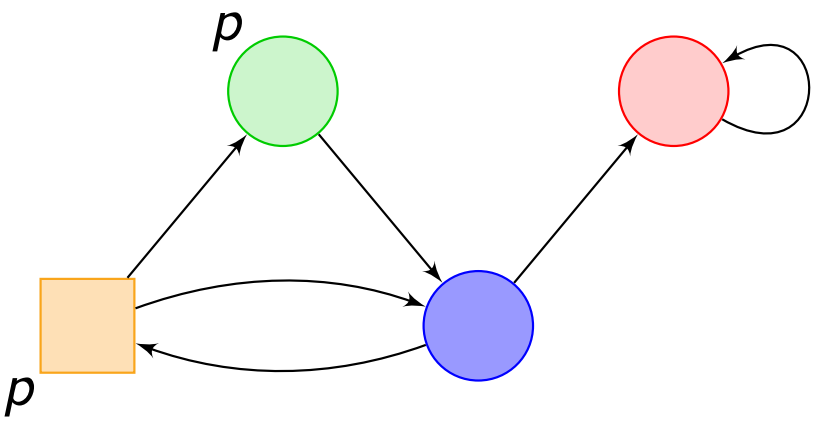

- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{F}$
- $\langle\langle \Box \rangle\rangle\ \mathbf{F}$
- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{G}(\underbrace{\langle\langle \Box \rangle\rangle\ \mathbf{F}}_{p})$

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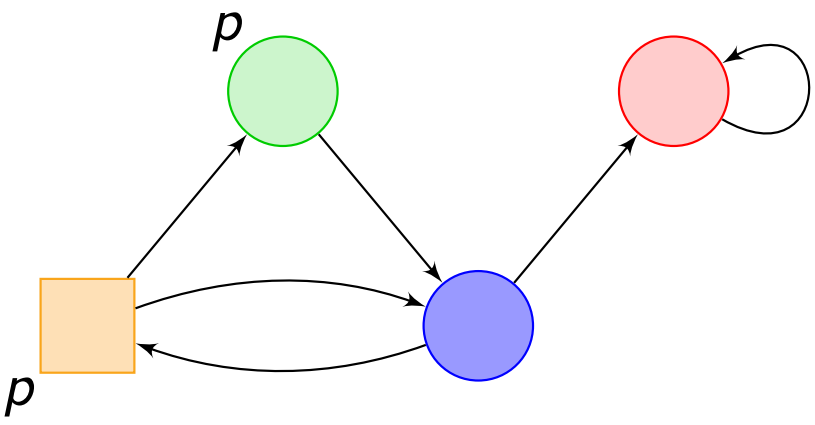

- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{F}$
- $\langle\langle \Box \rangle\rangle\ \mathbf{F}$
- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{G}(\underbrace{\langle\langle \Box \rangle\rangle\ \mathbf{F}}_{p}) \equiv \langle\langle \bigcirc \rangle\rangle\ \mathbf{G}\ p$

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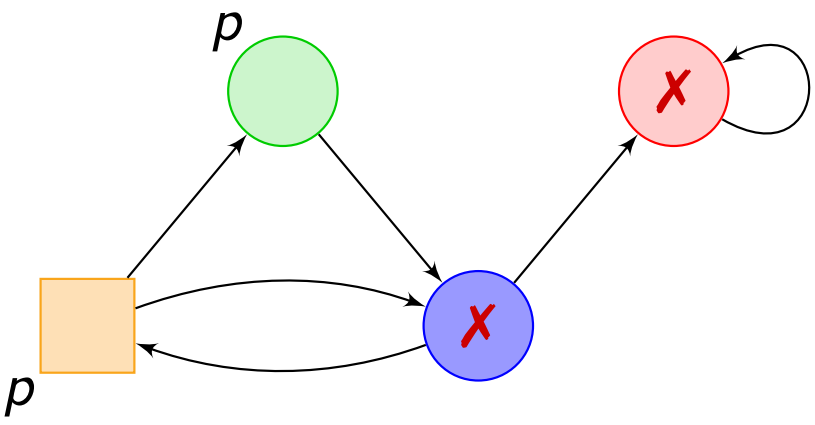

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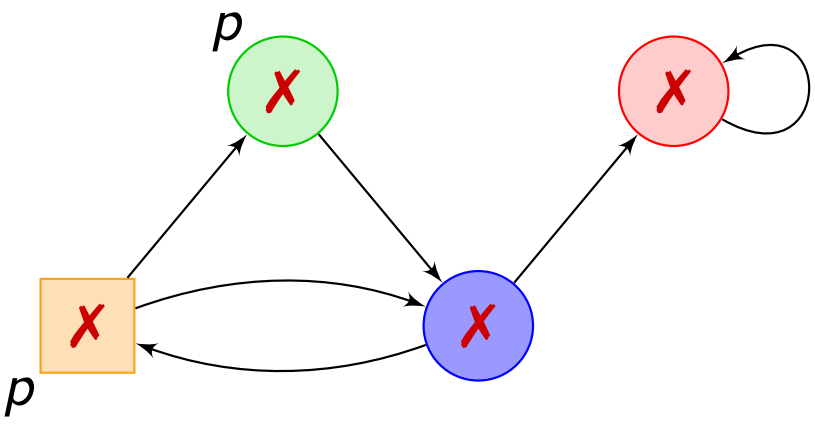

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- $\langle\langle \Box \rangle\rangle\ \mathbf{F}$ ●
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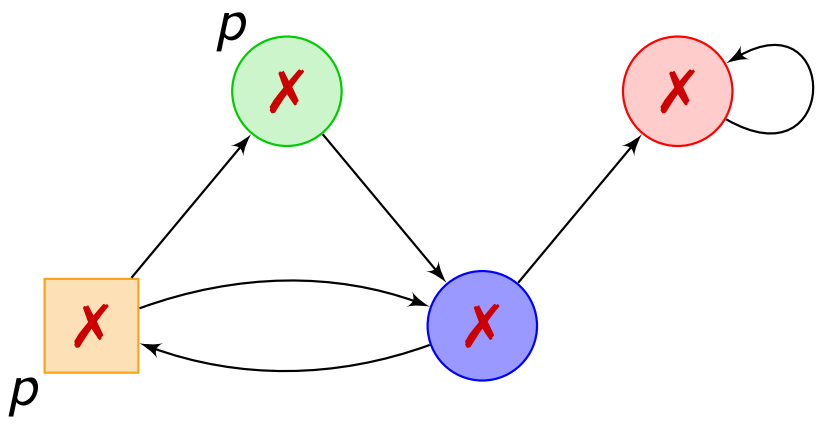

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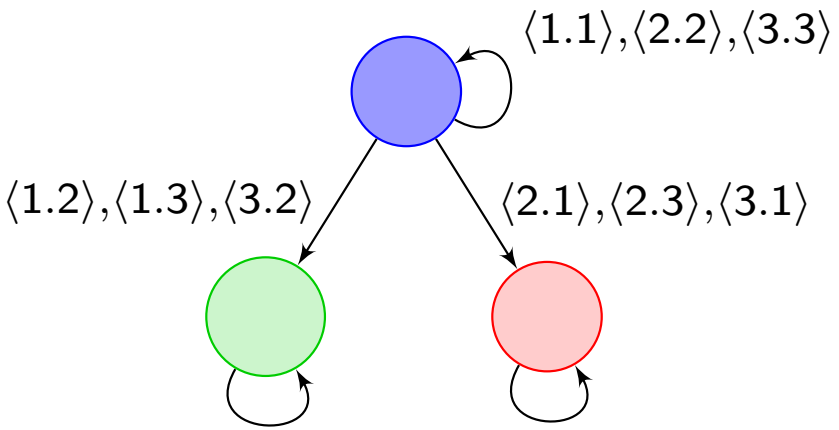
- $\langle\langle \Box \rangle\rangle\ \mathbf{F}$
- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{G}(\underbrace{\langle\langle \Box \rangle\rangle\ \mathbf{F}}_{p}) \equiv \langle\langle \bigcirc \rangle\rangle\ \mathbf{G}\ p$

Theorem ([AHK02])

Model checking ATL over turn-based games is PTIME*-complete.*

[AHK02] Alur, Henzinger, Kupferman. Alternating-time Temporal Logic. J. ACM, 2002.

Model checking ATL [AHK02]



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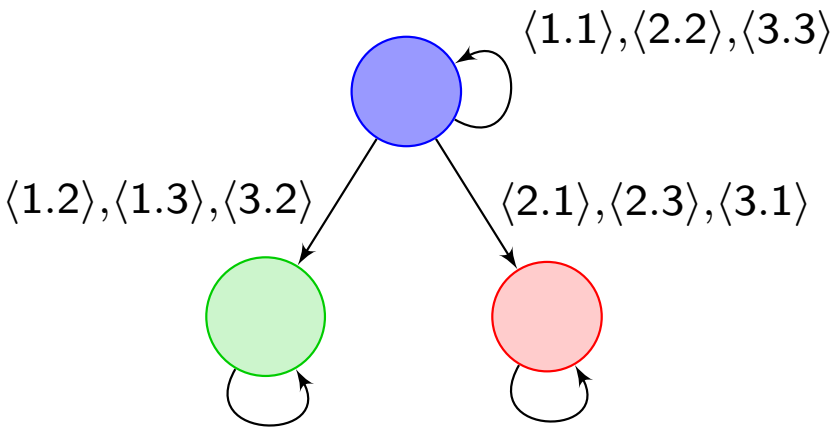

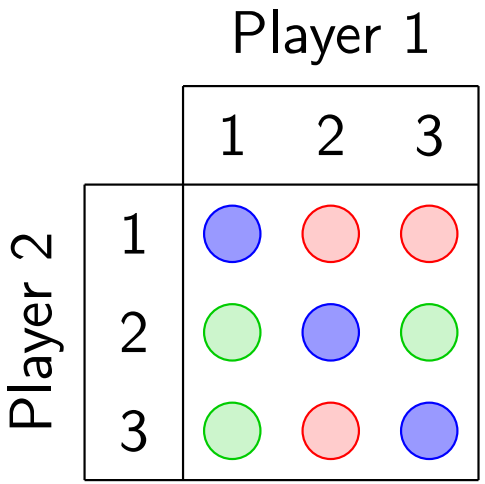

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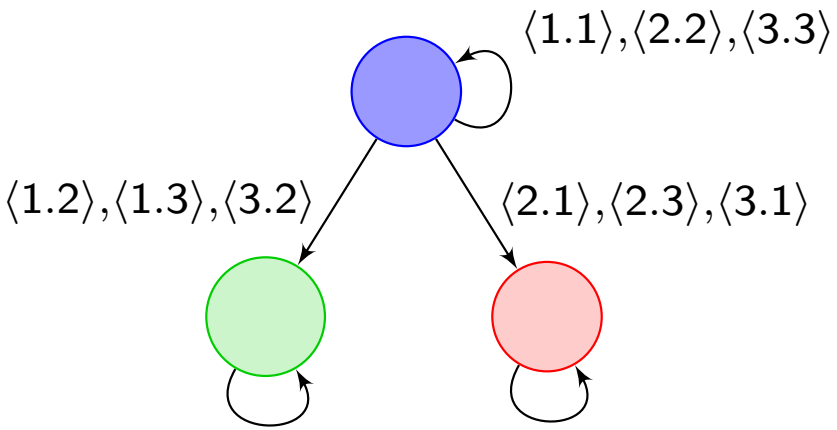

