TOAST : Théorie des jeux, Outils de l'automatique, de l'Algorithmique et du Signal pour les Télécommunications

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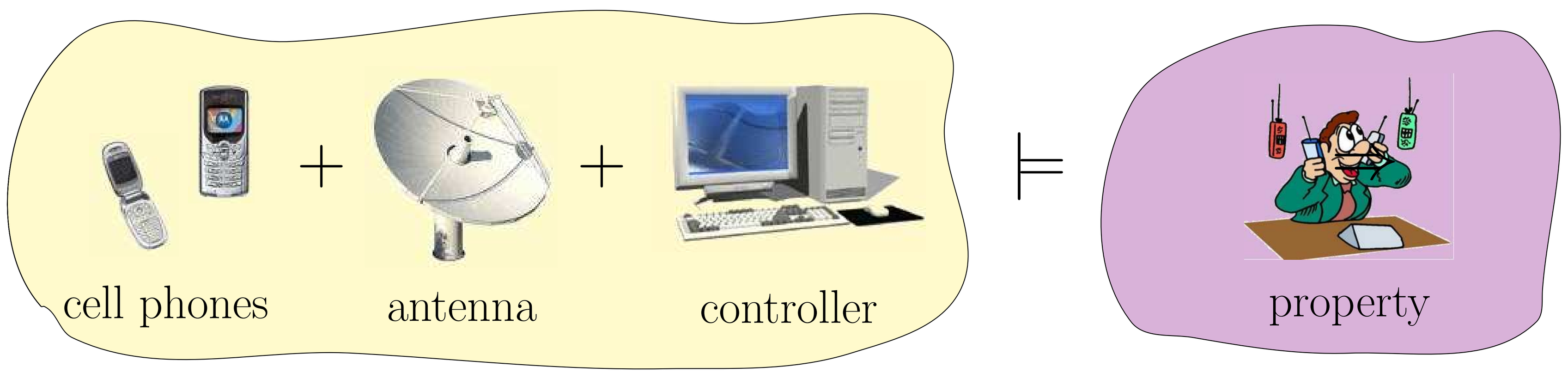
The power control problem in data networks has become a critical optimization problem because nowadays the telecommunication resource is scarce and is shared everywhere by more and more users of greedy applications. This optimization problem is non-convex, because of the presence of these multiple users with their own objectives associated. The user satisfaction is represented by a quality of service, based on signal to interference ratio, and a cost term on user's power. A user is able to maintain the communication if its utility function reaches a lower threshold.

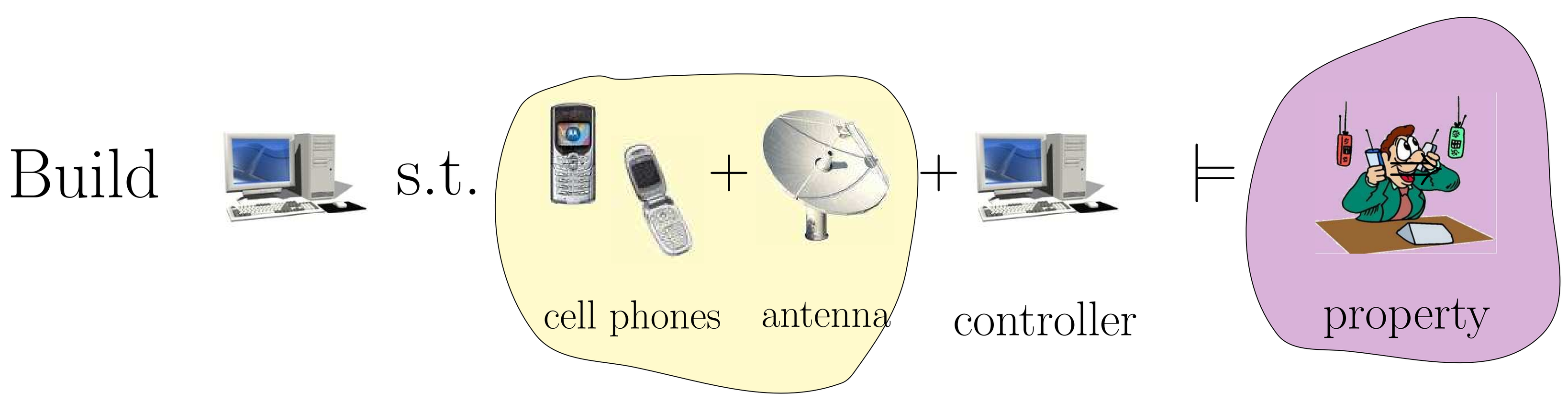
In the static case, the water-filling provides a well-known and elegant solution, corresponding to a Nash equilibrium in the framework of Game Theory. To cope with time-varying number of users, or with the evolution of the channel characteristics, we propose a new approach using an automata formulation of the game is presented in this paper. This approach is not only based on the research of an equilibrium, but also on the verification that some quantitative properties hold in the game.

