ATL with strategy contexts

— Expressiveness and model checking —

Arnaud Da Costa[1], François Laroussinie[2], Nicolas Markey[1]

[1] LSV, CNRS & ENS Cachan, France
[2] LIAFA, CNRS & Univ. Paris7-Diderot, France

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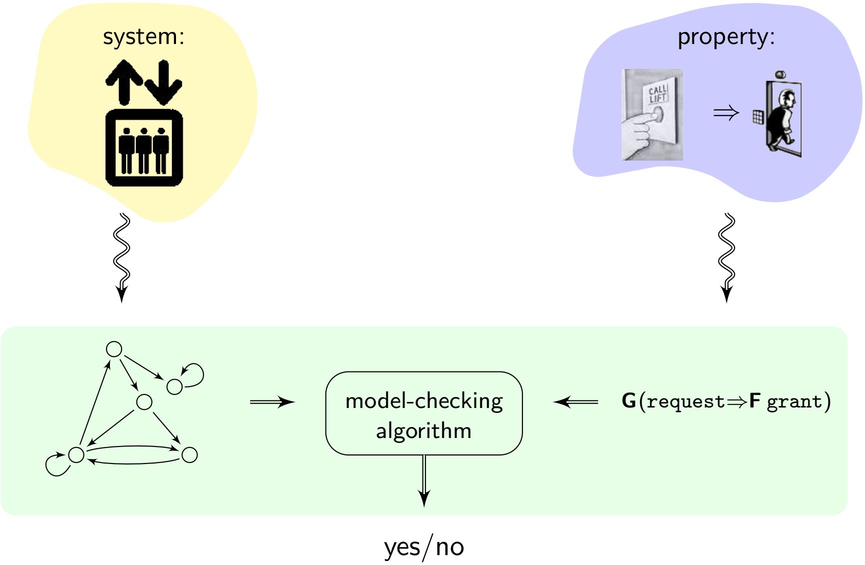
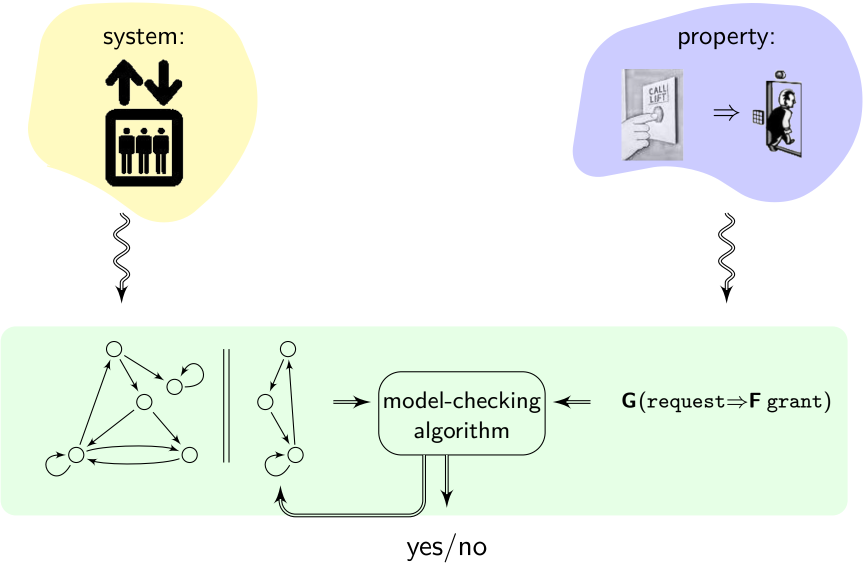
Model checking



Model checking and control



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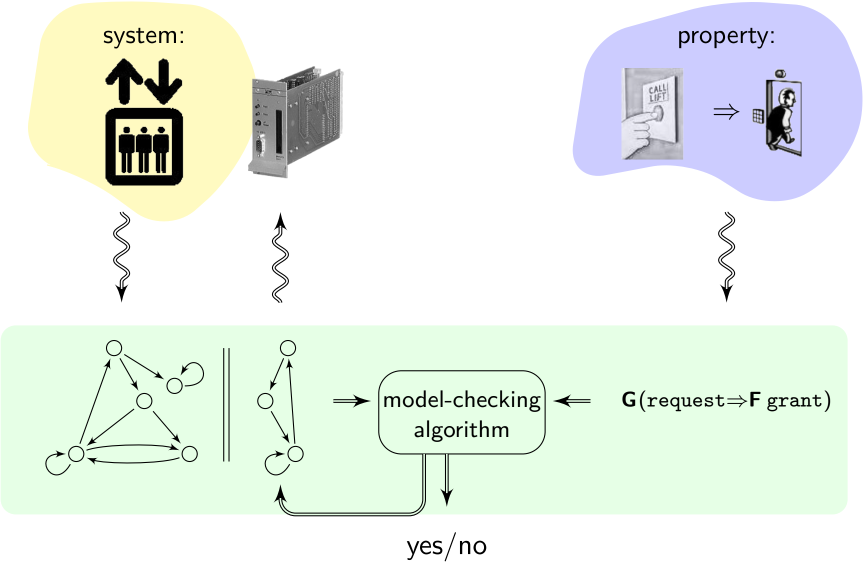