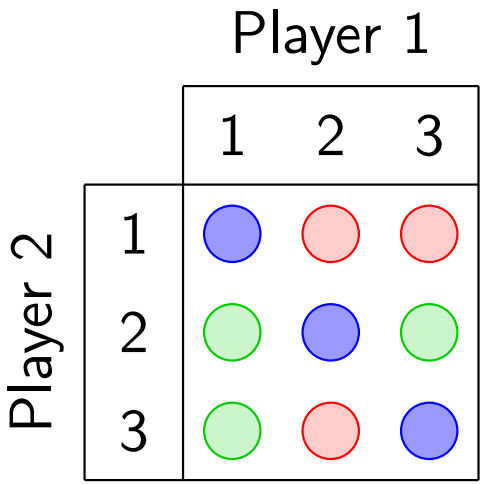

$\langle\langle 1 \rangle\rangle$ **F** 

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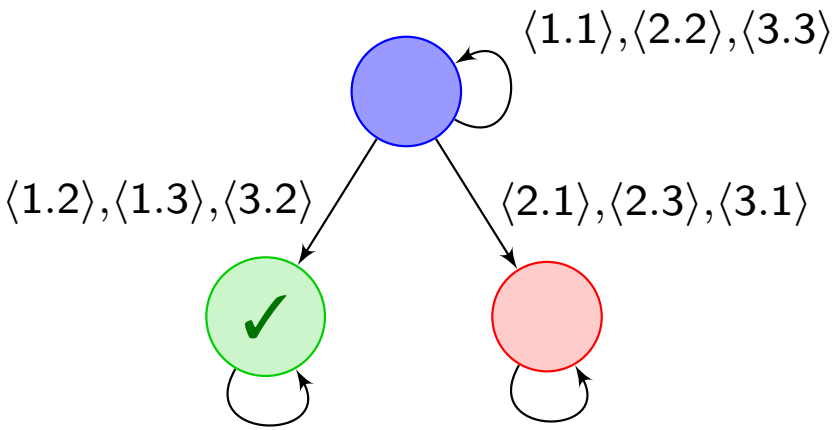

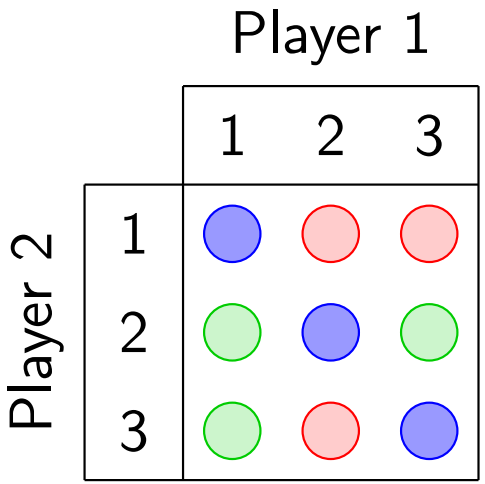

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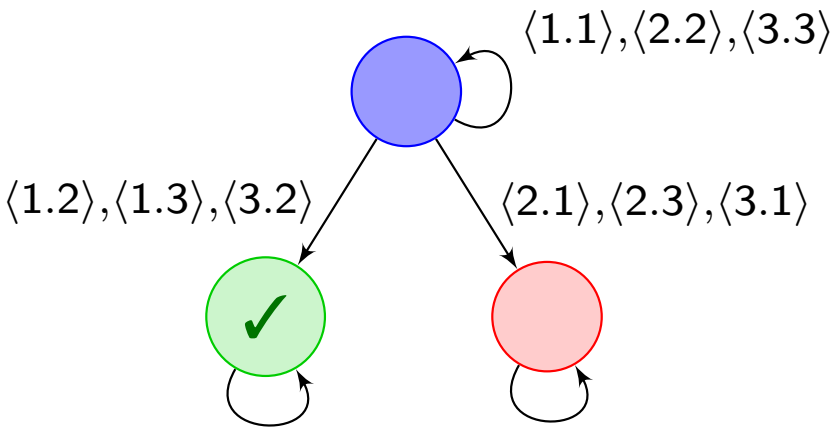

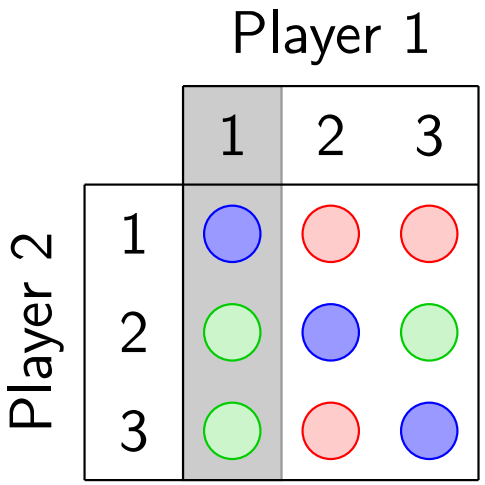

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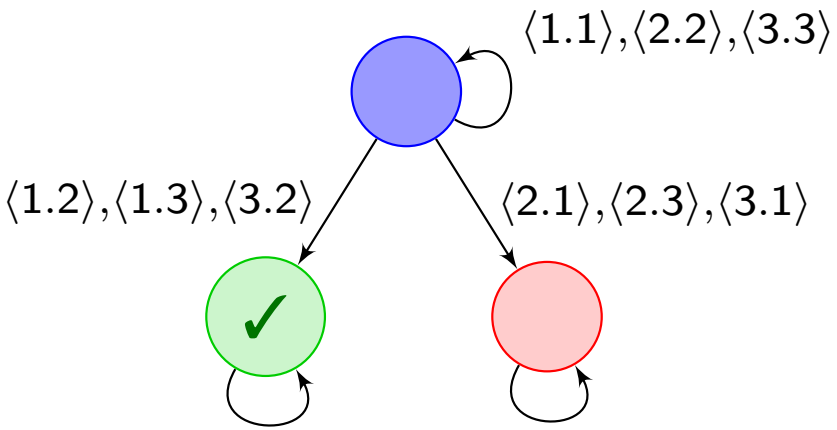

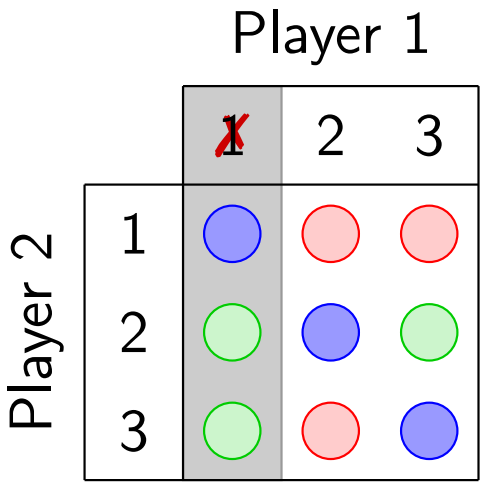

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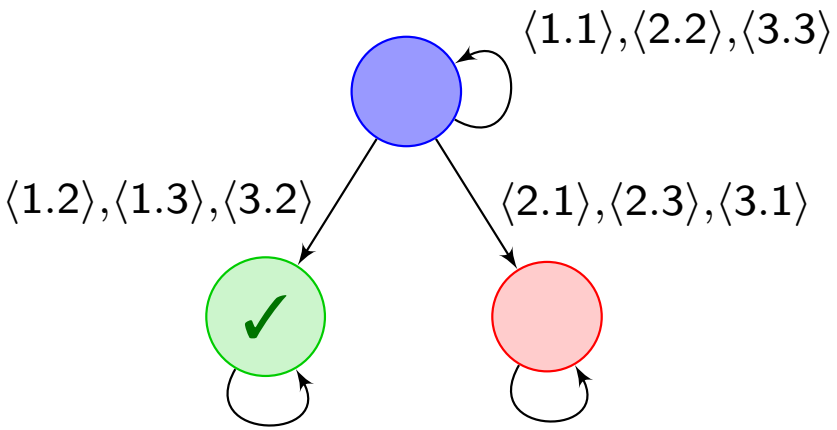

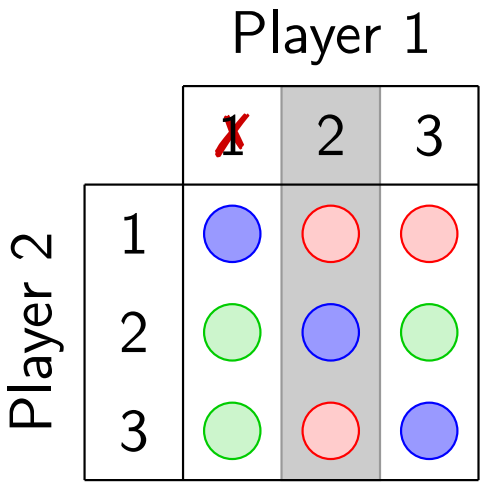

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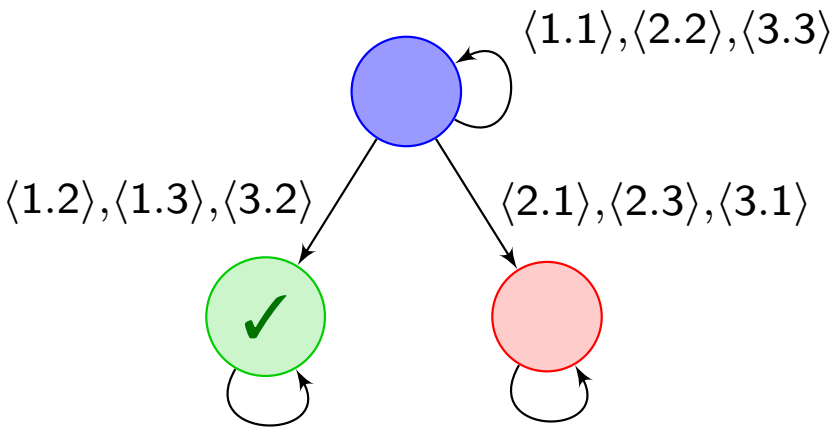