Control and Model-Checking

Verification:

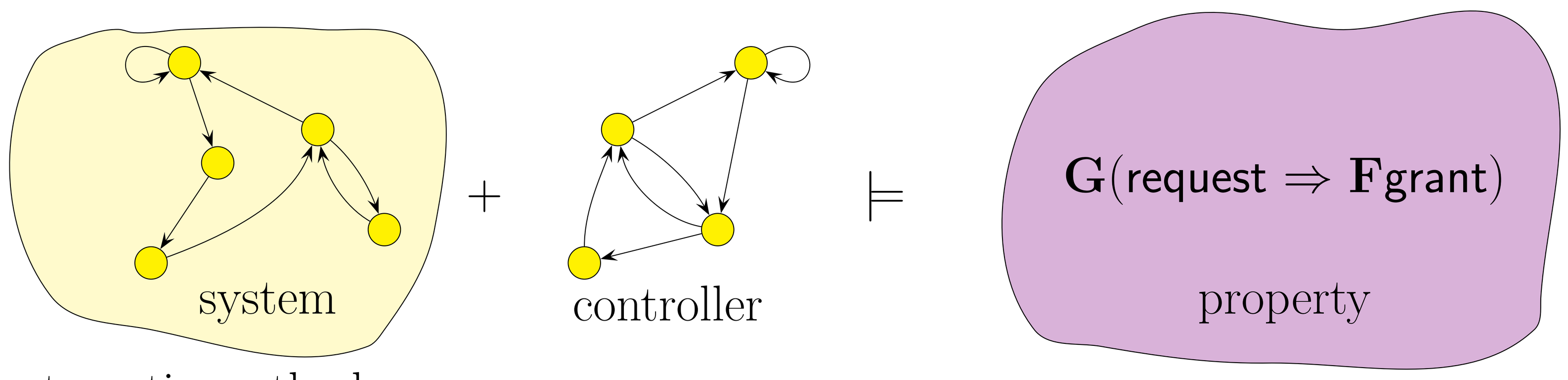


Controller synthesis:



Model-checking approach:

- use formal models and specification languages:



- automatic methods

From Control to Games

Control theory	Game theory
system under study	game board
agents (environment)	players (opponent)
control algorithm	strategy computation
controller synthesis	synthesis of winning strategy

