

ATL with strategy contexts

— Expressiveness and model checking —

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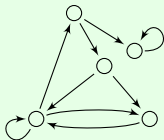
October 25, 2011

Model checking

system:



property:



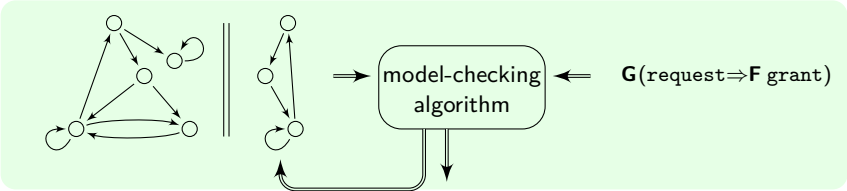
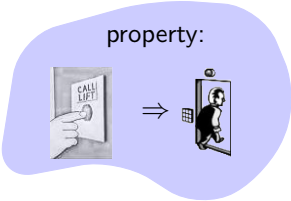
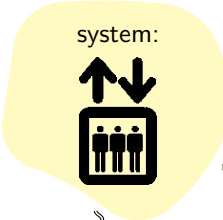
model-checking
algorithm



$G(\text{request} \Rightarrow F \text{ grant})$

yes/no

Model checking and control

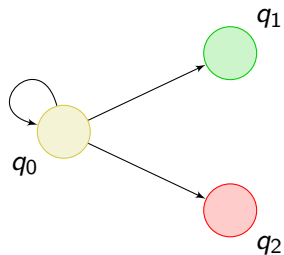


Reasoning about open systems

Concurrent games

A **concurrent game** is made of

- a transition system;

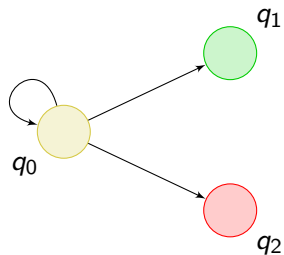


Reasoning about open systems

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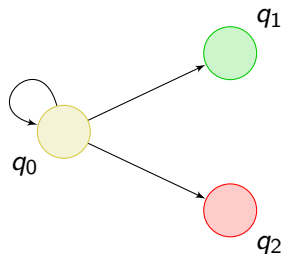


Reasoning about open systems

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A **concurrent game** is made of

- a transition system;
- a set of agents;
- a table indicating the transition to be taken given the actions of the players.



		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

Reasoning about open systems

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Turn-based games

A **turn-based game** is a game where only one agent plays at a time.

Reasoning about open systems

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Turn-based games

A **turn-based game** is a game where only one agent plays at a time.

Definition

A **strategy for Player i** is a function associating, with each finite play ρ of the game, a possible move for Player i from $\text{last}(\rho)$.

Alternating-time temporal logic

Definition

ATL extends CTL with *strategy quantifiers*:

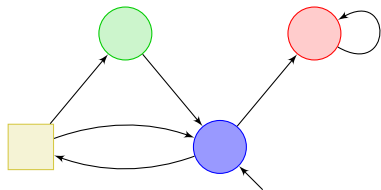
$$\langle\langle A \rangle\rangle \phi \quad \Leftrightarrow \quad A \text{ has a strategy } \sigma \text{ to enforce } \phi$$

(along all the outcomes)

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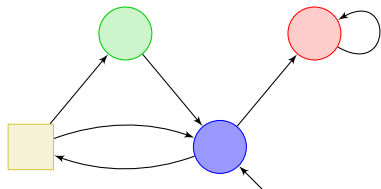
$$\langle\langle A \rangle\rangle \phi \iff A \text{ has a strategy } \sigma \text{ to enforce } \phi \\ \text{(along all the outcomes)}$$


✓ $\langle\langle O \rangle\rangle \mathbf{F} \text{ } \circ \equiv \langle\langle O \rangle\rangle \text{true } \mathbf{U} \text{ } \circ$