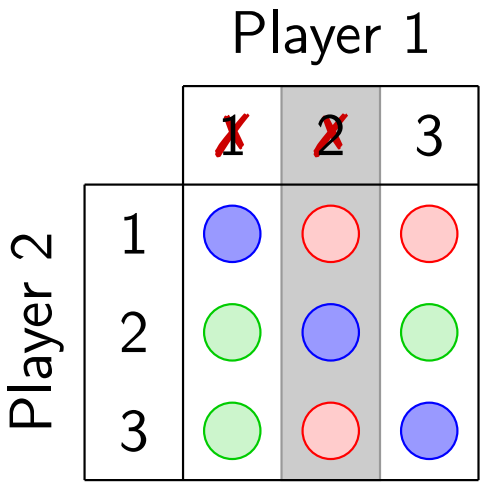

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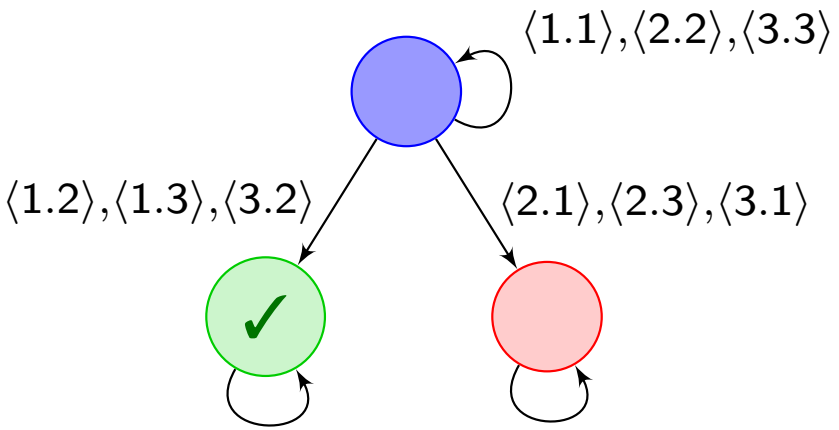

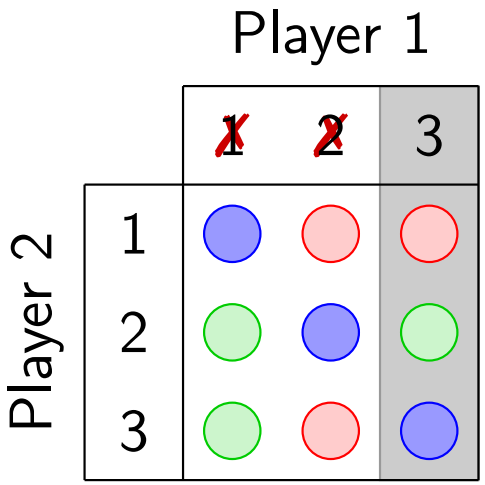

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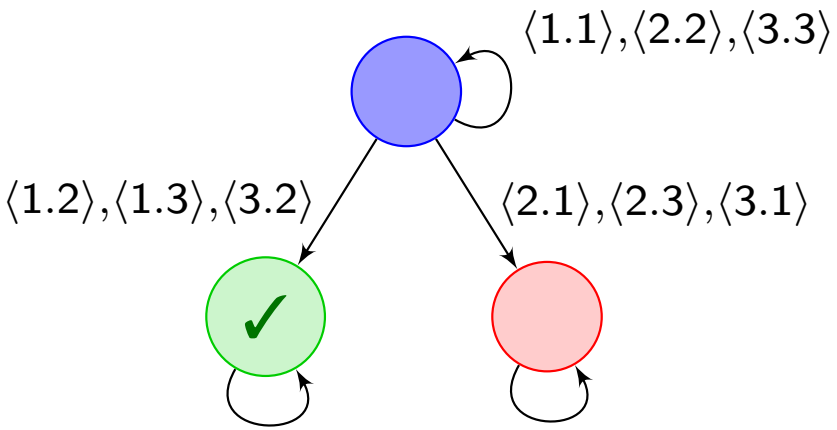

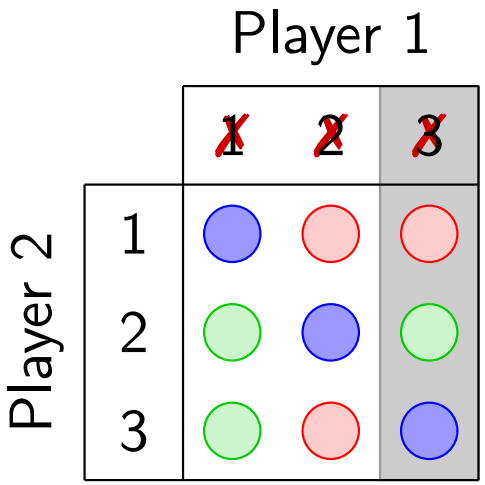

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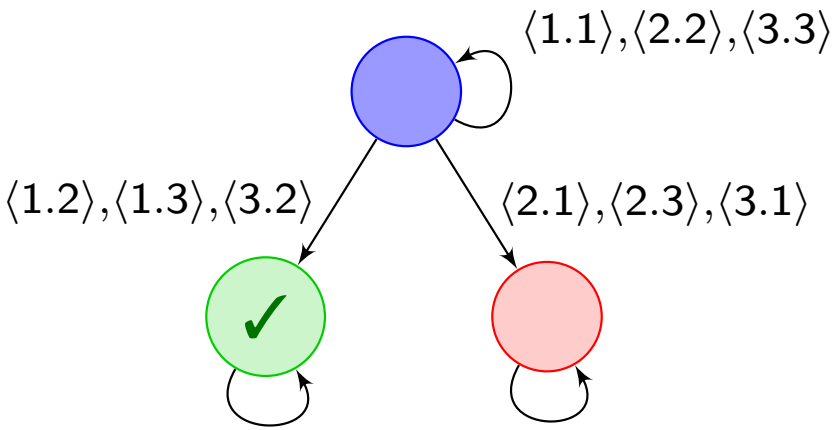

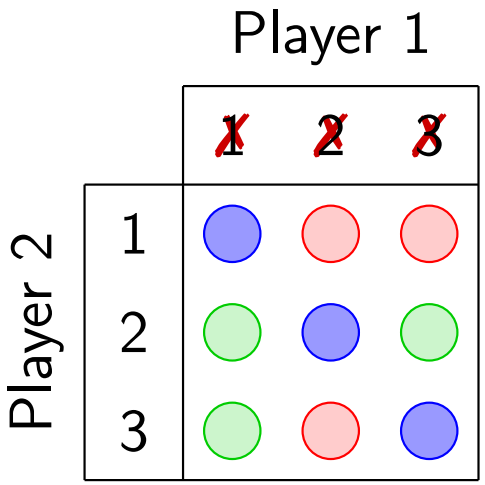

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Theorem ([AHK02])

Model checking ATL over concurrent games is PTIME*-complete.*

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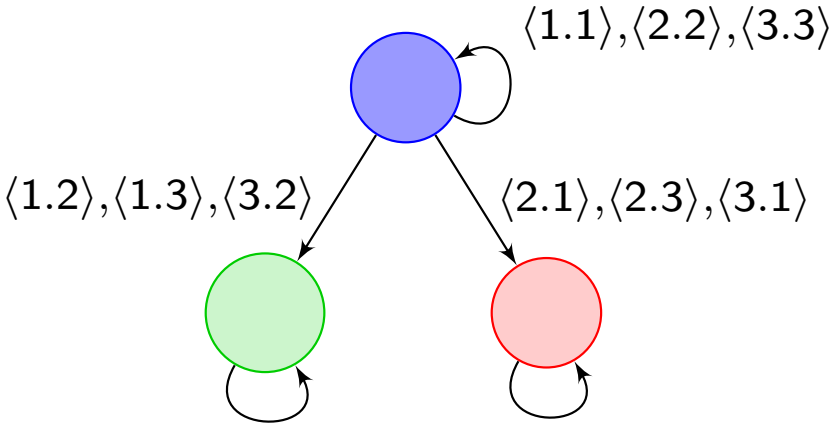
Model checking ATL [AHK02]



[AHK02] Alur, Henzinger, Kupferman. Alternating-time Temporal Logic. J. ACM, 2002.
[LMO07] Laroussinie, Markey, Oreiby. On the Expressiveness and Complexity of ATL. CONCUR, 2007.

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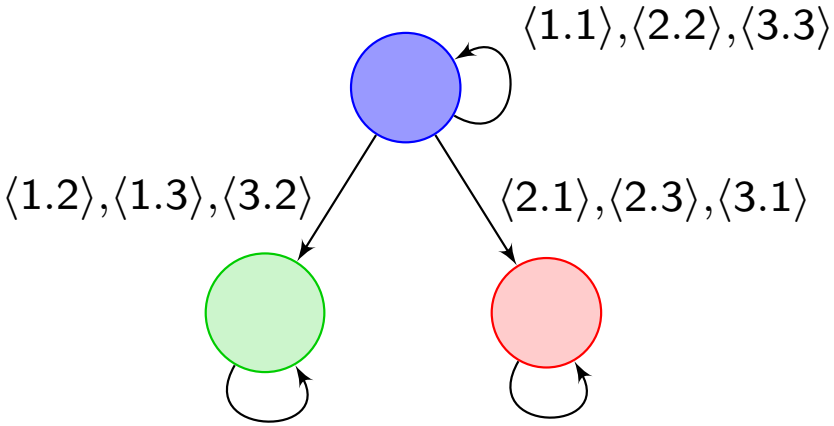

- if $m_1 = m_2$ goto ;
- otherwise, if $m_1 = 1$ or $m_2 = 2$ goto ;
- otherwise, goto ;

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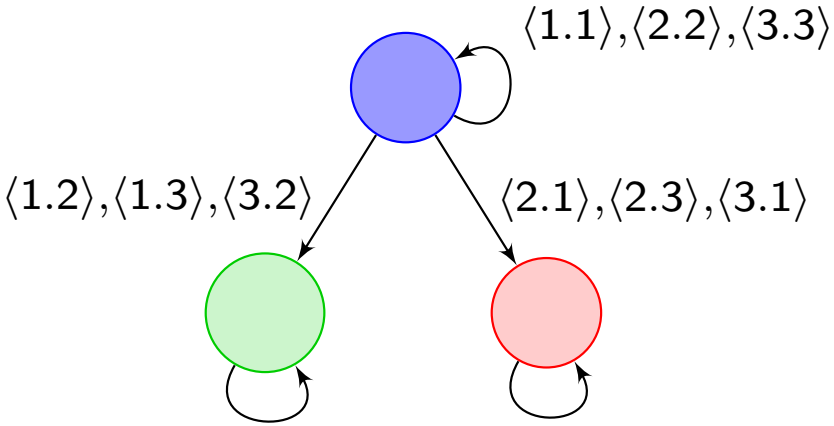

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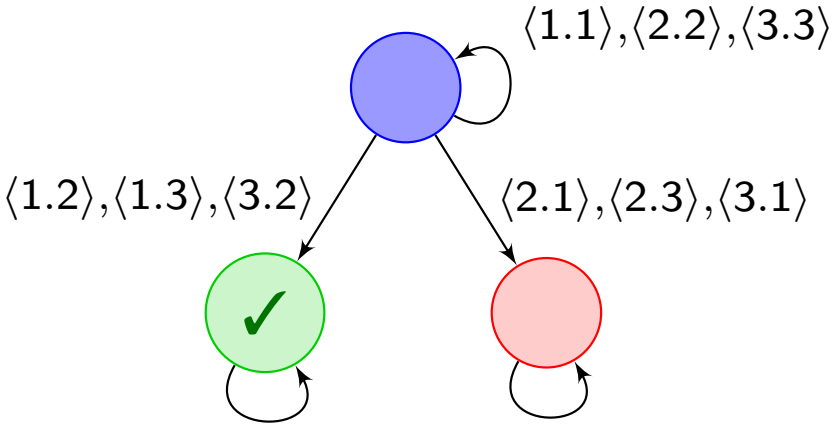