Game models

Definition

Concurrent game structures (CGS):

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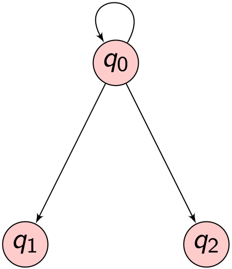
Concurrent game structures (CGS):

- labelled transition system;



Game models

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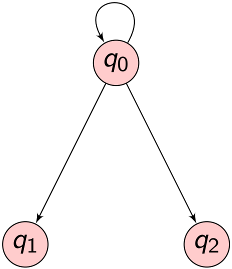
Concurrent game structures (CGS):

- labelled transition system;
- for each state, a table indicating the transitions to be taken depending on the choices of the players.



Player 2 (columns); Player 1 (rows)

q_0	p	r	s
p	q_0	q_1	q_2
r	q_2	q_0	q_1
s	q_1	q_2	q_0

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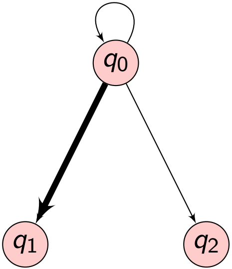

Player 2

Player 1

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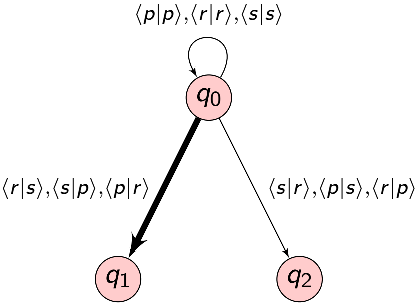

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Game models

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Concurrent game structures (CGS):

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Remark

Turn-based games form a subclass of CGSs where at each state, all the moves are equivalent for all but one player.

Game models

Definition

Concurrent game structures (CGS):

- labelled transition system;
- for each state, a table indicating the transitions to be taken depending on the choices of the players.

Definition

A *strategy for Player i* is a function associating, with each finite play ρ of the game, a possible move for Player i from $\mathrm{last}(\rho)$.

Alternating-time temporal logic

Definition

ATL extends CTL with *strategy quantifiers*:

$$\langle\langle A \rangle\rangle\, \phi \quad \Leftrightarrow \quad A \text{ has a strategy } \sigma \text{ to enforce } \phi$$
$$\text{(along all the outcomes)}$$

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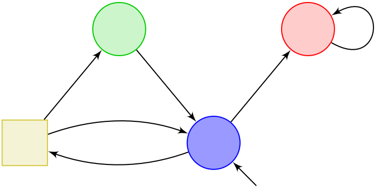

$\checkmark$ $\langle\!\langle \bigcirc \rangle\!\rangle$ **F** $\bigcirc$

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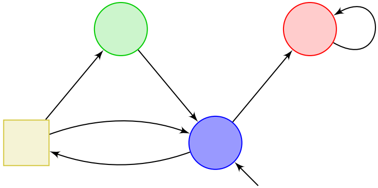

✓ $\langle\langle \bigcirc \rangle\rangle$ **F** ○

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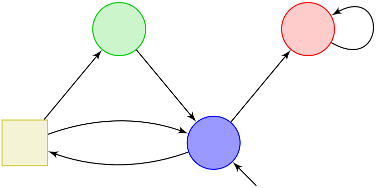

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Theorem

ATL model checking is PTIME*-complete.*

ATL with strategy contexts

Definition

ATL_{sc} has two new strategy quantifiers: $\langle\cdot A\cdot\rangle\,\phi$ and $\rangle\!\cdot A\cdot\!\langle\,\phi$.

- $\langle\cdot A\cdot\rangle$ is similar to $\langle\langle A\rangle\rangle$ but assigns the corresponding strategy to A for evaluating ϕ;
- $\rangle\!\cdot A\cdot\!\langle$ drops the assigned strategies for A.