Alternating-time temporal logic

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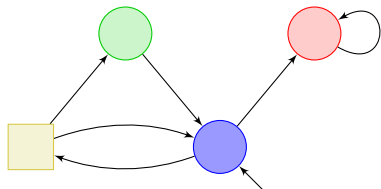
✓ $\langle\langle \circ \rangle\rangle \mathbf{F} \text{ (red circle)} \equiv \langle\langle \circ \rangle\rangle \text{true } \mathbf{U} \text{ (red circle)}$

✗ $\langle\langle \square \rangle\rangle \mathbf{F} \text{ (green circle)}$

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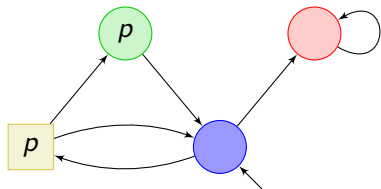
✗ $\langle\langle \square \rangle\rangle \mathbf{F} \text{ } \circ$

$\langle\langle \circ \rangle\rangle \mathbf{G}(\langle\langle \square \rangle\rangle \mathbf{F} \text{ } \circ)$

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$$\checkmark \quad \langle\langle \circ \rangle\rangle \mathbf{F} \text{red} \equiv \langle\langle \circ \rangle\rangle \text{true} \mathbf{U} \text{red}$$

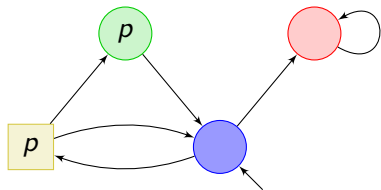
$$\times \quad \langle\langle \square \rangle\rangle \mathbf{F} \text{green}$$

$$\langle\langle \circ \rangle\rangle \mathbf{G} \left(\underbrace{\langle\langle \square \rangle\rangle \mathbf{F} \text{green}}_p \right) \equiv \langle\langle \circ \rangle\rangle \mathbf{G} p$$

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Alternating-time temporal logic

Definition

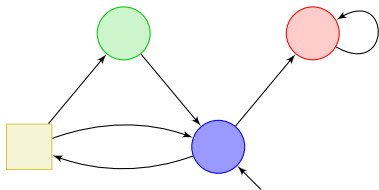
ATL extends CTL with *strategy quantifiers*:

$$\langle\langle A \rangle\rangle \phi \quad \Leftrightarrow \quad A \text{ has a strategy } \sigma \text{ to enforce } \phi \\ \text{(along all the outcomes)}$$

Theorem

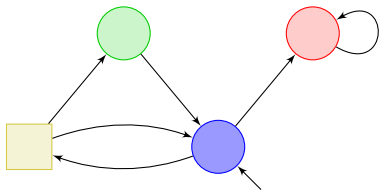
ATL model checking is PTIME-complete.

Another semantics: ATL with strategy contexts [BDLM09]

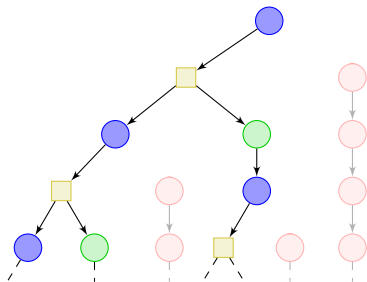


$\langle\langle \circ \rangle\rangle \mathbf{G}(\langle\langle \square \rangle\rangle \mathbf{F} \circ)$

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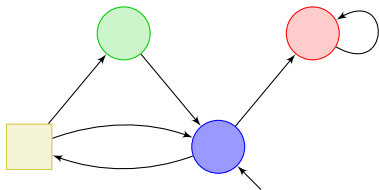
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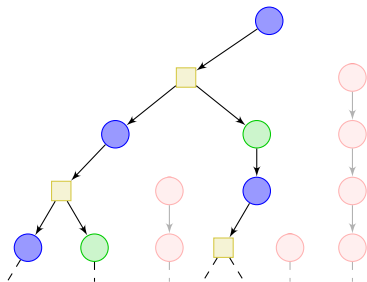
Evaluate the formula on the execution tree:

- apply a strategy of Player \bigcirc ;

Another semantics: ATL with strategy contexts [BDLM09]



$$\langle\langle \bigcirc \rangle\rangle \mathbf{G}(\langle\langle \square \rangle\rangle \mathbf{F} \bigcirc)$$



Evaluate the formula on the execution tree:

- apply a strategy of Player \bigcirc ;
- in the remaining tree, check that Player \square can always enforce a visit to \bigcirc .

