Quantified-CTL model checking

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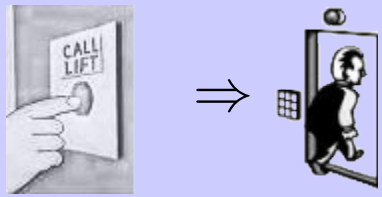
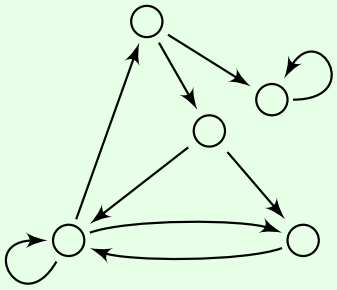
Model checking

system:

property:





model-checking algorithm

$\mathbf{G}(\texttt{request} \Rightarrow \mathbf{F}\,\texttt{grant})$

yes/no

Computation-Tree Logic (CTL)

Definition

$$\text{CTL} \ni \varphi ::= \bigcirc \mid \varphi \vee \varphi \mid \neg\varphi \mid \mathbf{E}\,\mathbf{X}\,\varphi \mid \mathbf{E}\,\mathbf{G}\,\varphi \mid \mathbf{E}\varphi\ \mathbf{U}\ \varphi$$

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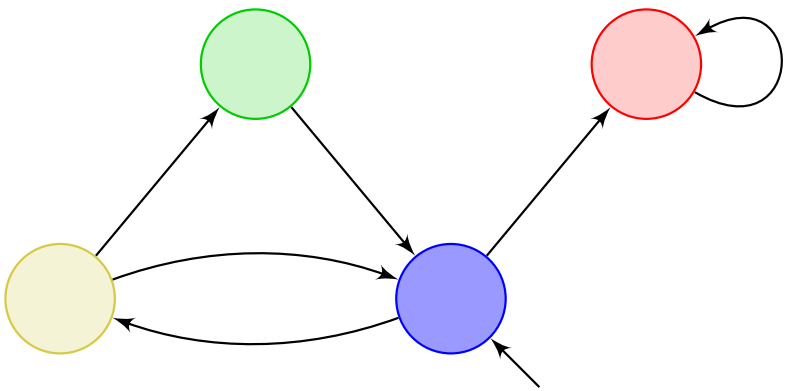

✓ $\mathbf{E}(\texttt{true}\ \mathbf{U}\ \bigcirc) \equiv \mathbf{E}\,\mathbf{F}\,\bigcirc$

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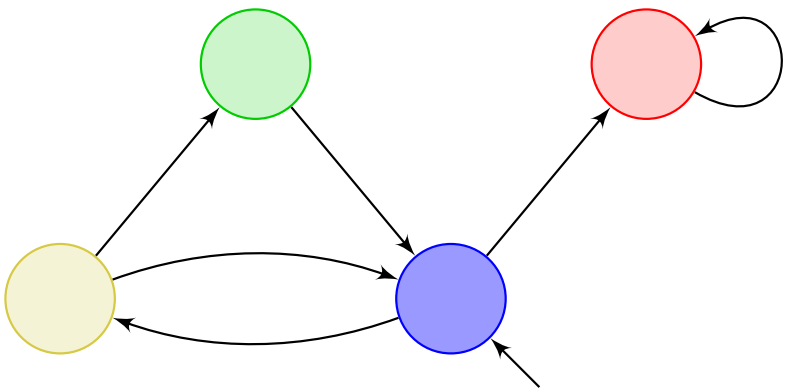

- ✓ $\mathbf{E}(\texttt{true}\ \mathbf{U}\ \bigcirc) \equiv \mathbf{E}\,\mathbf{F}\,\bigcirc$
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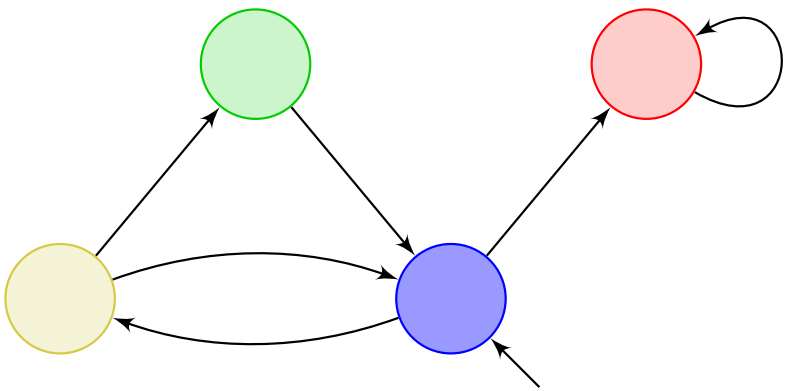

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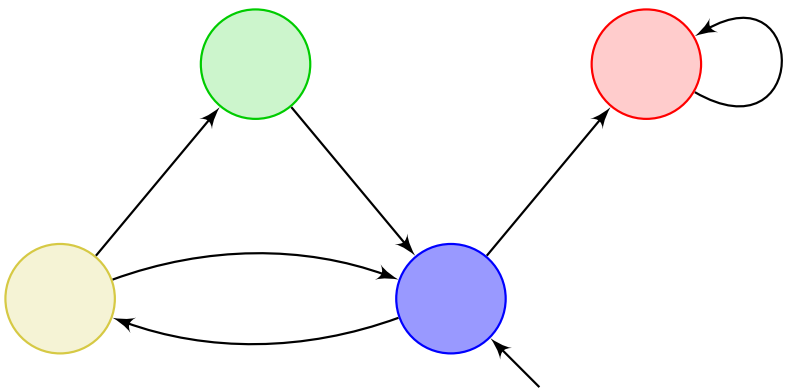

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- ✓ $\mathbf{EG}\,\neg\bigcirc \equiv \neg(\mathbf{AF}\,\neg\bigcirc)$
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- ✓ $\mathbf{EG}(\neg\bigcirc \wedge \mathbf{EF}\,\bigcirc)$

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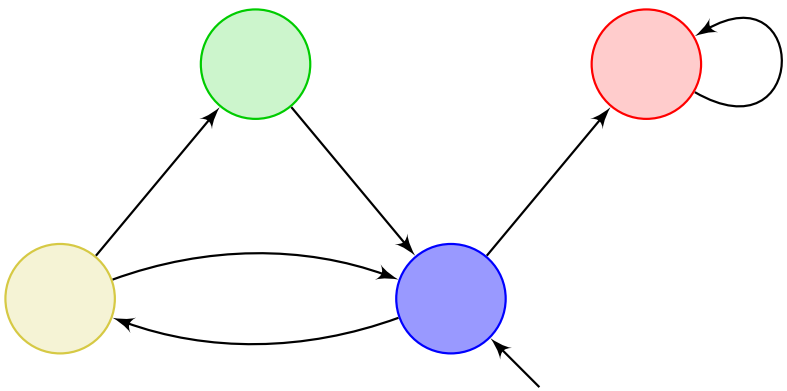