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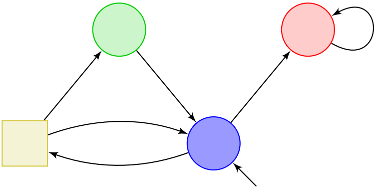

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Definition

$$
\begin{aligned}
G, s \models_F \langle\cdot A\cdot\rangle\,\phi \quad &\Leftrightarrow \quad \exists\sigma_A.\ \forall\rho' \in \mathrm{Out}(s, F[A \mapsto \sigma_A]). \\
&\qquad\qquad G, \rho' \models_{F_{[A\mapsto\sigma_A]}} \phi \\
G, \rho \models_F \phi\ \mathbf{U}\ \psi \quad &\Leftrightarrow \quad \exists j.\ G, \rho_{\geq j} \models_{\mathrm{shift}(F, \rho[0..j])} \psi \\
&\qquad \text{and } \forall 0 \leq k < j.\ G, \rho_{\geq k} \models_{\mathrm{shift}(F, \rho[0..k])} \phi
\end{aligned}
$$

Outline of the talk

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Example of ATL_{sc} formulas

- ATL_{sc} encompases ATL:

$$\widehat{\langle\!\langle A \rangle\!\rangle\,\phi} \equiv \llparenthesis \text{Agt} \rrparenthesis\ \langle\cdot A \cdot\rangle\ \hat{\phi}$$

- Client-server interactions for accessing a shared resource:

$$\langle\cdot\text{Server}\cdot\rangle\ \mathbf{G}\left[\begin{array}{l} \displaystyle\bigwedge_{c\in\text{Clients}} \langle\cdot c\cdot\rangle\ \mathbf{F}\,\text{access}_c \\ \wedge \\ \displaystyle\neg \bigwedge_{c\neq c'} \text{access}_c \wedge \text{access}_{c'} \end{array}\right]$$

- (Boolean-objective, pure-strategy) Nash equilibria:

$$\langle\cdot A_1, \ldots, A_n\cdot\rangle \bigwedge_i \left(\langle\cdot A_i\cdot\rangle\,\phi_{A_i} \Rightarrow \phi_{A_i}\right)$$

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Expressiveness of ATL_{sc}

Theorem

- *The* $⦉A⦊$ *-operator is superfluous;*
- ATL_{sc} *is as expressive as* ATL^{*}_{sc}*;*
- ATL_{sc} *is strictly more expressive than* ATL^{*}*.*

Expressiveness of ATL_{sc}

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- *The* $\langle\!\!\langle A \rangle\!\!\rangle$ *-operator is superfluous;*
- ATL_{sc} *is as expressive as* ATL^*_{sc}*;*
- ATL_{sc} *is strictly more expressive than* ATL^**.*

Proof.

- The first statement is obtained (roughly) by replacing $\langle\!\!\langle A \rangle\!\!\rangle$ with $[\cdot A \cdot]$, which is the dual of $\langle \cdot A \cdot \rangle$.
- The second statement is obtained (roughly) by inserting $\langle \cdot \varnothing \cdot \rangle$ between any two nested temporal modalities.

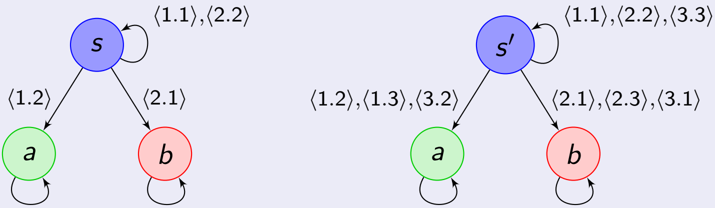
Expressiveness of ATL_{sc}

Theorem

- *The ⧚A⧛-operator is superfluous;*
- *ATL_{sc} is as expressive as ATL^*_{sc};*
- *ATL_{sc} is strictly more expressive than ATL^*.*

Proof.

- $\langle\cdot 1\cdot\rangle\,(\,\langle\cdot 2\cdot\rangle\ \mathbf{X}\,a \wedge \langle\cdot 2\cdot\rangle\ \mathbf{X}\,b)$.



s and s' are alternating-bisimilar, hence undistinguishable by ATL^*.