ATL with strategy contexts

Definition

ATL_{sc} has two new strategy quantifiers: $\langle \cdot A \cdot \rangle \phi$ and $\langle\langle A \rangle\rangle \phi$.

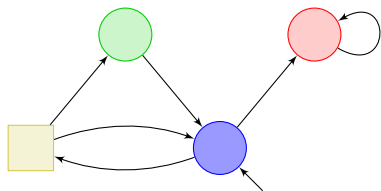
- $\langle \cdot A \cdot \rangle$ is similar to $\langle\langle A \rangle\rangle$ but **assigns** the corresponding strategy to A for evaluating ϕ ;
- $\langle\langle A \rangle\rangle$ drops the assigned strategies for A .

ATL with strategy contexts

Definition

ATL_{sc} has two new strategy quantifiers: $\langle \cdot A \cdot \rangle \phi$ and $\llbracket A \rrbracket \phi$.

- $\langle \cdot A \cdot \rangle$ is similar to $\llbracket A \rrbracket$ but **assigns** the corresponding strategy to A for evaluating ϕ ;
- $\llbracket A \rrbracket$ drops the assigned strategies for A .



✓ $\langle \cdot \circ \cdot \rangle \mathbf{F} \circ$

✗ $\langle \cdot \square \cdot \rangle \mathbf{F} \circ$

✓ $\langle \cdot \circ \cdot \rangle \mathbf{G}(\langle \cdot \square \cdot \rangle \mathbf{F} \circ)$

Outline of the talk

- 1 Introduction
- 2 Related approaches
- 3 Expressiveness issues
- 4 Model checking
- 5 Conclusions

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Related approaches

- **ATL with commitment** (van der Hoek, Jamroga, Wooldridge, 2005) extends ATL with an operator which restricts the behaviour of some players to a fixed (**memoryless**) strategy.
ATL with irrevocable strategies (Ågostnes, Goranko, Jamroga, 2008) is a similar extension to ours, but with a different way of handling the strategy contexts. Again, only investigated in the memoryless case.

Related approaches

- **ATL with commitment** (van der Hoek, Jamroga, Wooldridge, 2005)
ATL with irrevocable strategies (Ågostnes, Goranko, Jamroga, 2008)
- **QD μ** (Pinchinat, 2007): extension of the μ -calculus with a *decision modality*. A strategy is a labelling of a tree whose directions are the set of decisions of the agents (only works for a subclass of CGSs).

Related approaches

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- **$QD\mu$** (Pinchinat, 2007)
- **Stochastic Game Logic** (Baier, Brázdil, Größer, Kučera, 2007):
same extension as ours, in a probabilistic setting: games are turn-based and stochastic. Model checking is undecidable (both for deterministic and mixed strategies), but decidable when restricting to memoryless strategies.

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- **Strategy logic** (Chatterjee, Henzinger, Piterman, 2007):
first-order quantification over strategies. Nested formulas must be closed. Defined and studied only on 2-player turn-based games. Algorithm similar to ours but in a simpler setting (non-elementary complexity).

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- **Strategy logic** (Mogavero, Murano, Vardi, 2010): new version of SL with separate strategy quantifications and strategy assignments. Model-checking in 2EXPTIME-complete over the full class of n -player CGSs. Satisfiability is undecidable.

Related approaches

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- **(Basic) Strategy Interaction Logic** (Wang, Huang, Yu, 2011):
same approach as ours. PSPACE algorithm when nested formulas are required to be closed.