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Theorem

CTL model checking is PTIME*-complete.*

Quantified-CTL

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$$\text{QCTL} \ni \varphi ::= \bigcirc \mid \varphi \vee \varphi \mid \neg \varphi \mid \mathbf{E}\,\mathbf{X}\,\varphi \mid \mathbf{E}\,\mathbf{G}\,\varphi \mid \mathbf{E}\varphi\ \mathbf{U}\ \varphi \mid \exists \bigcirc.\ \varphi$$

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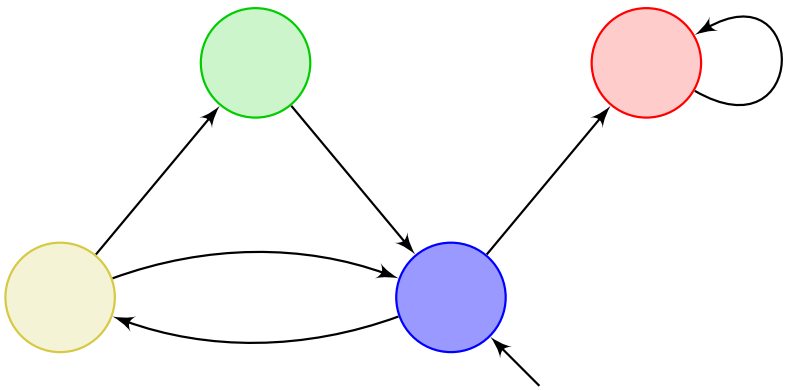

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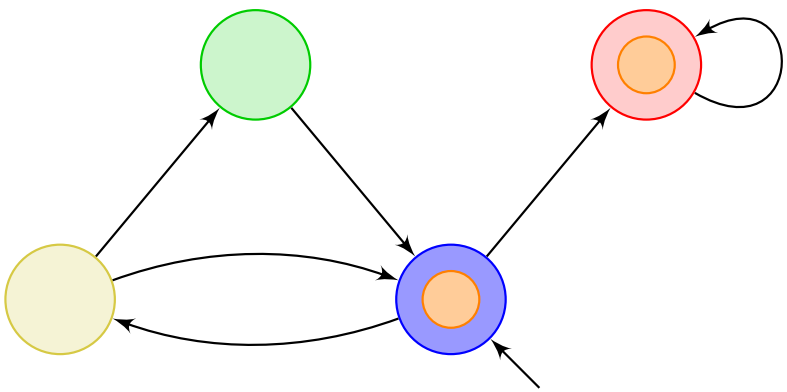

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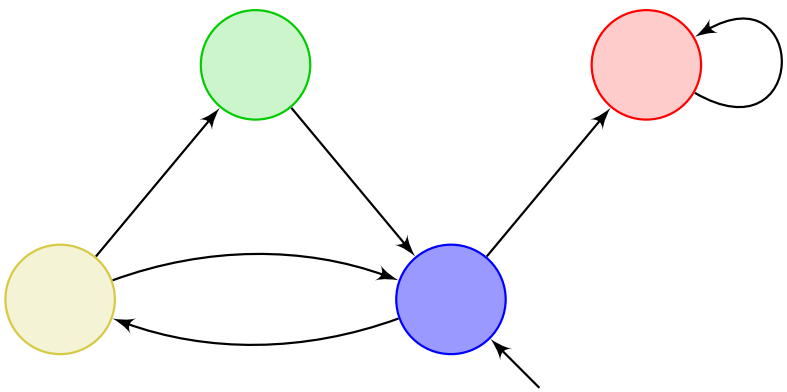

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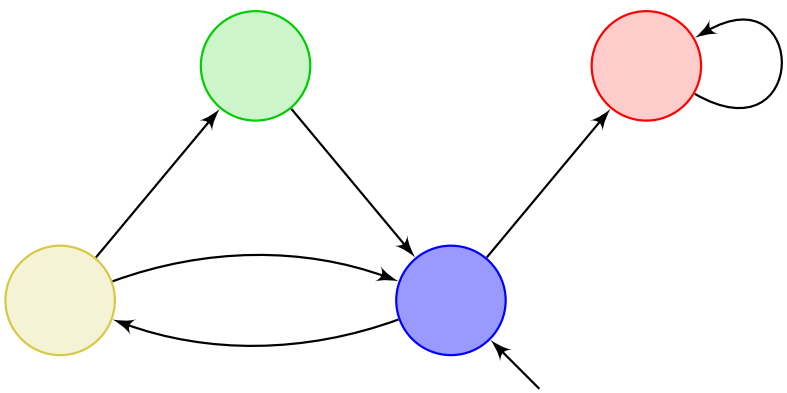

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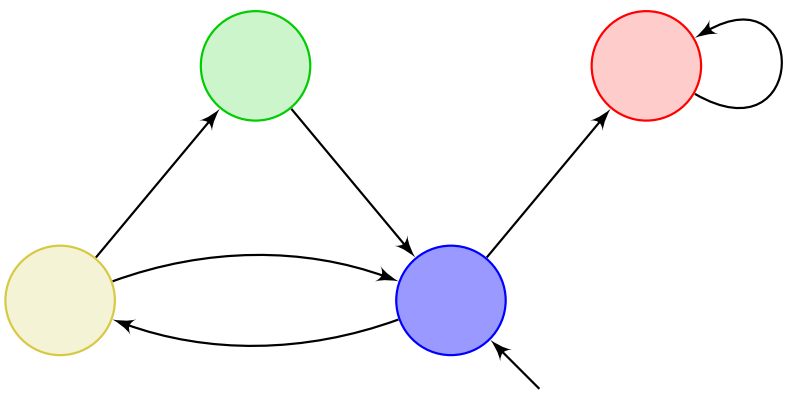

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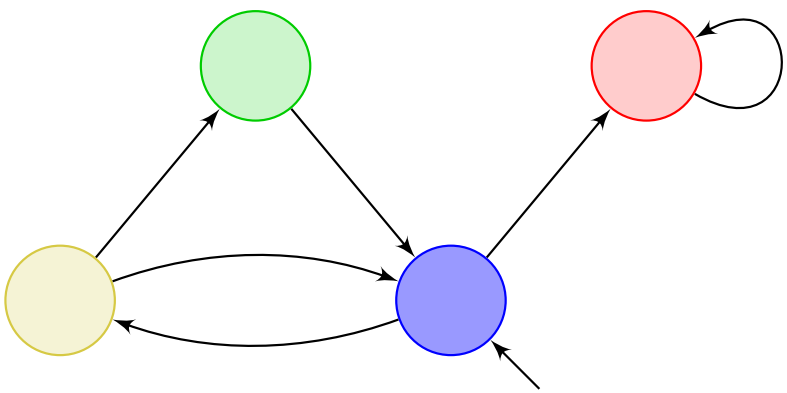