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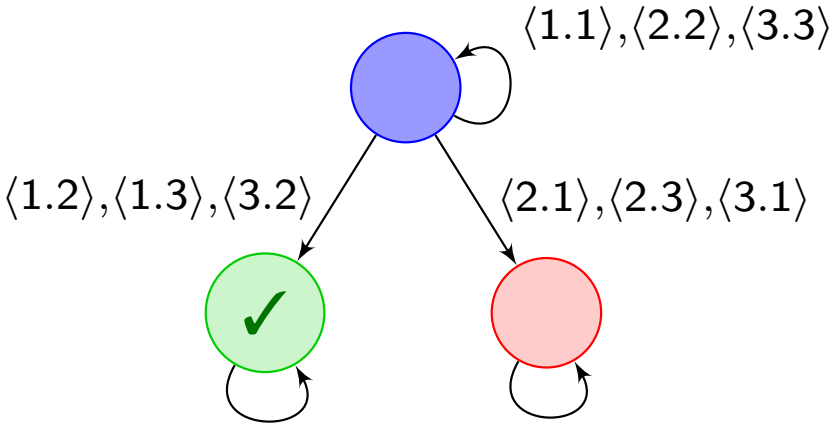

- if $m_1 = m_2$ goto ;
- otherwise, if $m_1 = 1$ or $m_2 = 2$ goto ;
- otherwise, goto ;

$\langle\langle 1 \rangle\rangle$ **F**

$\rightsquigarrow$ for each move of Player 1, need to solve several instances of SAT

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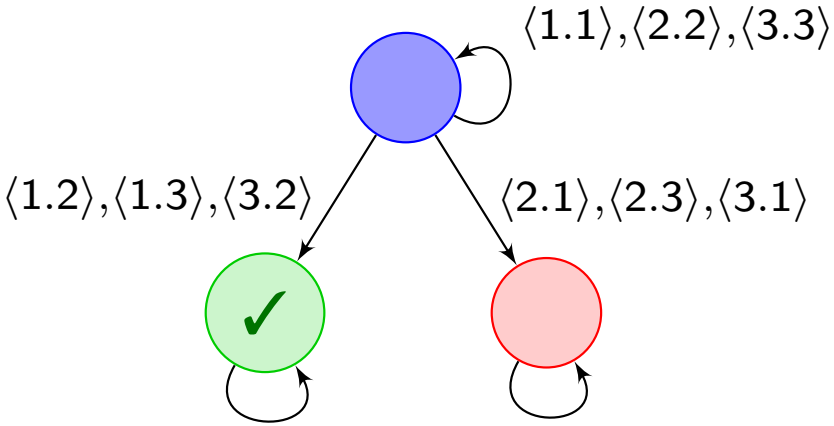

- if $m_1 = m_2$ goto ●;
- otherwise, if $m_1 = 1$ or $m_2 = 2$ goto ●;
- otherwise, goto ●;

$\langle\langle 1 \rangle\rangle$ **F** ●

$\rightsquigarrow$ for each move of Player 1, need to solve several instances of SAT

Theorem ([LMO07])

Model checking ATL over concurrent games with symbolic transition table is Δ_3^P-complete (hence in PSPACE, NP*-hard).*

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The full logic ATL*

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Theorem ([AHK02])

Model checking ATL is 2EXPTIME-complete.*

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Proof

Games with LTL objectives can be solved in 2EXPTIME:

- LTL formula to deterministic parity automaton:
  - size doubly-exponential;
  - number of priorities exponential.
- solve parity game in $O(|Q|^{|p|})$.

[AHK02] Alur, Henzinger, Kupferman. Alternating-time Temporal Logic. J. ACM, 2002.

Behavioural equivalence for ATL

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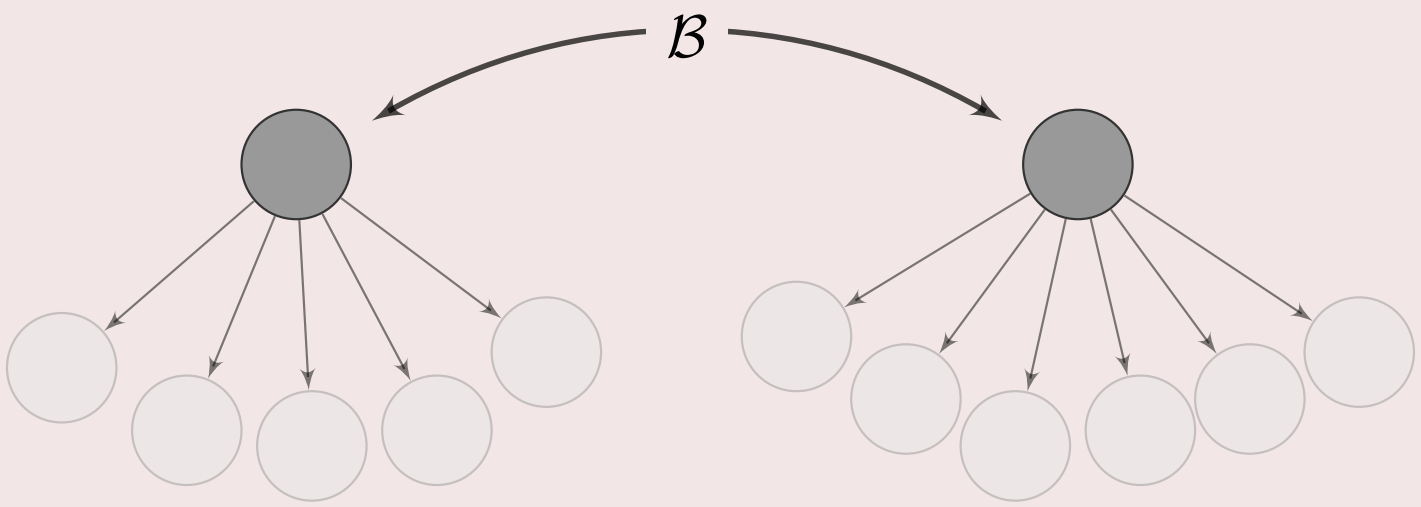
A relation $\mathcal{B} \subseteq Q \times Q$ is an alternating bisimultion if



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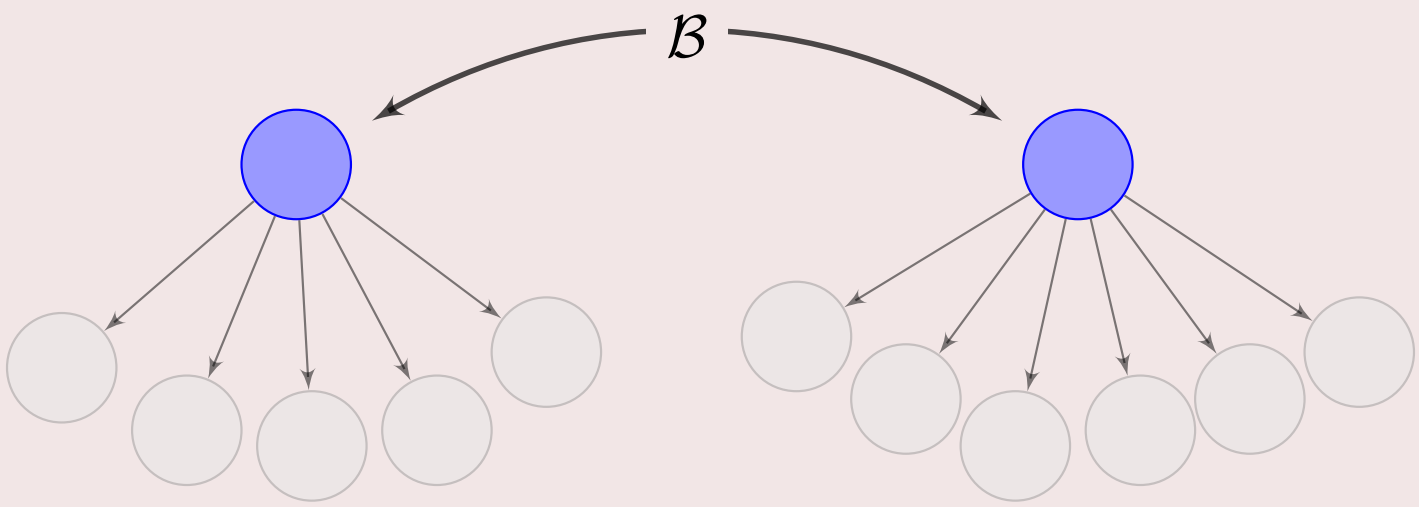

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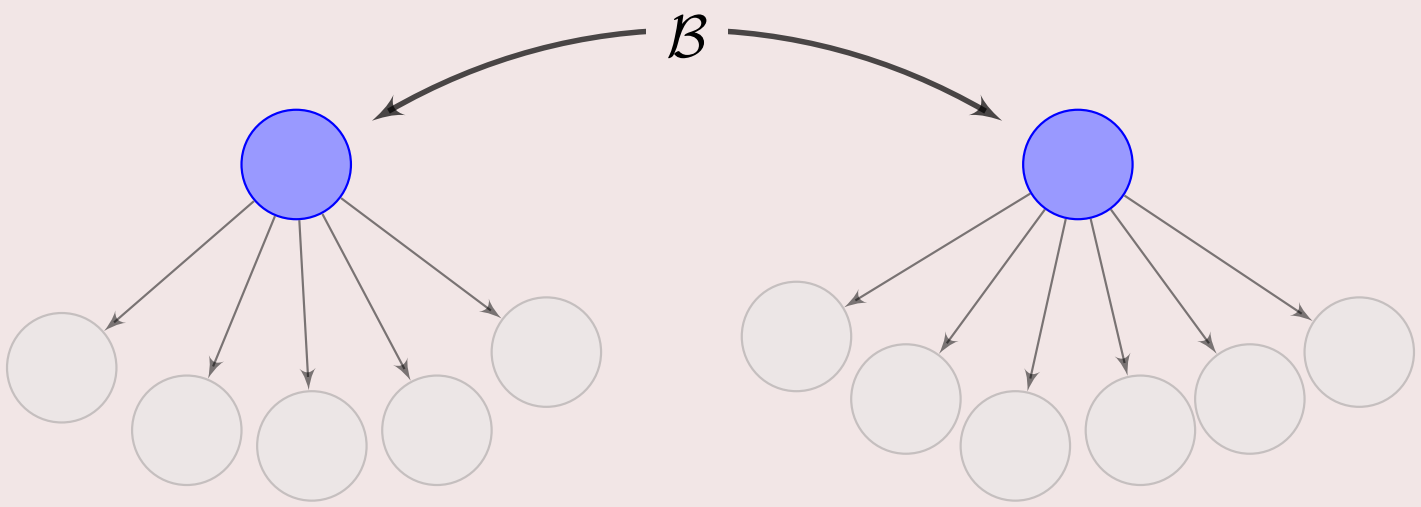