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What ATL_{sc} can express

- All ATL^* properties;

What ATL_{SC} can express

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- Client-server interactions for accessing a shared resource:

$$\langle \cdot \text{Server} \cdot \rangle \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \text{Clients}} \langle \cdot c \cdot \rangle \mathbf{F} \text{access}_c \\ \neg \bigwedge_{c \neq c'} \text{access}_c \wedge \text{access}_{c'} \end{array} \right]$$

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- Existence of Nash equilibria:

$$\langle A_1, \dots, A_n \rangle \bigwedge_i (\langle A_i \cdot \rangle \varphi_{A_i} \Rightarrow \varphi_{A_i})$$

What ATL_{sc} can express

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- Existence of Nash equilibria:

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- Existence of dominating strategy:

$$\langle A \rangle [B] (\neg \varphi \Rightarrow [A] \neg \varphi)$$

Expressiveness of ATL_{sc}

Theorem ([BDLM09])

- *The $\langle\langle A \rangle\rangle$ -operator is superfluous;*
- *ATL_{sc} is strictly more expressive than ATL .*

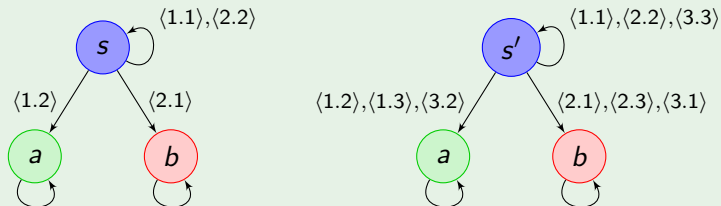
Expressiveness of ATL_{sc}

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Proof

$\langle 1 \cdot \rangle (\langle 2 \cdot \rangle \mathbf{X} a \wedge \langle 2 \cdot \rangle \mathbf{X} b)$.



s and s' are alternating-bisimilar, hence undistinguishable by ATL^* .

Expressiveness of ATL_{sc}

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Theorem

- ATL_{sc} is as expressive as ATL_{sc}^* .

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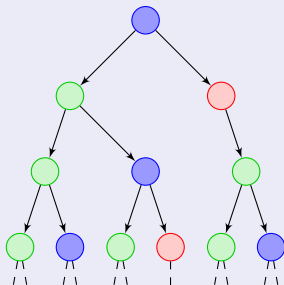
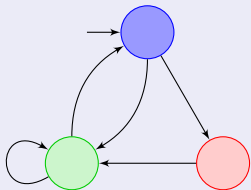
Insert extra $\langle\cdot\rangle$ between nested modalities.

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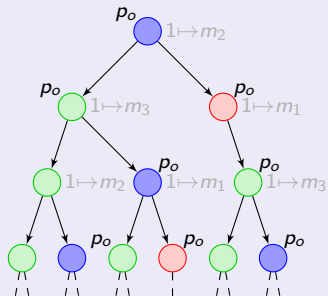
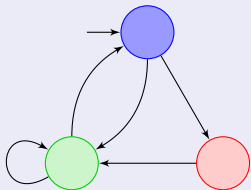
Model checking ATL_{sc}

Tree-automata approach



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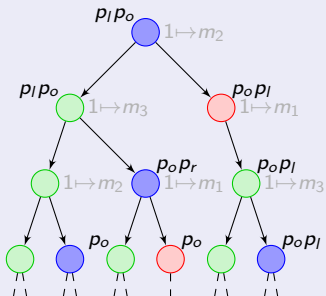
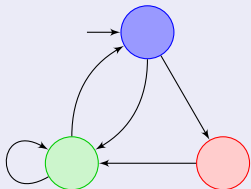
Tree-automata approach



- We can mark outcomes corresponding to selected strategies;

Model checking ATL_{sc}

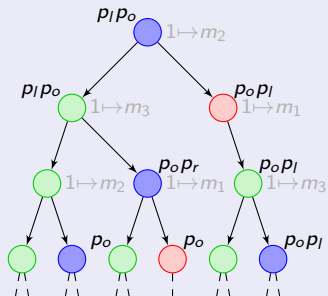
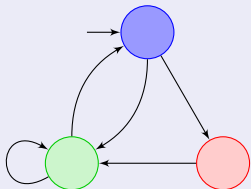
Tree-automata approach