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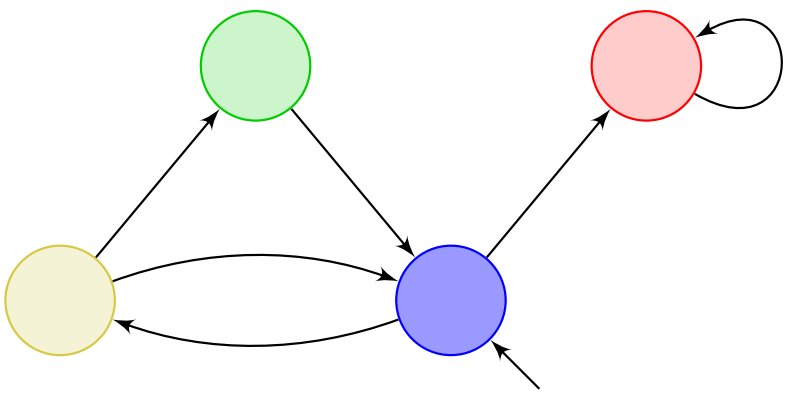

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× in the *structure* semantics

✓ in the *tree* semantics

●-equivalent structures

Definition

Given a Kripke structure $\mathcal{S} = \langle W, R, \ell \rangle$ with $\ell \colon W \to 2^{\mathsf{AP}}$ and an atomic proposition ●, we define $\mathcal{S}_{\bullet}$ as the Kripke structure $\langle W, R, \ell' \rangle$ with $\ell'(w) = \ell(w) \setminus \{\bullet\}$.

Definition

Two Kripke structures $\mathcal{S} = \langle W, R, \ell \rangle$ and $\mathcal{S}' = \langle W', R', \ell' \rangle$ are ●-equivalent if $\mathcal{S}_{\bullet} = \mathcal{S}'_{\bullet}$.

Example

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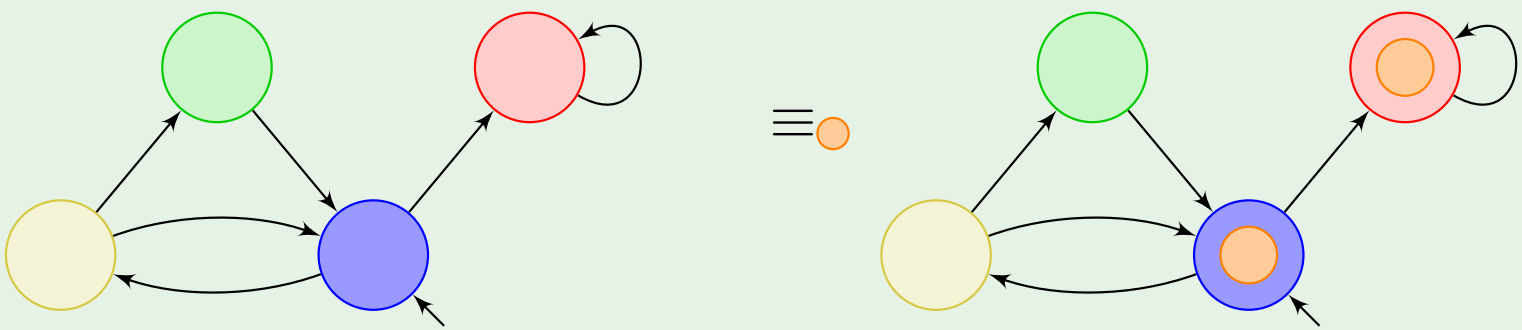

Semantics of QCTL

Definition (structure semantics)

The structure semantics of QCTL extends that of CTL with

$$\mathcal{S}, q \models_s \exists \bigcirc. \varphi \quad \Leftrightarrow \quad \mathcal{S}', q \models_s \varphi$$

for some $\mathcal{S}' \equiv_{\bigcirc} \mathcal{S}$.

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Related work

Linear-time

- studied by Sistla, Vardi, Wolper (1987);
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Branching-time

- Kupferman (1995):
  - model-checking is NP-c. (resp. EXPTIME-c.) with quantifier depth 1 for structure (resp. tree) semantics;
- Kupferman, Madhusudan, Thiagarajan, Vardi (2000):
  - model-checking is 2-EXPTIME-complete with quantifier depth 2 (tree semantics);
- French (2003):
  - expressiveness: QCTL and QCTL* are equally expressive for the tree semantics.

Alternating-time Temporal Logic (ATL)

Definition

ATL extends CTL with *strategy quantifiers*:

$$\text{ATL} \ni \varphi ::= \bigcirc \mid \varphi \vee \varphi \mid \neg\varphi \mid \langle\langle A \rangle\rangle \mathbf{X}\,\varphi \mid \langle\langle A \rangle\rangle \varphi\ \mathbf{U}\ \varphi \mid \langle\langle A \rangle\rangle \neg(\varphi\ \mathbf{U}\ \varphi)$$

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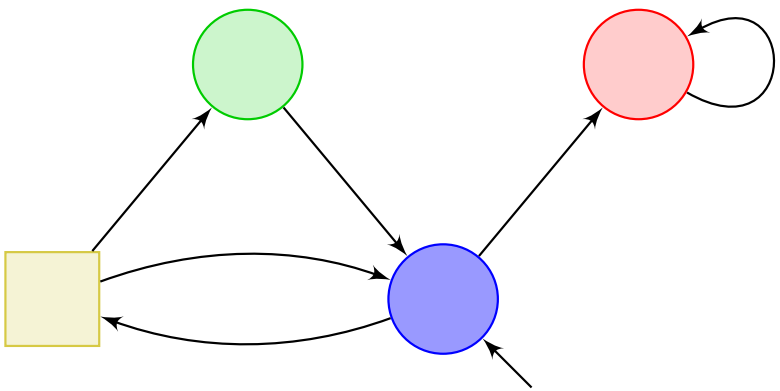