Outline of the talk

Related approaches

- ATL with commitment (van der Hoek, Jamroga, Wooldridge, 2005) extends ATL with an operator which restricts the behaviour of some players to a fixed (memoryless) strategy.
  ATL with irrevocable strategies (Ågostnes, Goranko, Jamroga, 2008) is a similar extension to ours, but with a different of handling the strategy context. Again, only investigated in the memoryless case.

Related approaches

- ATL with commitment (van der Hoek, Jamroga, Wooldridge, 2005)
  ATL with irrevocable strategies (Ågostnes, Goranko, Jamroga, 2008)

- QDμ (Pinchinat, 2007): extension of the μ-calculus with a *decision modality*. A strategy is a labelling of a tree whose directions are the set of decisions of the agents (hence only works for ATSs).

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- QDμ (Pinchinat, 2007)

- Stochastic Game Logic (Baier, Brázdil, Größer, Kučera, 2007): same extension as ours, in a probabilistic setting: games are turn-based stochastic games. Model checking is undecidable (both deterministic and mixed strategies), but becomes decidable when restricting to memoryless strategies.

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- Strategy logic (Chatterjee, Henzinger, Piterman, 2007): first-order quantification over strategies. Nested formulas must be closed. Defined only on 2-player turn-based games. Algorithm similar to ours but in a simpler setting.

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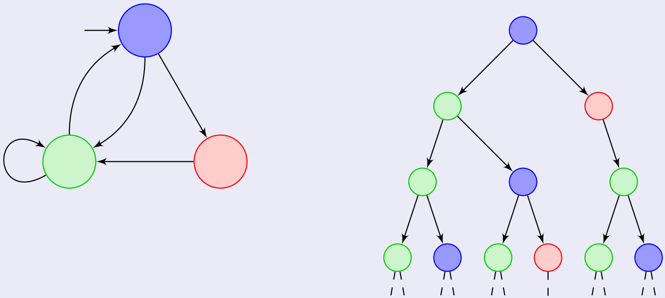
- Strategy logic (Chatterjee, Henzinger, Piterman, 2007)
- Strategy logic (Mogavero, Murano, Vardi, 2010): new version of SL with separate strategy quantifications and strategy assignments. Model-checking in 2EXPTIME over the full class of *n*-player CGSs. Satisfiability is undecidable.

Outline of the talk

Model checking ATL$_{sc}$

Tree-automata approach



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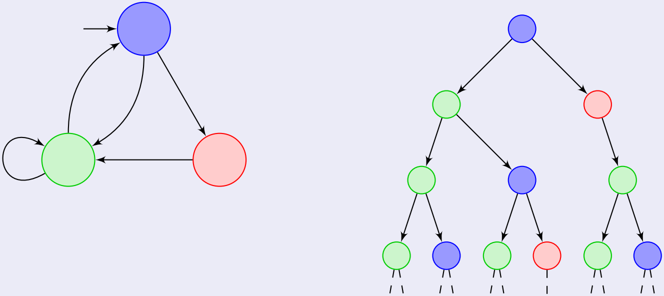

- The unwinding tree is accepted by a deterministic tree automaton;

Model checking ATL$_{sc}$

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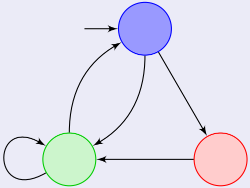

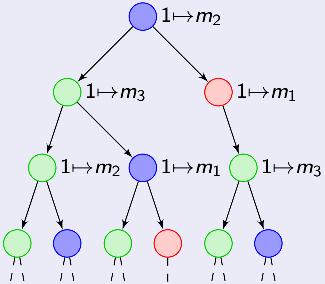

- A strategy is encoded as a labelling of the unwinding tree;

Model checking ATL$_{sc}$

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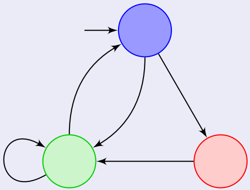

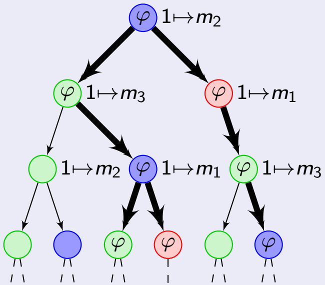

- We can mark outcomes corresponding to selected strategies, and check that they satisfy subformula φ;

Model checking ATL$_{sc}$

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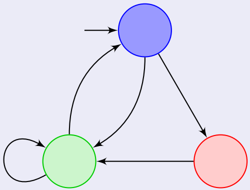
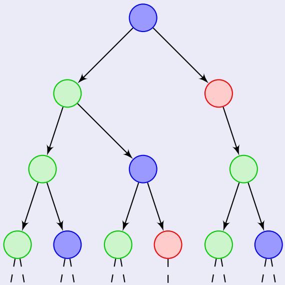




- We can build a tree automaton accepting all trees that *can be labelled* with correct strategies. This requires turning the alternating tree automaton into a non-deterministic one, which yields an exponential-size automaton.