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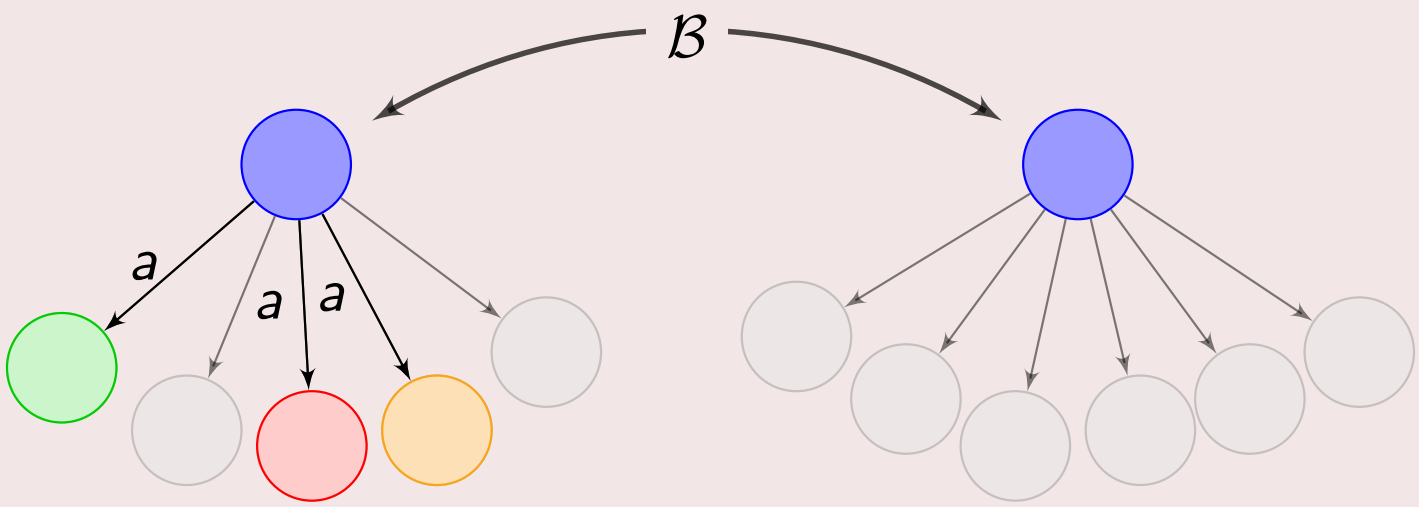

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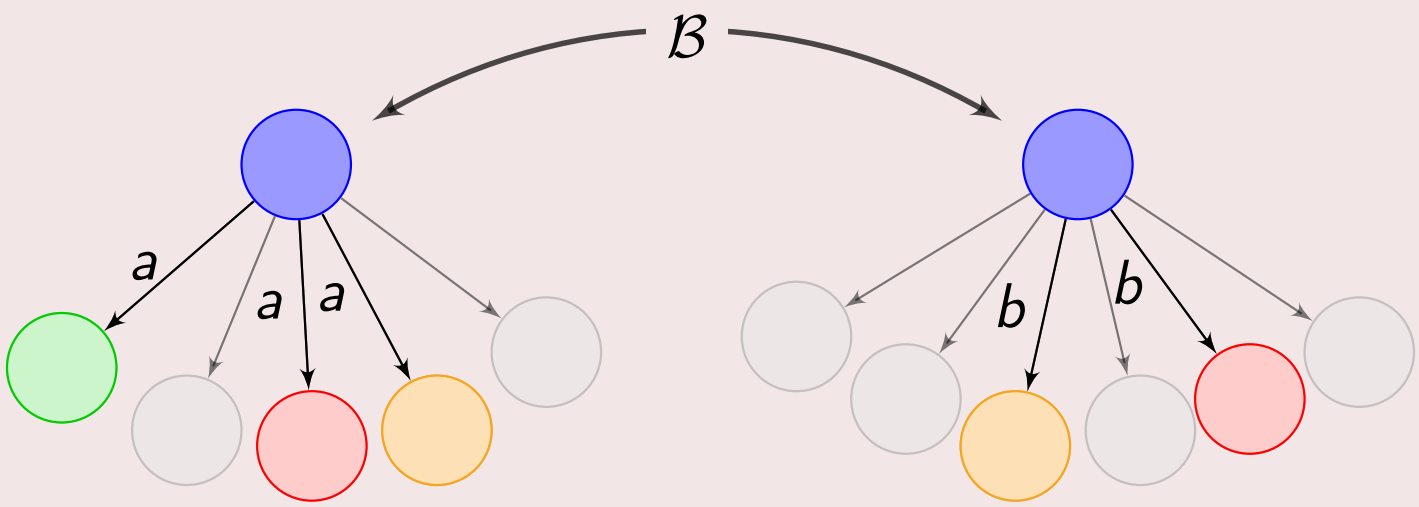

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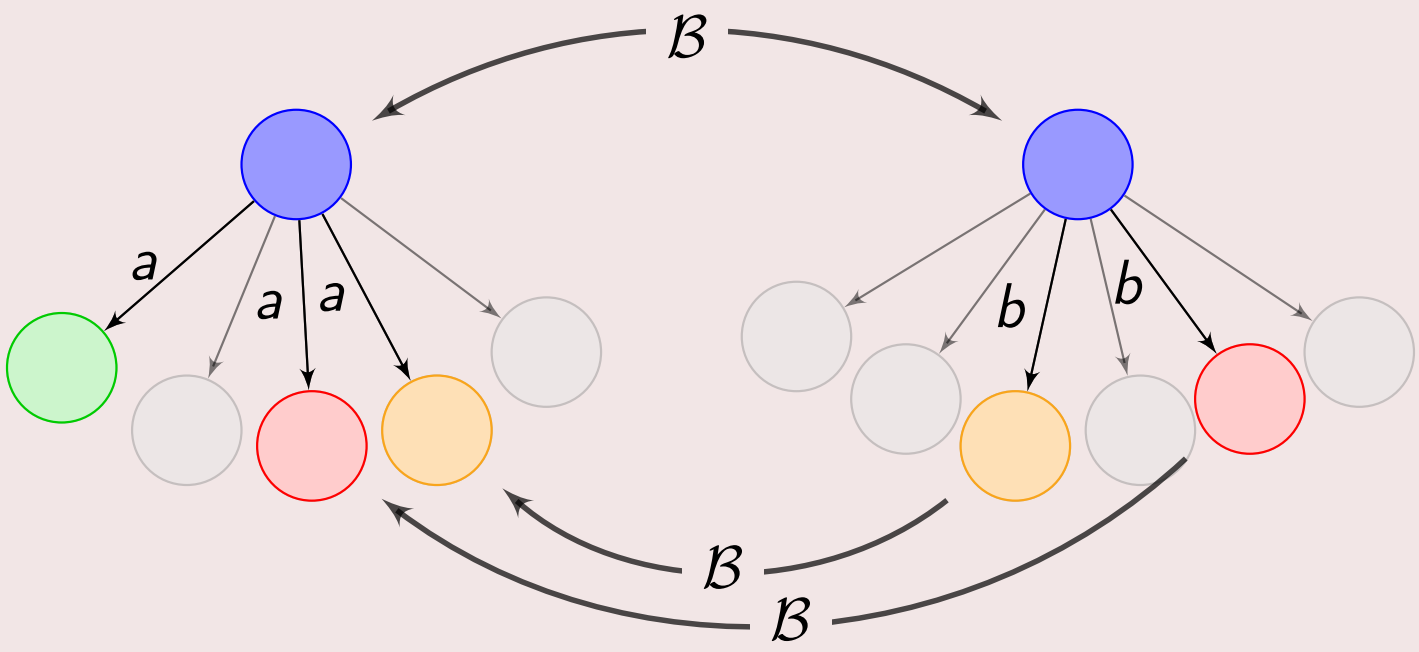

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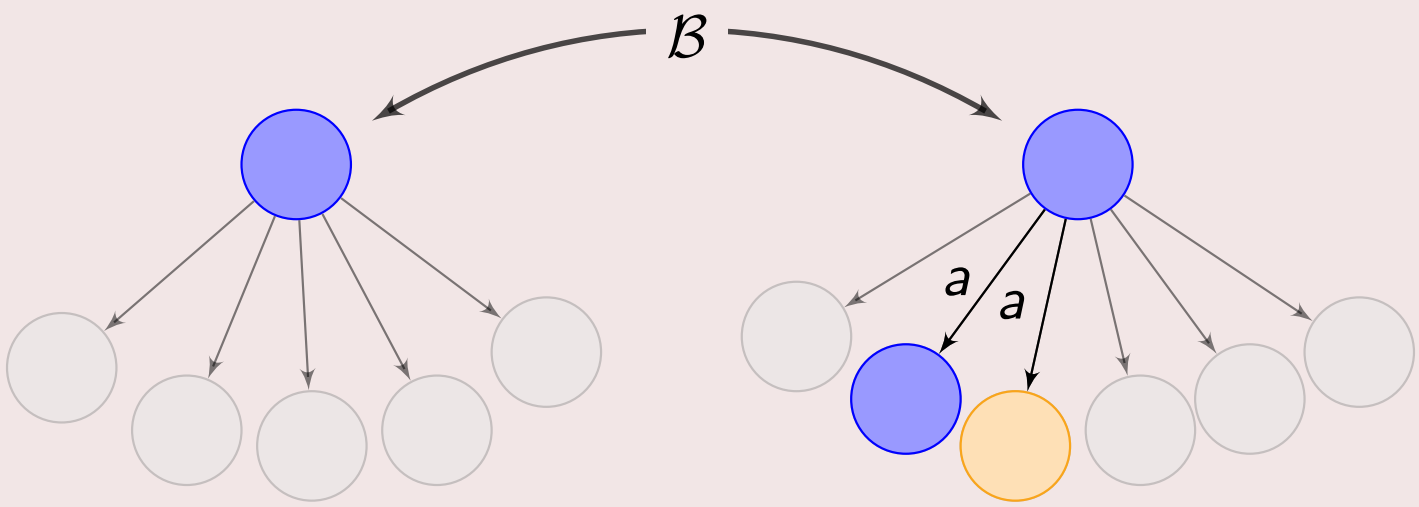