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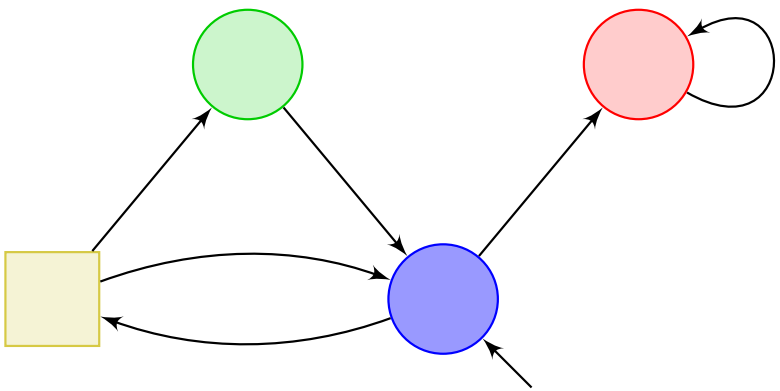

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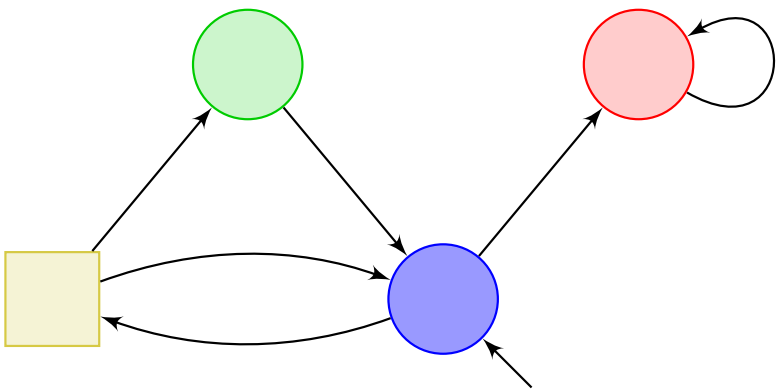

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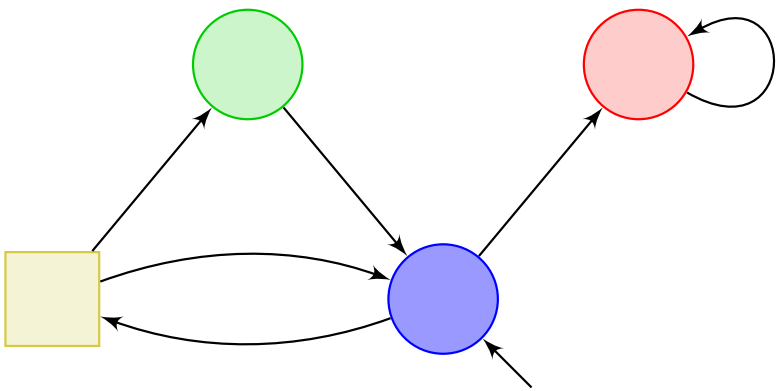

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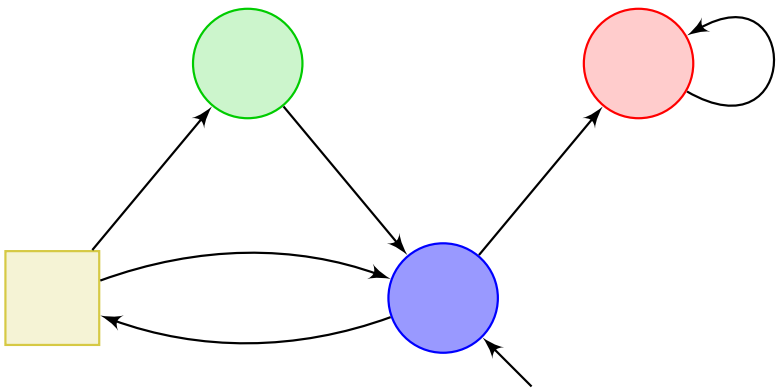

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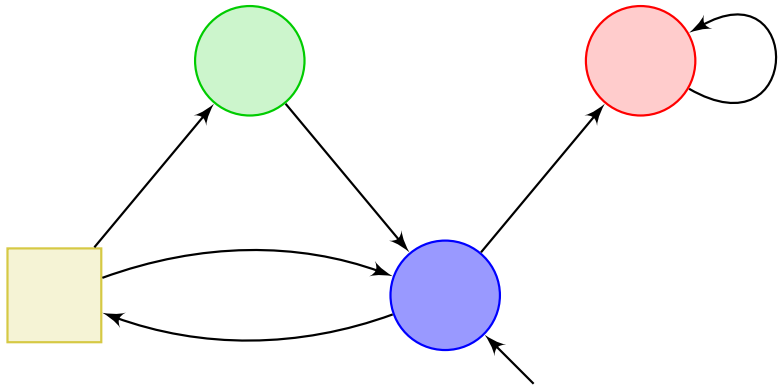
Theorem

ATL model checking is PTIME*-complete.*

ATL with strategy contexts

ATL_{sc} has the same syntax as ATL, but different semantics:



$\langle\langle\bigcirc\rangle\rangle$ **G**($\langle\langle\square\rangle\rangle$ **F** ●)

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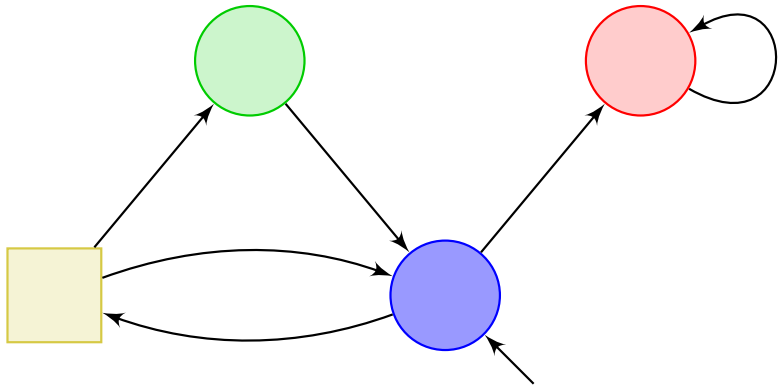

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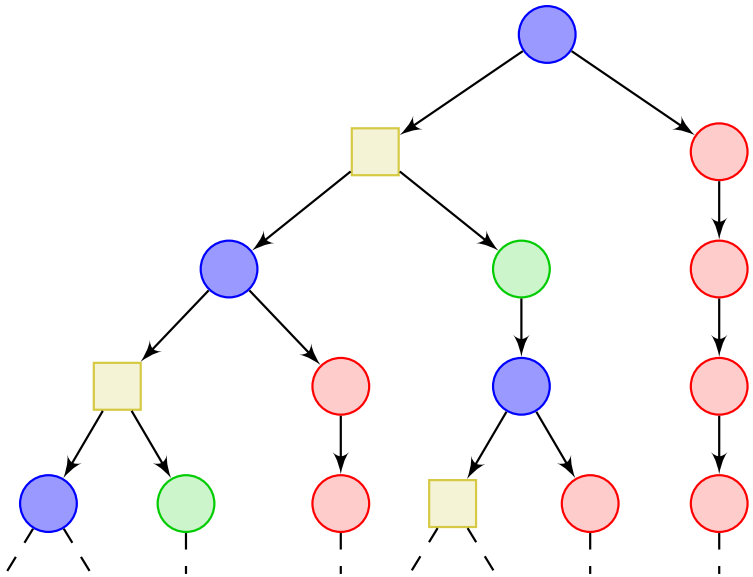