- We mark the tree with extra propositions p_l and p_r , and require that it satisfies $\mathbf{A}(\mathbf{G} p_o \Rightarrow p_l \mathbf{U} p_r)$;

Model checking ATL_{sc}

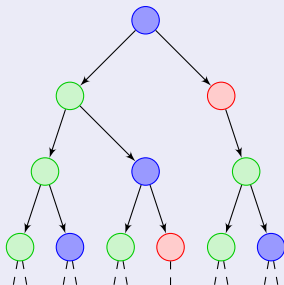
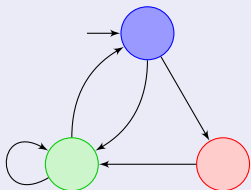
Tree-automata approach



- We require that subtrees rooted at a p_l or p_r node is accepted by the automaton for φ or φ' , respectively;

Model checking ATL_{sc}

Tree-automata approach



- We can build a tree automaton accepting all trees that *can be labelled* with correct strategies. This requires turning the alternating tree automaton into a non-deterministic one, which yields an **exponential-size** automaton.

Model checking ATL_{sc}

Theorem

Given a CGS \mathcal{C} , a state l_0 and an ATL_{sc} formula φ , we can build an alternating parity tree automaton \mathcal{A} s.t.

$$\mathcal{L}(\mathcal{A}) \neq \emptyset \quad \Leftrightarrow \quad \mathcal{C}, l_0 \models_{\emptyset} \varphi.$$

\mathcal{A} has size d -exponential, where d is the maximal number of nested quantifiers.

Theorem

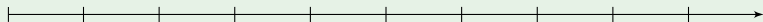
Model-checking ATL_{sc} can be achieved in **d -EXPTIME**, where d is the maximal number of nested quantifiers in the formula.

Hardness

QLTL extends LTL with quantification over atomic propositions:

Example

$$\forall a. \exists b. \mathbf{G}(b \Leftrightarrow \mathbf{X} a)$$



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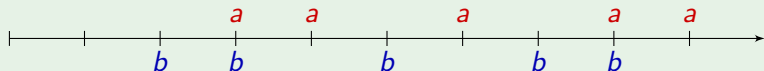


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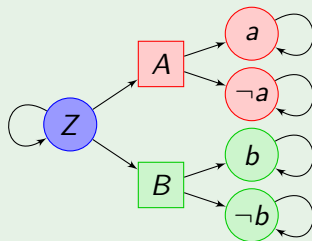
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Hardness

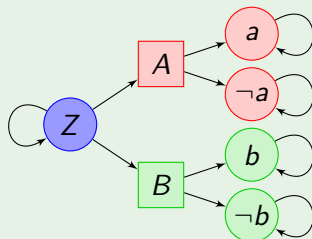
Example



$$[A] \langle B \rangle \left[\mathbf{G} \bullet \Rightarrow \mathbf{G}(\langle Z \rangle \mathbf{X} \mathbf{X} \bullet \Leftrightarrow \mathbf{X} \langle Z \rangle \mathbf{X} \mathbf{X} \bullet) \right]$$

Hardness

Example



$$[A] \langle B \rangle \left[\mathbf{G} \text{ (blue circle)} \Rightarrow \mathbf{G} (\langle Z \rangle \mathbf{X} \mathbf{X} \text{ (green circle)} \Leftrightarrow \mathbf{X} \langle Z \rangle \mathbf{X} \mathbf{X} \text{ (red circle)}) \right]$$

Theorem

ATL_{sc} model checking is $(d-1)$ -EXPSPACE-hard for formulas having at most d nested quantifiers.

Conclusions

- Our results on ATL_{SC} :
 - ATL_{SC} is a **natural semantical extension** of the popular ATL;
 - ATL_{SC} is **much more expressive**: equilibria, client-server interactions... Well-suited for non-zero-sum objectives;
 - There is a price for this expressiveness: **high complexity** of the model-checking algorithm.

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- Future works:
 - links between ATL_{SC} and QCTL;
 - study satisfiability of ATL_{SC} ;
 - behavioural equivalence for ATL_{SC} .