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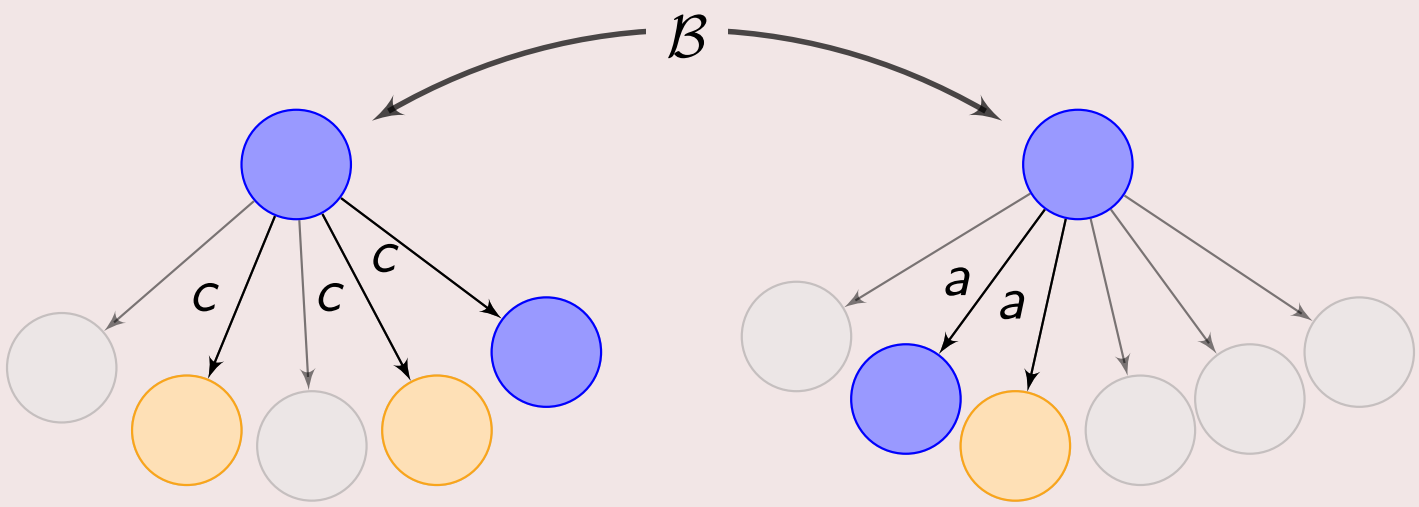

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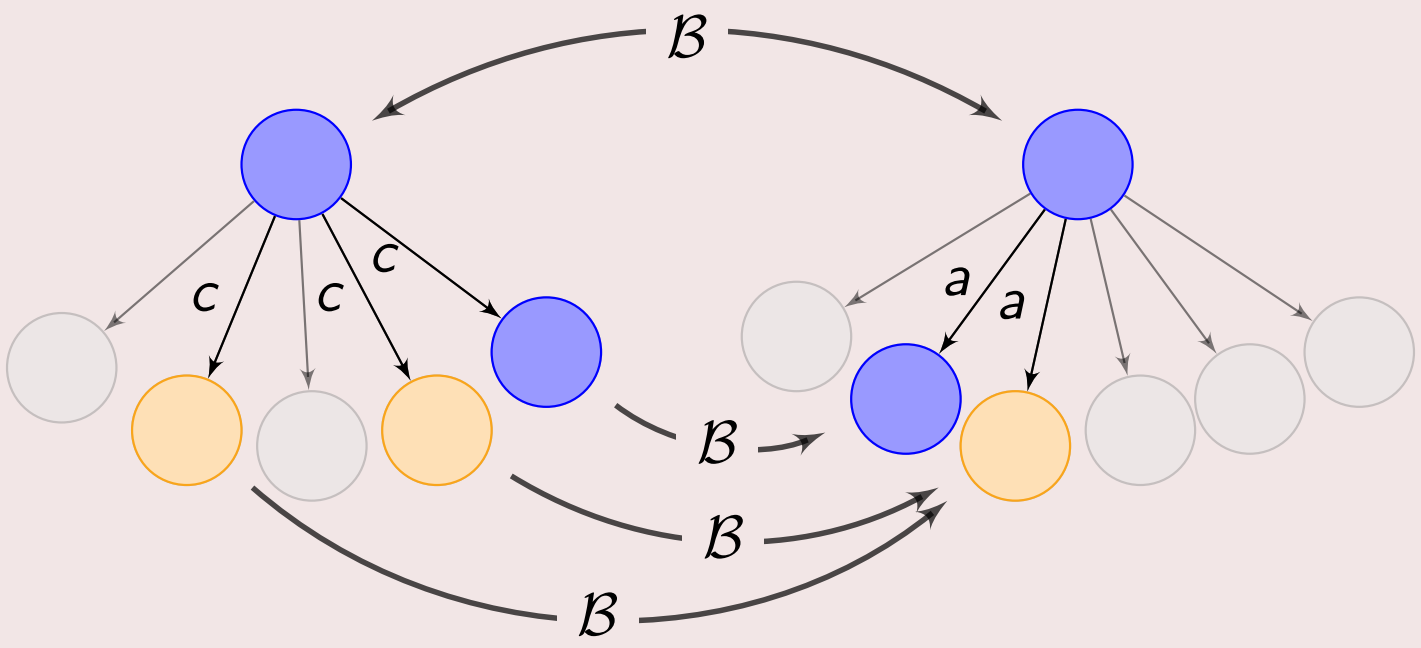

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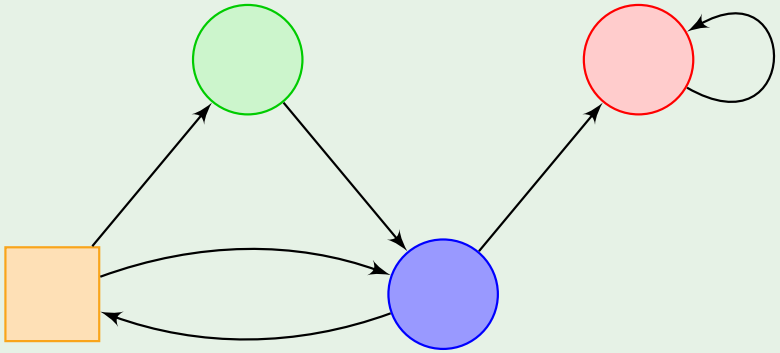
Theorem ([AHKV98])

(The initial states of) two concurrent game structures (with finitely many moves) satisfy the same ATL formulas if, and only if, they are alternating-bisimilar.

[AHKV98] Alur, Henzinger, Kupferman, Vardi. Alternating refinement relations. CONCUR, 1998.

ATL with strategy contexts [BDLM09,DLM10]

Example



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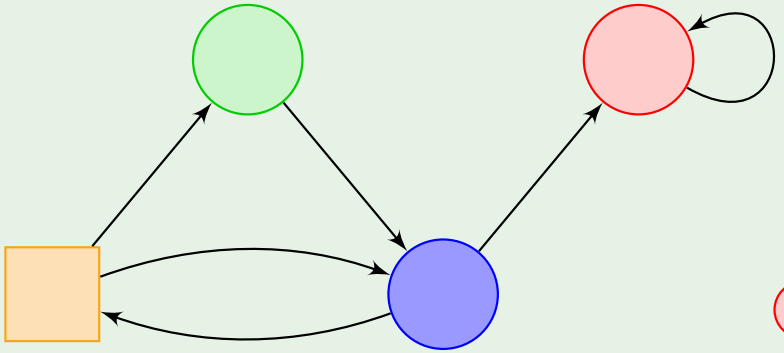

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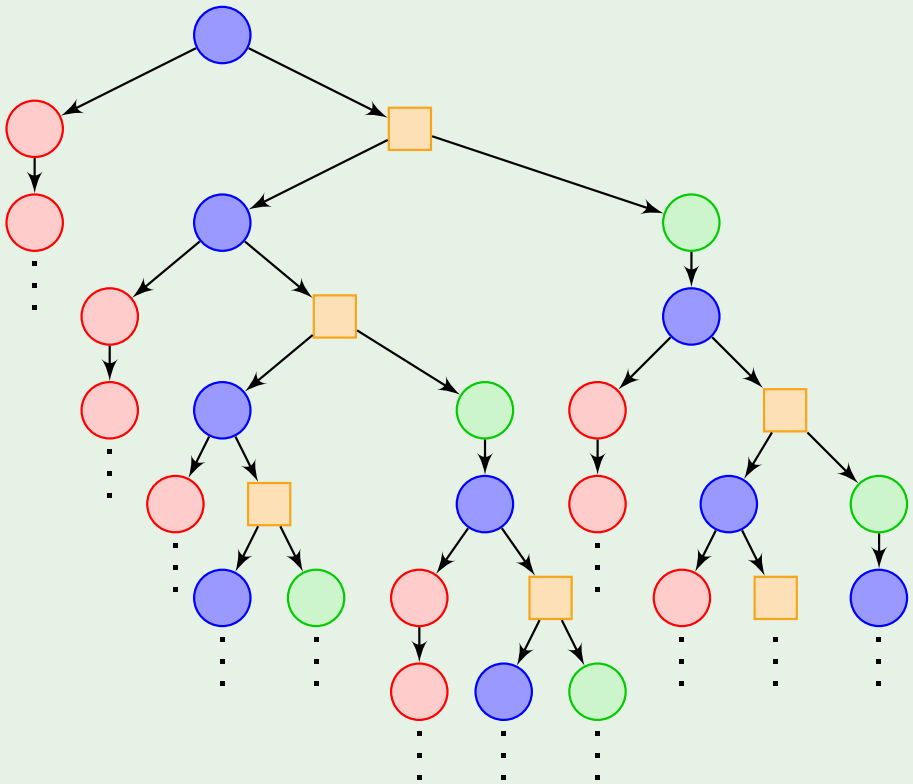

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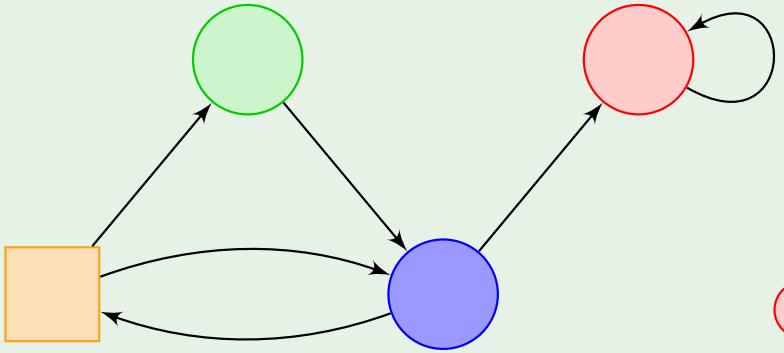

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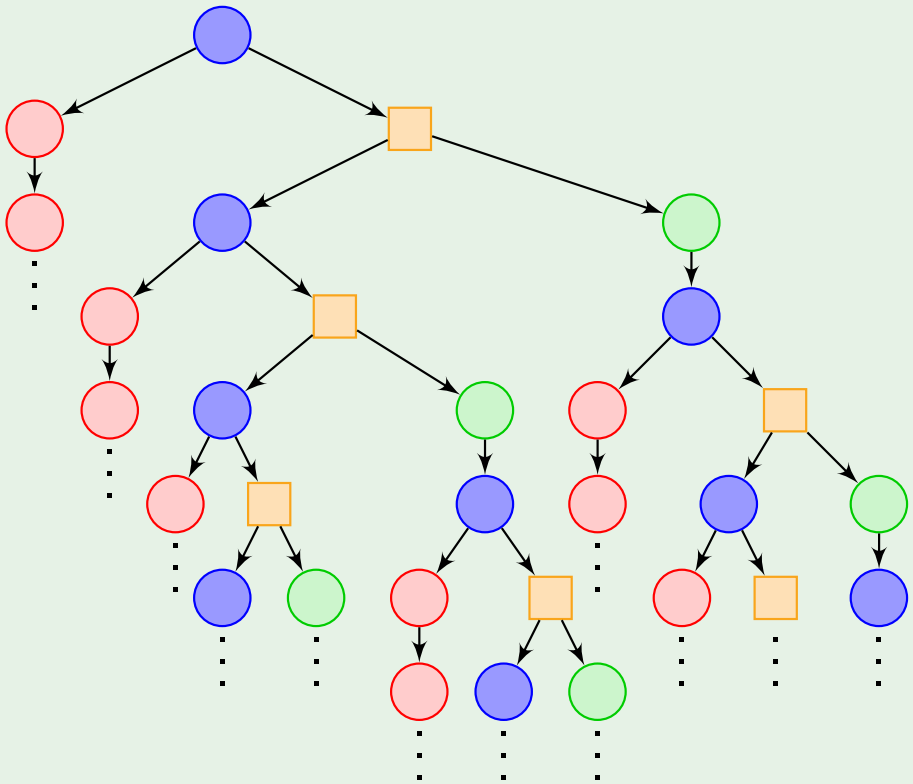