Evaluate the formula on the execution tree:

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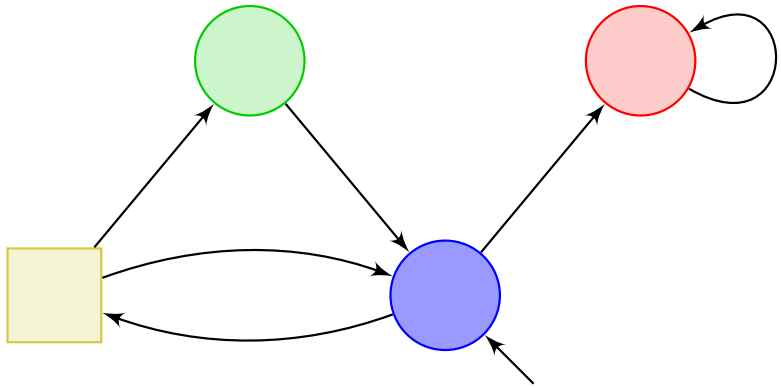

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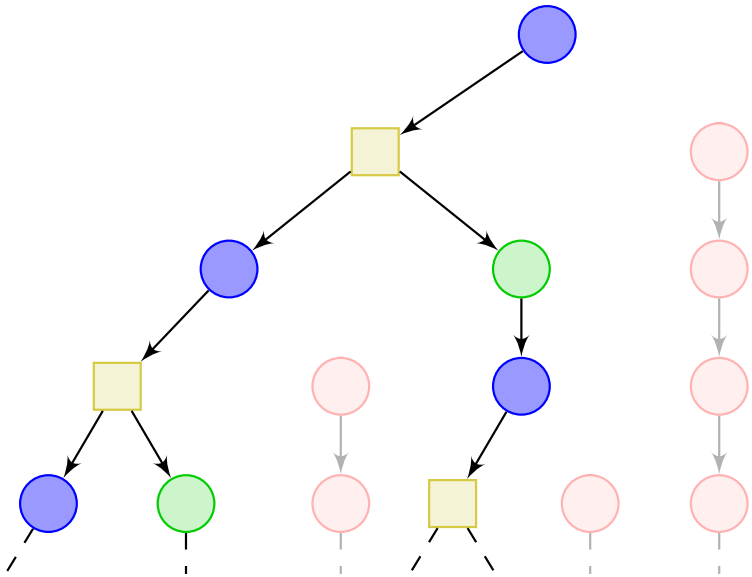

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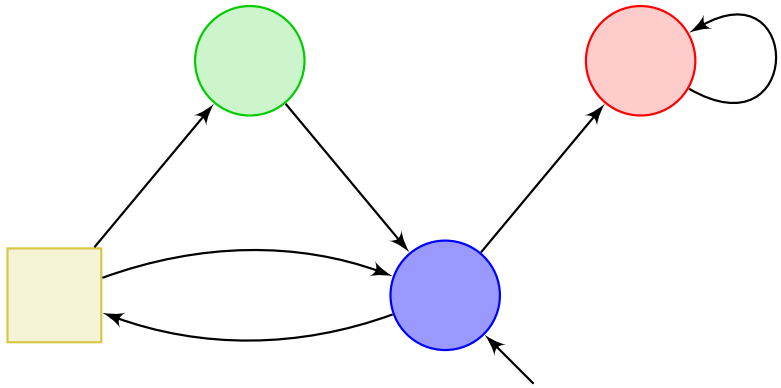

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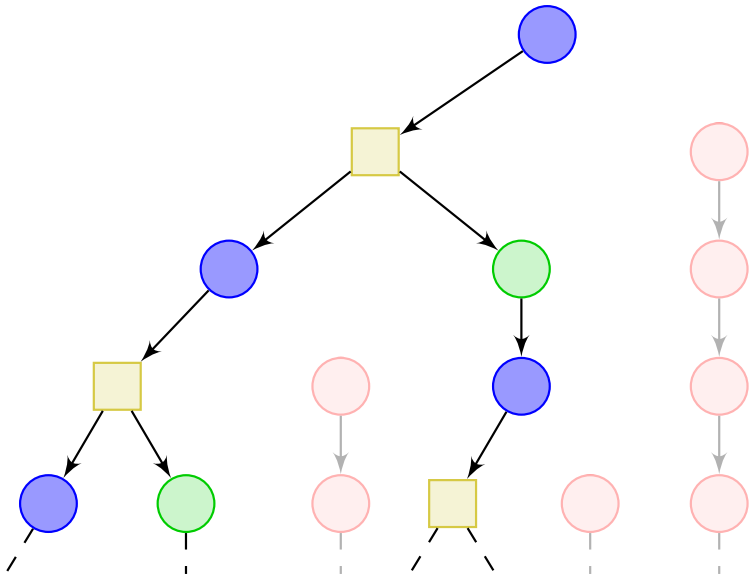

Evaluate the formula on the execution tree:

- apply a strategy of Player $\bigcirc$;
- in the remaining tree, check that Player $\square$ can always enforce a visit to  .

ATL with strategy contexts

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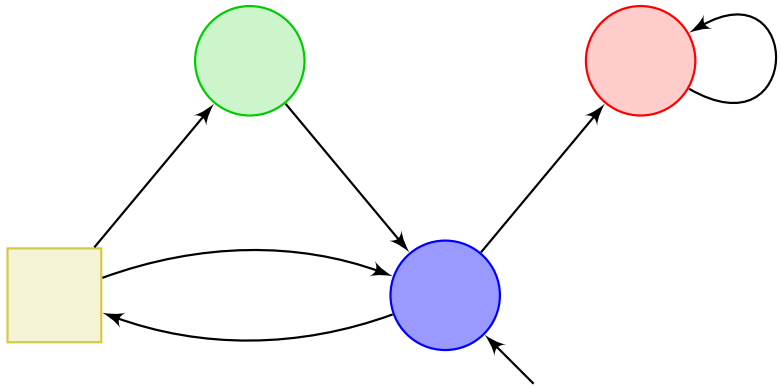