Model checking ATL_{sc}

Theorem

Given a CGS $\mathcal{C}$, a state ℓ_0 and an ATL_{sc} formula φ, we can build an APT $\mathcal{A}$ s.t.

$$\mathcal{L}(\mathcal{A}) \neq \varnothing \quad \Leftrightarrow \quad \mathcal{C}, \ell_0 \models_\varnothing \varphi.$$

$\mathcal{A}$ has size d-exponential, where d is the maximal number of nested quantifiers.

Theorem

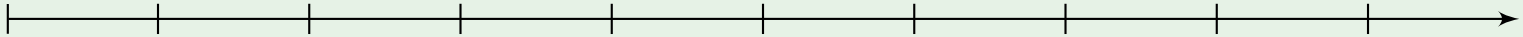
Model-checking ATL_{sc} can be achieved in $(d+1)$-EXPTIME, where d is the maximal number of nested quantifiers in the formula.

Recent advances: hardness

QPTL extends LTL with quantification over atomic propositions:

Example

$$\forall a.\ \exists b. \quad \mathbf{G}(b \Leftrightarrow \mathbf{X}\ a)$$



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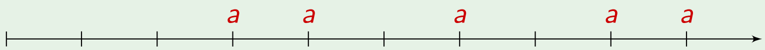

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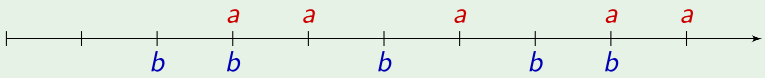

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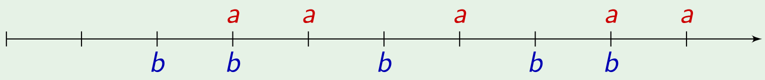
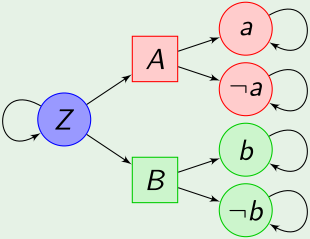


Theorem (SVW87)

Satisfiability of a QPTL formula is k-EXPSPACE-complete, where k is the alternation-depth of the formula.

Recent advances: hardness

Example



$$[\cdot A \cdot]\ \langle \cdot B \cdot \rangle \left[\mathbf{G}\ \bigcirc \Rightarrow \mathbf{G}(\ \langle \cdot Z \cdot \rangle\ \mathbf{X}\,\mathbf{X}\ \textcircled{b} \Leftrightarrow \mathbf{X}\ \langle \cdot Z \cdot \rangle\ \mathbf{X}\,\mathbf{X}\ \textcircled{a}) \right]$$

Recent advances: hardness

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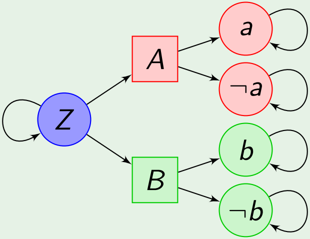

$$[\cdot A\cdot]\ \langle\cdot B\cdot\rangle\ \Big[\mathbf{G}\ \bigcirc \Rightarrow \mathbf{G}(\ \langle\cdot Z\cdot\rangle\ \mathbf{X}\,\mathbf{X}\ \textcircled{b} \Leftrightarrow \mathbf{X}\ \langle\cdot Z\cdot\rangle\ \mathbf{X}\,\mathbf{X}\ \textcircled{a})\Big]$$

Theorem

*ATL_{sc} model checking is k-*EXPSPACE*-hard for formulas with $k+1$ nested quantifiers.*

Conclusions

- Our results on ATL_{sc}:
  - ATL_{sc} is a natural semantical extension of the popular ATL;
  - ATL_{sc} is much more expressive: equilibria, client-server interactions... Very interesting for non-zero-sum objectives;
  - There is a price for this expressiveness: high complexity of the model-checking algorithm.

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  - There is a price for this expressiveness: high complexity of the model-checking algorithm.

- Future works:
  - close the complexity gap in the model-checking problem;
  - study satisfiability of ATL_{sc}.