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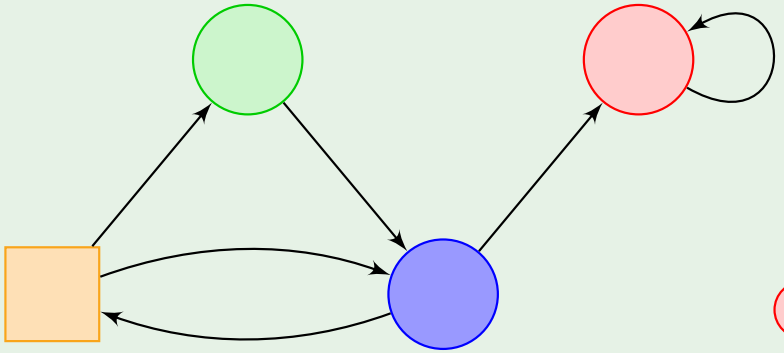

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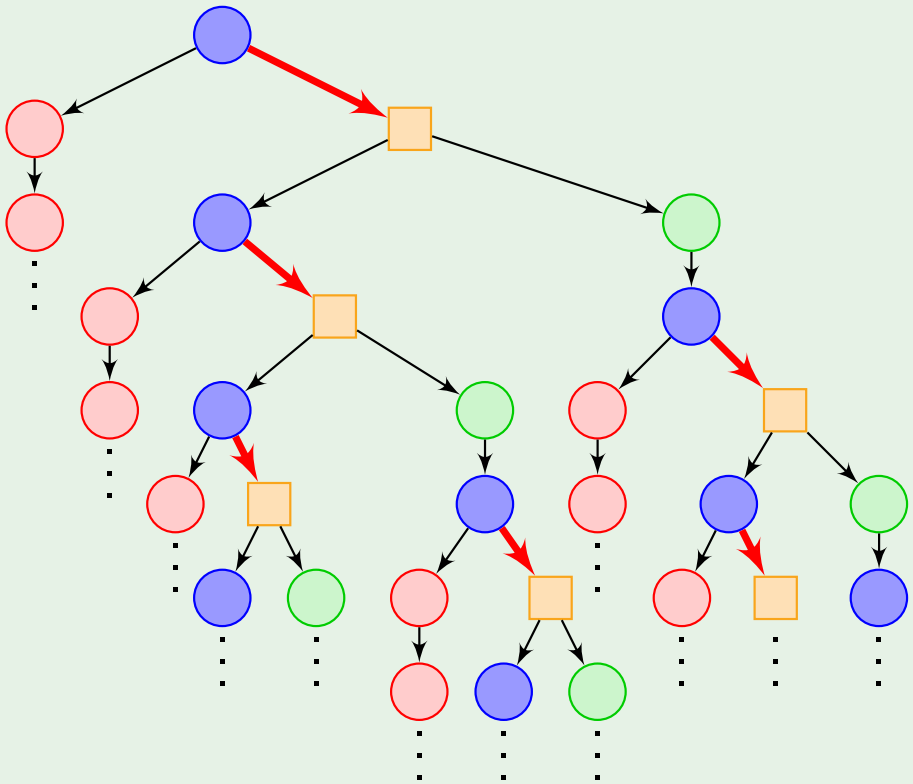

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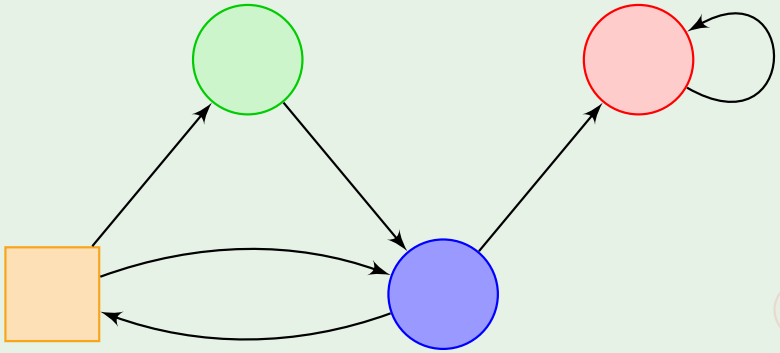

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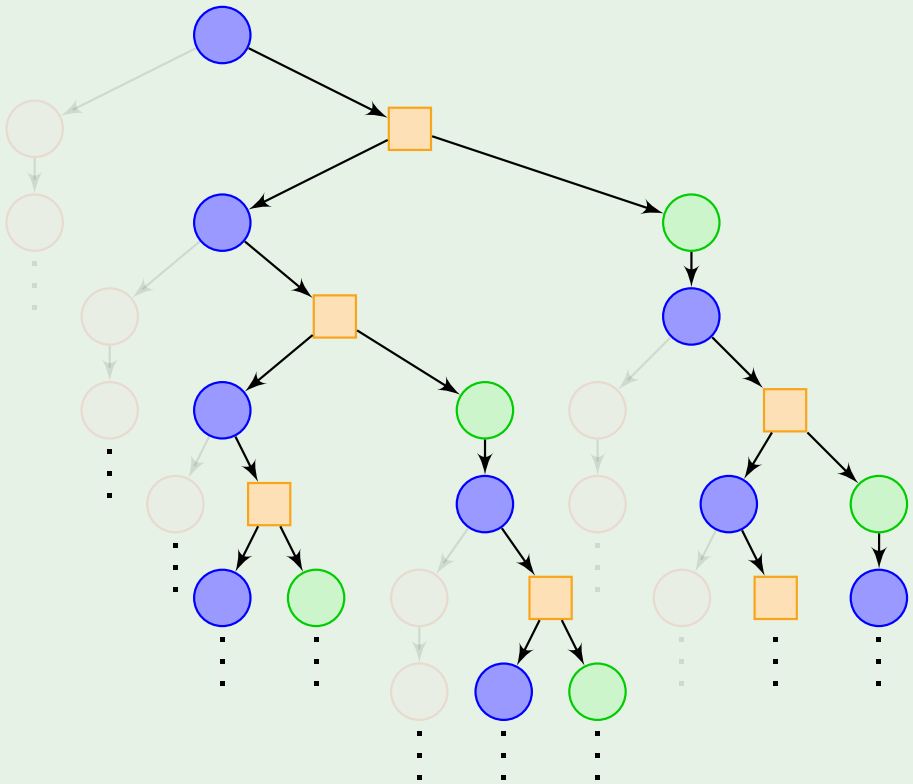

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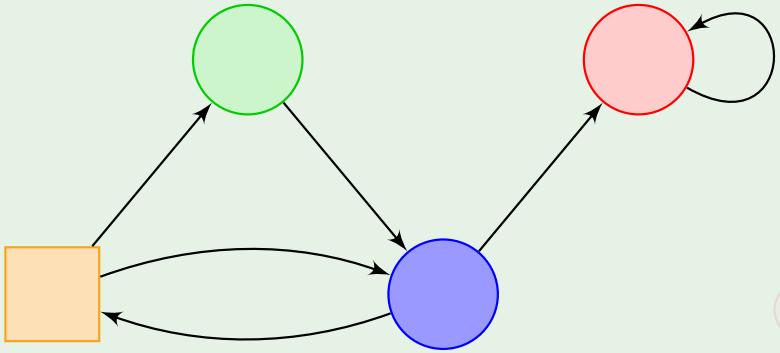

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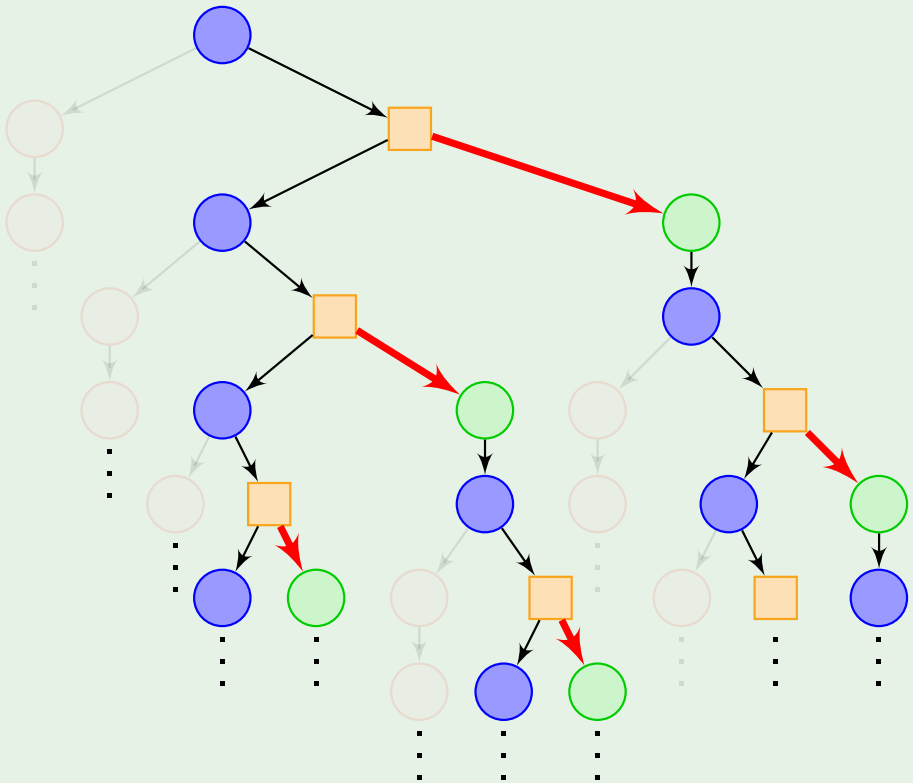

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- $[\cdot A\cdot]$ is dual to $\langle\cdot A\cdot\rangle$:

$$[\cdot A\cdot]\,\varphi \equiv \neg\,\langle\cdot A\cdot\rangle\,\neg\,\varphi$$

Semantics of $\langle\cdot A\cdot\rangle\,\varphi$

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Semantics of ATL strategy quantifier:

$$\mathcal{G}, \bigcirc \models \langle\langle A \rangle\rangle\, \varphi \quad \Leftrightarrow \quad \exists \sigma_A.\ \forall \pi \in \mathsf{Out}(\bigcirc, \sigma_A).\ \pi \models \varphi$$

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- formulas evaluated in a context (a strategy for some coalition);
- context initially empty;
- newly selected strategies added to the context:

$$\sigma_A \circ \sigma_B : \quad \begin{array}{ll} a \mapsto \sigma_A(a) & \text{if } a \in A \setminus B \\ b \mapsto \sigma_B(b) & \text{if } b \in B \setminus A \\ c \mapsto \sigma_A(c) & \text{if } c \in B \cap A \end{array}$$

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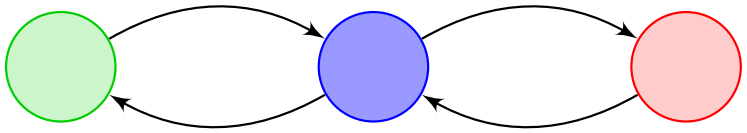

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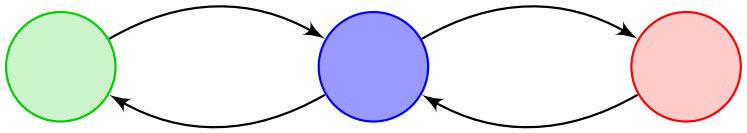

$\rightsquigarrow$ memory needed!