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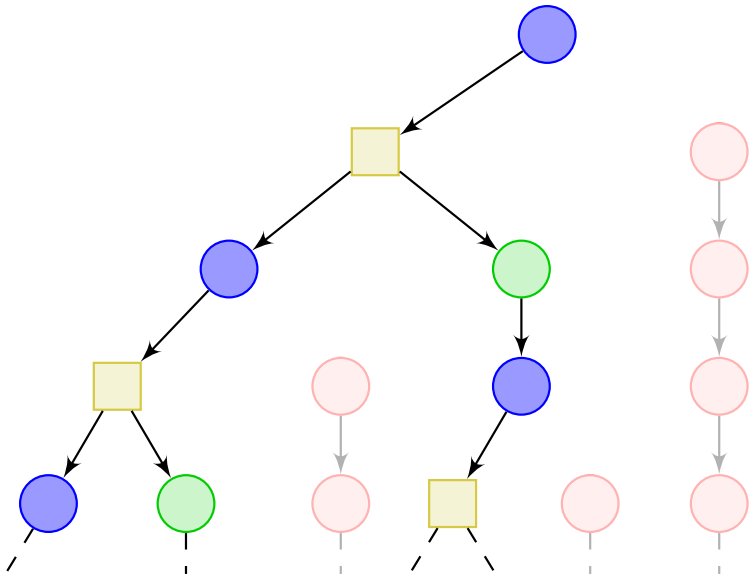


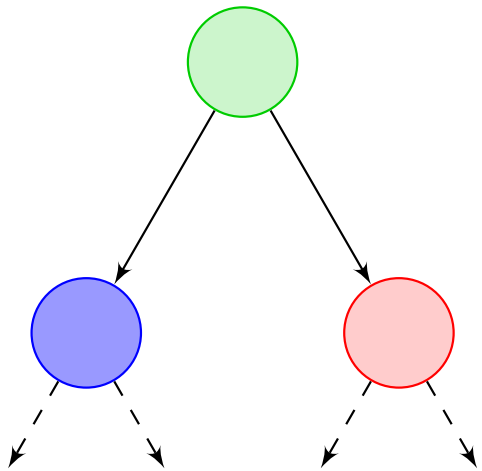
Evaluate the formula on the execution tree:

- apply a strategy of Player $\bigcirc$;
- in the remaining tree, check that Player $\Box$ can always enforce a visit to  .

From ATL_{sc} to QCTL

ATL_{sc} and QCTL are very tightly connected:

strategy = labelling of configurations with the action to be played.



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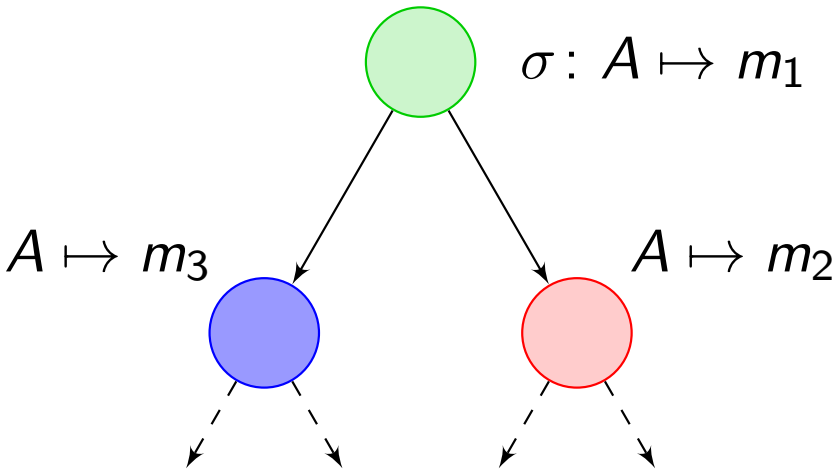

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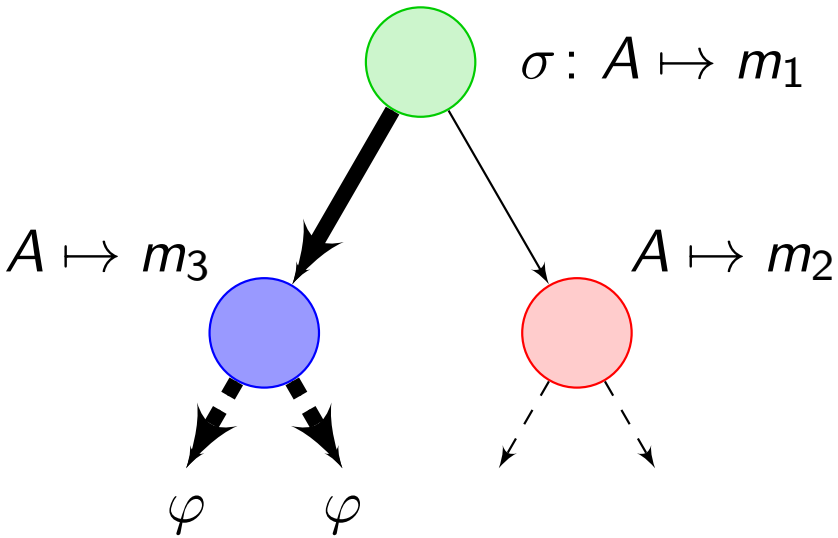

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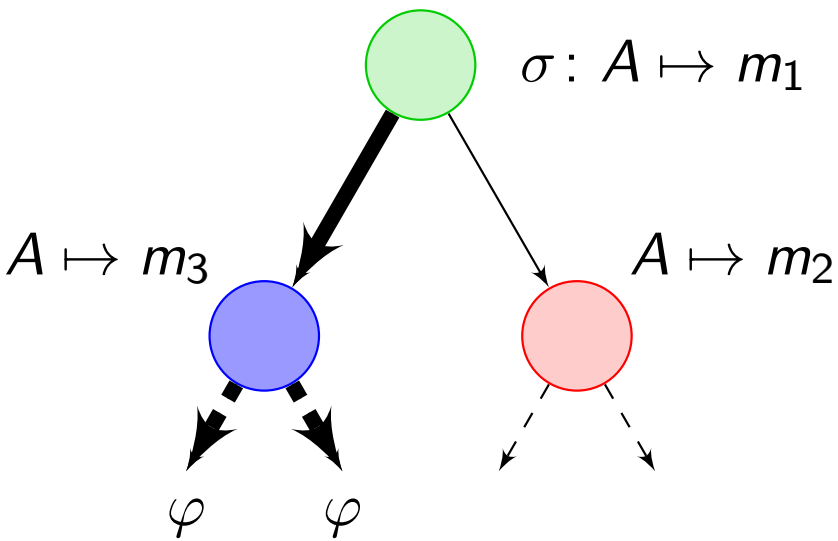