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- existence of a correct controller for the elevator:

$$\langle\cdot\mathrm{ctrl}\cdot\rangle\ \mathbf{G}\left[\begin{array}{l} \bigwedge_{i,j} \langle\cdot\mathrm{user}\cdot\rangle_0\ \mathbf{F}\left(\mathrm{op}_i \wedge \mathbf{F}\ \mathrm{op}_j\right) \\ \wedge \\ \bigwedge_{i\in\{0,1,2\}} \mathrm{op}_i \Rightarrow \mathrm{serv}_i) \end{array}\right]$$

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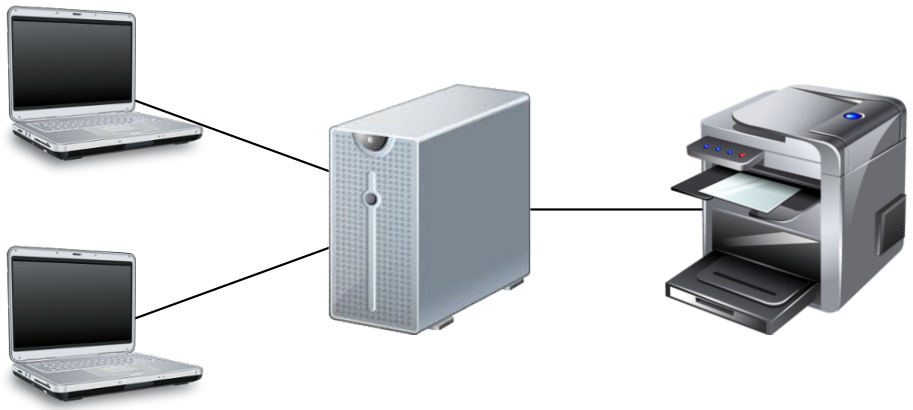
- existence of a correct controller for the elevator:
- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server} \cdot\rangle\ \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \mathrm{Clients}} \langle\cdot c \cdot\rangle\ \mathbf{F}\ \mathrm{access}_c \\ \wedge \\ \neg \bigwedge_{c \neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$



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- Existence of dominating strategy:

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More expressiveness results

Theorem

ATL_{sc} is strictly more expressive than ATL.

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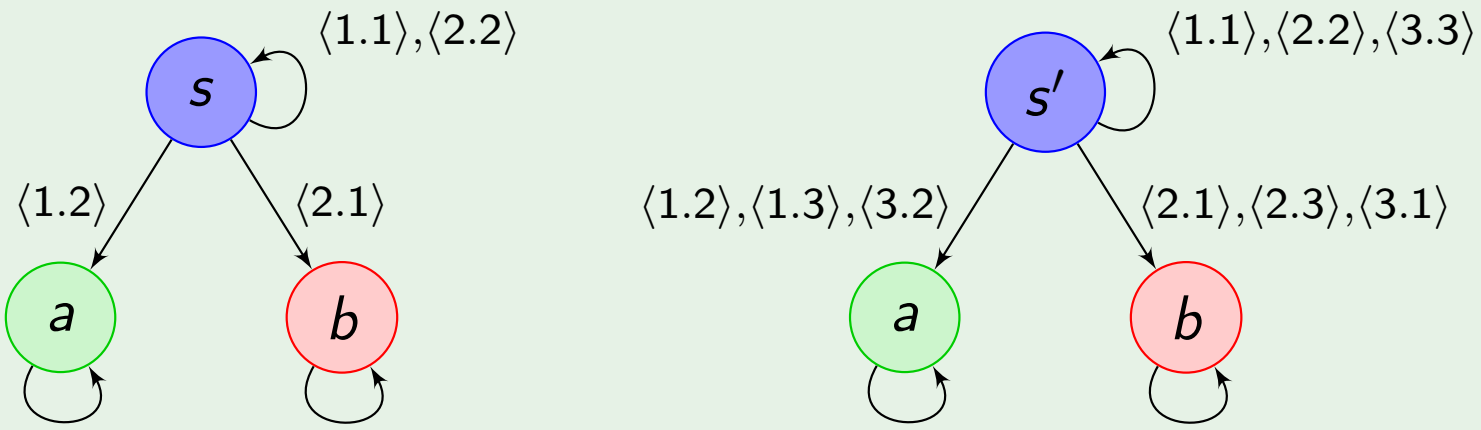

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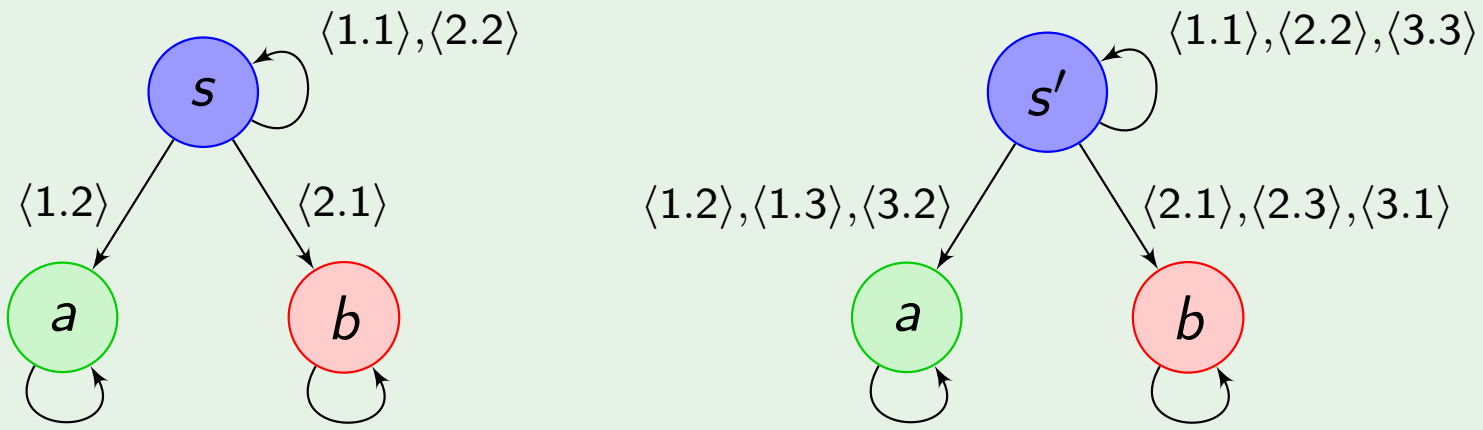

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- for full context, insert $\langle\cdot\varnothing\cdot\rangle$ between temporal modalities:

$$\mathbf{G}\,\mathbf{F}\,\varphi \equiv \langle\cdot\varnothing\cdot\rangle\,\mathbf{G}\,\langle\cdot\varnothing\cdot\rangle\,\mathbf{F}\,\varphi$$

Related works – Strategy logic [CHP07,MMV10]

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Explicit manipulation of strategies

- first-order quantification over strategies;

$$\exists\sigma.\ \varphi(\sigma) \qquad \textit{there is a strategy } \sigma \textit{ such that } \varphi \textit{ holds}$$

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$$\mathsf{assign}(A \mapsto \sigma)\varphi \qquad \textit{if } A \textit{ plays } \sigma \textit{ then } \varphi \textit{ holds}$$

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Some other related approaches

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(Basic) strategy interaction logic [WSH15]

- ATL augmented with strategy interaction quantifiers:

$$\langle\langle A \rangle\rangle \, (\, \langle\langle +B \rangle\rangle \, \varphi_B \, \wedge \, \langle\langle +C \rangle\rangle \, \varphi_c)$$

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Proposition ([WSH15])

Model checking BSIL is PSPACE*-complete.*

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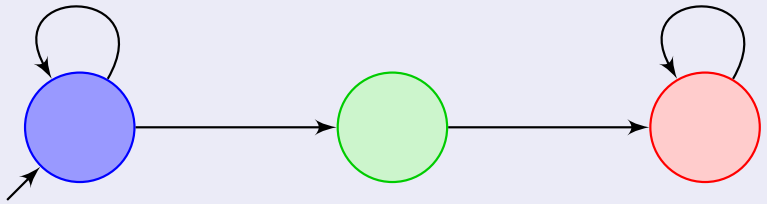
- $\langle\langle A \rangle\rangle\, \mathbf{G}(\, \langle\langle A \rangle\rangle\, \mathbf{X}$ ◯)



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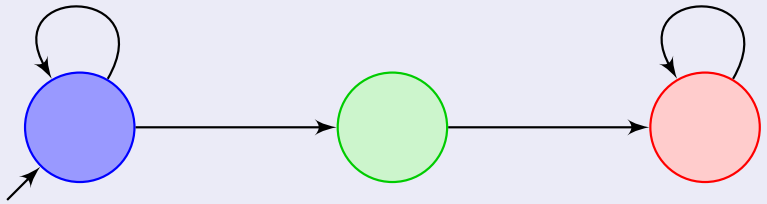

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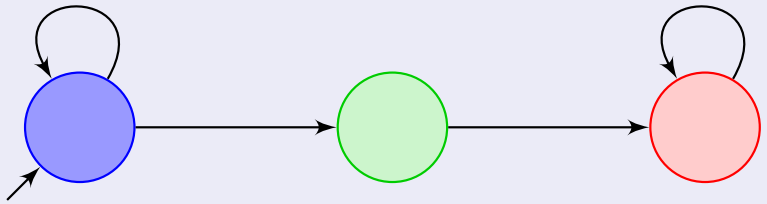

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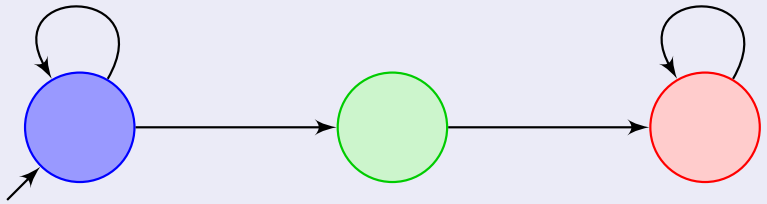

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Model checking ATL with irrevocable memoryless strategies is PSPACE*-complete.*

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For memoryless explicit strategy contexts, model checking ATLES is PTIME*-complete.*

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Model checking Stochastic Game Logic with memoryless randomized strategies is decidable in EXPSPACE.

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