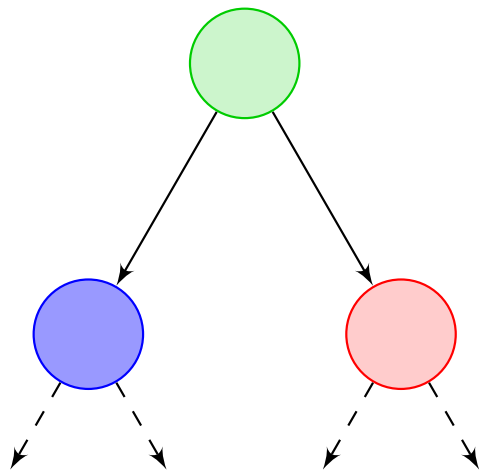

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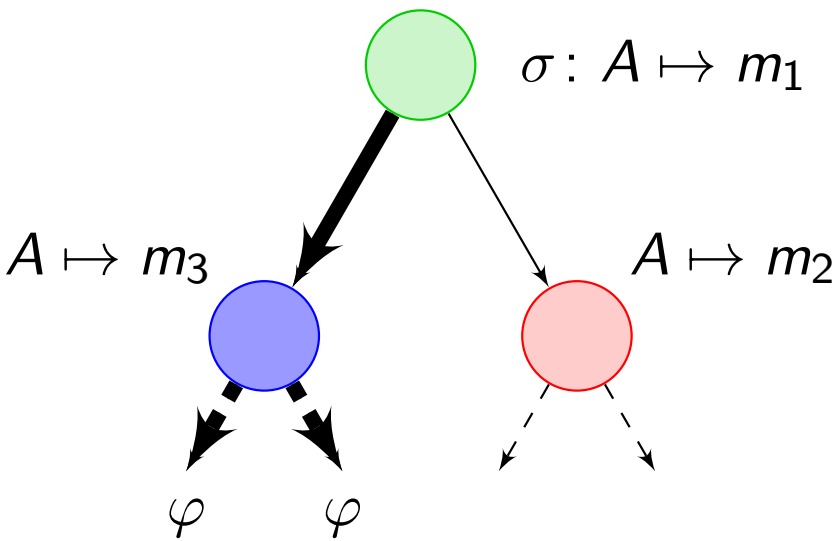

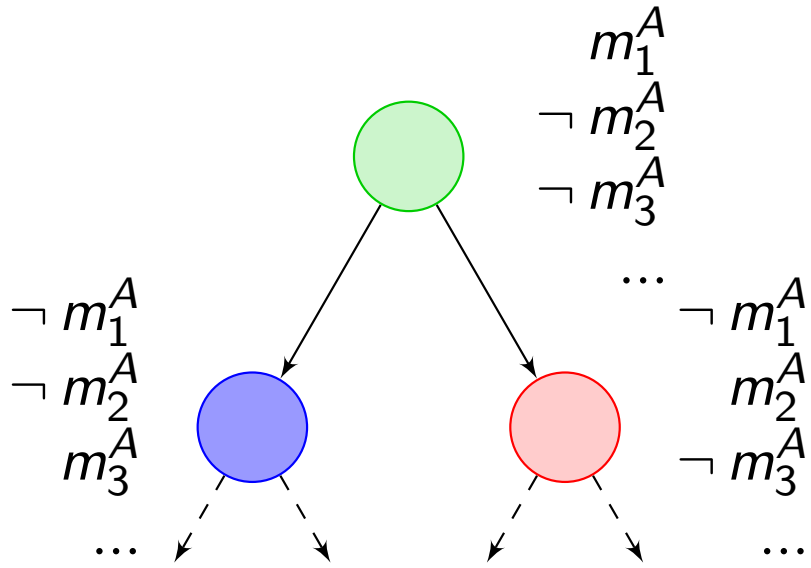

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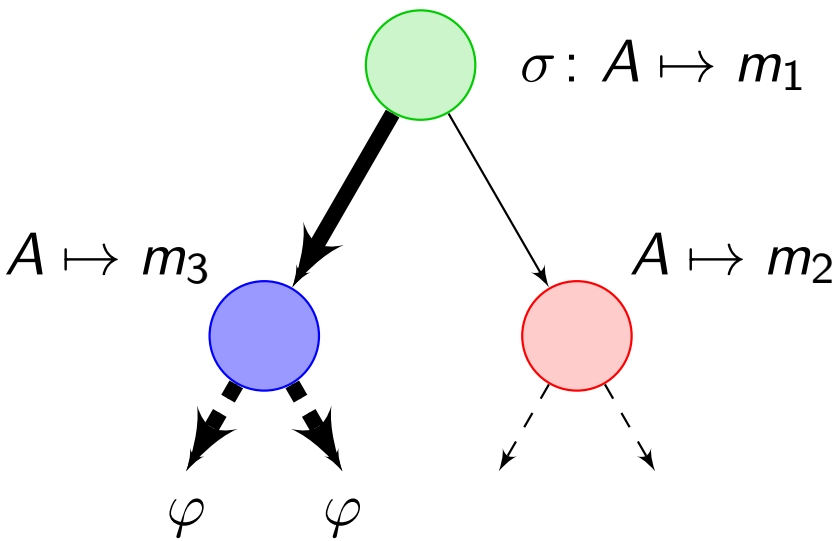

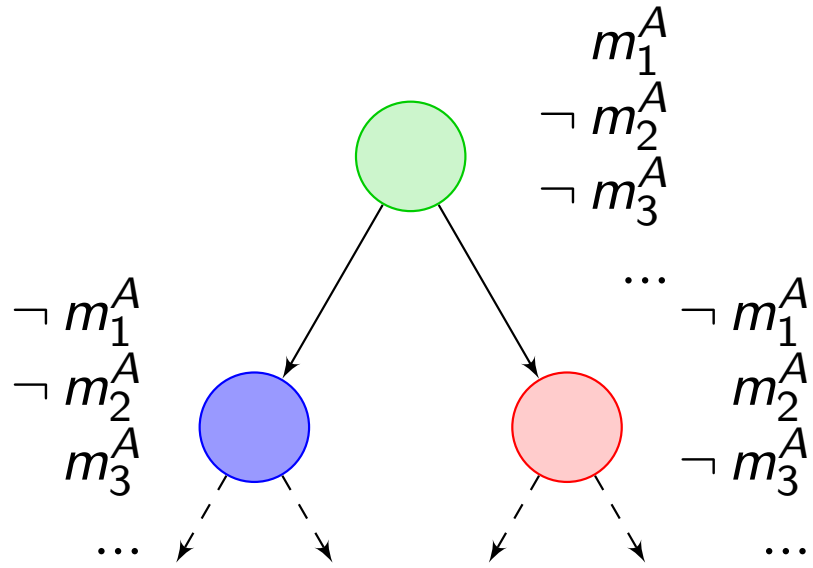

$\langle\langle A \rangle\rangle \varphi$

$\exists m_i^A.\ \mathbf{A}\,\mathbf{G}(\text{one}(m_1^A, ..., m_k^A)) \wedge$
$\mathbf{A}(\text{outcome} \Rightarrow \hat{\varphi})$

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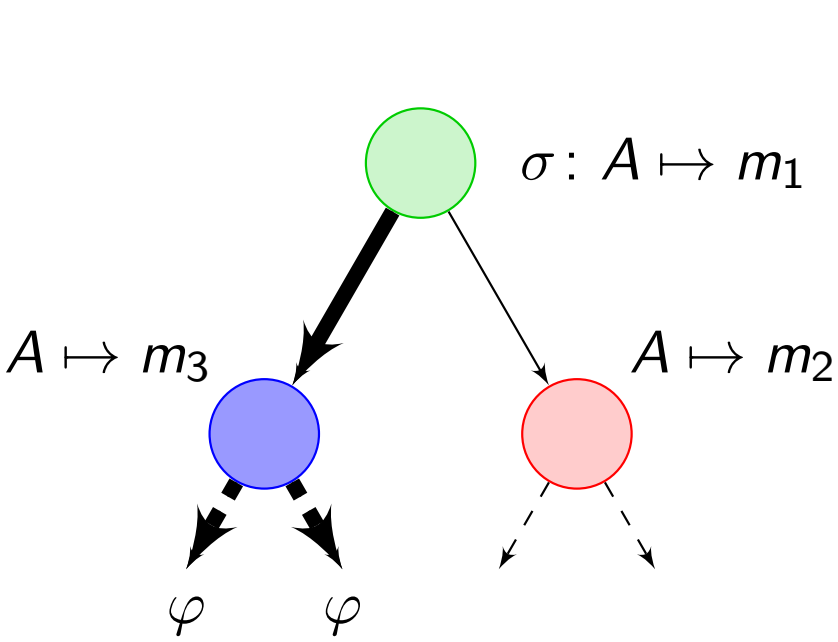

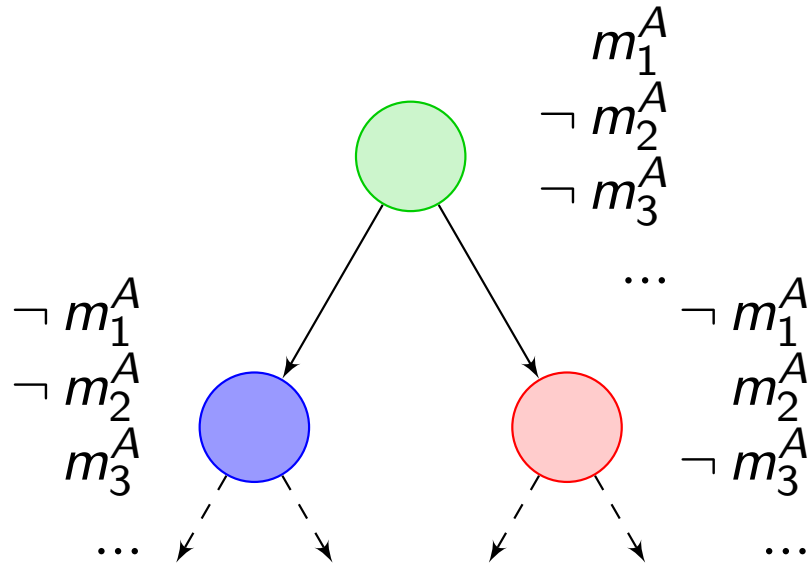

- tree semantics corresponds to ATL_{sc};
- structure semantics represents memoryless-strategy quantification.

Outline of the talk

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Expressiveness of QCTL and QCTL*

Theorem

QCTL and QCTL* are equally expressive.

Proof

QCTL can express μ-calculus, hence CTL*, hence QCTL*.

$$q \models \mu T.\varphi(T) \quad \Leftrightarrow \quad q \models \exists T.\Big[T \wedge \mathbf{A}\,\mathbf{G}(T \Leftrightarrow \varphi(T) \wedge \forall U.(\mathbf{A}\,\mathbf{G}(U \Leftrightarrow \varphi(U)) \Rightarrow \mathbf{A}\,\mathbf{G}(T \Rightarrow U))\Big]$$

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Expressiveness of QCTL

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QCTL and QCTL* are as expressive as MSO.

Remark

The following formula expresses Hamiltonicity (under structure semantics):

$$\mathbf{E}\,\mathbf{G}(\exists z.\, \forall z'.\, [\mathrm{state}(z) \wedge \mathrm{state}(z') \wedge z \wedge \neg z'] \Rightarrow \mathbf{X}(\neg z\ \mathbf{U}\ z'))$$

Using similar ideas, it can express Eulerianity, which cannot be expressed in MSO.

But these are not QCTL* formulas: in QCTL*, propositional quantifiers must be followed by path quantifiers.

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$$\mathbf{E}\,\mathbf{X}(\forall x.\varphi) \equiv \exists z.\forall x.\mathbf{E}\,\mathbf{X}(z \wedge \varphi)$$

Proof

Transform path quantification into propositional quantification.

- DAG-size linear in the DAG-size of the original formula.
- quantifier alternation linear in the quantifier depth of the original formula.

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Model checking under the structure semantics

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QCTL and QCTL* model checking is PSPACE*-complete.*

Proof

- PSPACE-hardness from QBF;
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Theorem

- *EQ^kCTL model checking is $\Sigma^{\mathrm{P}}_{k+1}$-complete.*
  EQ^kCTL is the fragment in prenex normal form, with at most k quantifier alternations, starting with existential quantification.
- *Q^kCTL model checking is $\Delta^{\mathrm{P}}_{k+1}$-complete.*
  Q^kCTL is the fragment with at most k quantifiers.

Model checking under the tree semantics

Theorem

- *$EQ^k CTL$ and $Q^k CTL$ model checking are* k-EXPTIME-*complete.*
- *$EQ^k CTL^*$ and $Q^k CTL^*$ model checking are* (k+1)-EXPTIME-*complete.*

Proof

Algorithm for $\mathbf{EQ^k CTL}$:

- build (exponential-size) tree automaton for CTL formula;
- use projection for existential quantification (universal quantification by complementing, with exponential blowup).
- build product with the Kripke structure, and check emptiness.

Hardness: encoding of $k-1$-exponential-space alternating Turing machine (Sistla, Vardi, Wolper (1987)).

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Future works

- other semantics for modelling finite-memory strategies?
- can QCTL help us find the right bisimulation notion that corresponds to ATL_{sc}?