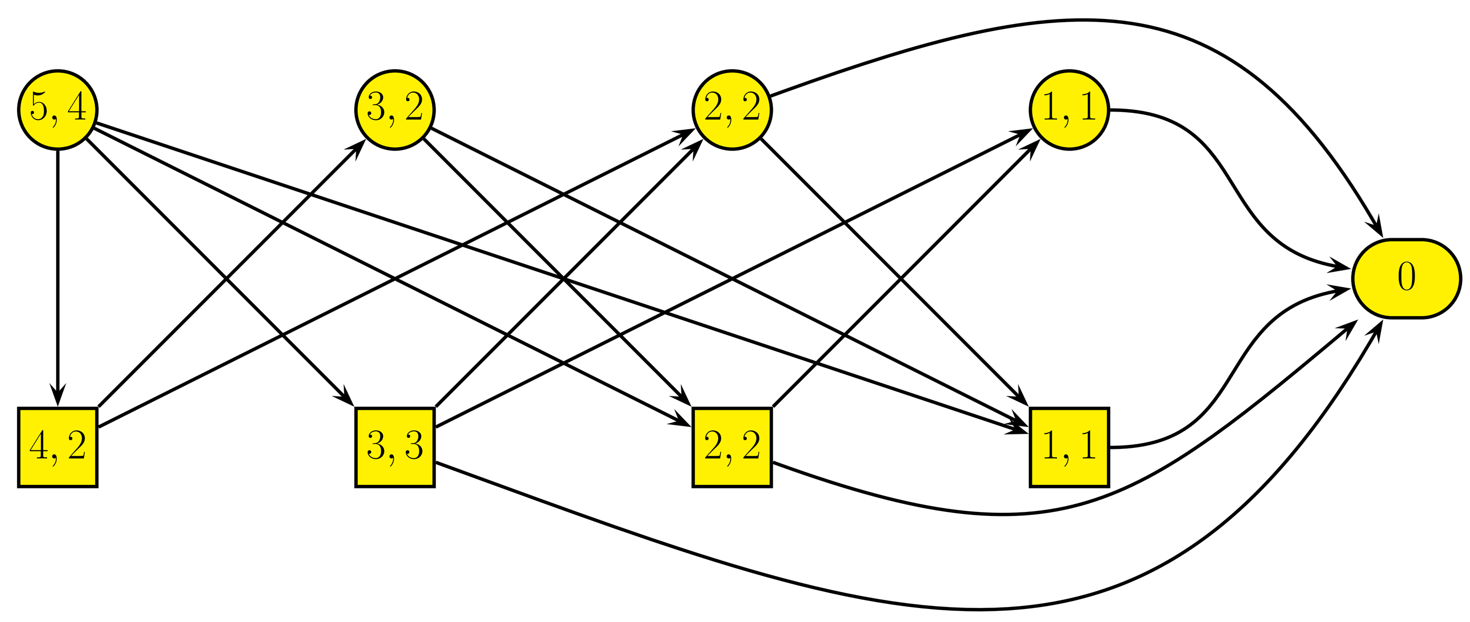
Example: (simplified) game of Nim.

Two players take, in turn, a certain number of matches, with two constraints:

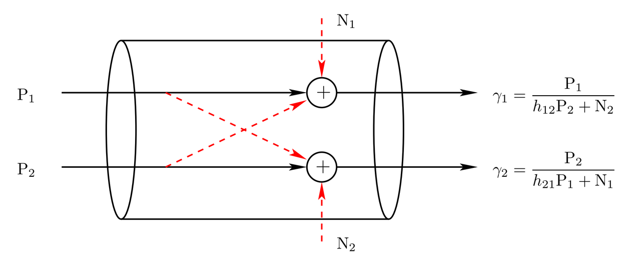
- starting with n matches, the first player takes between 1 and $n-1$ matches,
- then, in turn, each player takes between one and twice as many matches as the previous player just picked.

The player who takes the last match is declared the winner.

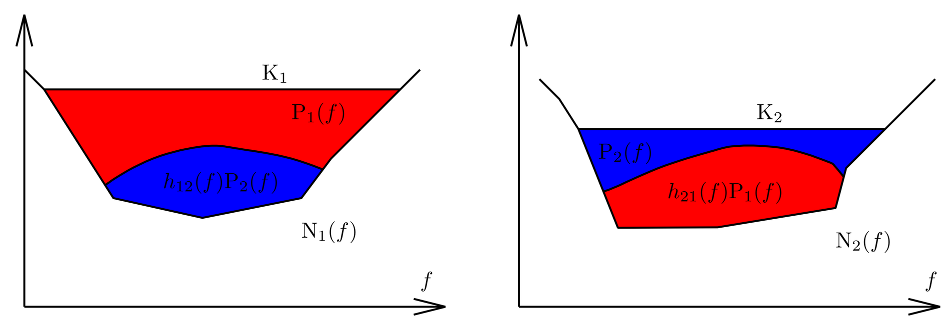
In the graph below, circle states "belong" to the first player, and square states to the opponent. The first number in each state is the number of matches left, and the second one is the maximal number of matches the player can pick. It can be observed that the first player has no winning strategy when he starts to play with 5 matches.



Multiuser power control



The transmitted signals, whose power is noted P_i, are corrupted by other signals and by environmental noise N_j. Each user aims to improve his Quality of Service (QoS) based on Signal Interference Noise Rate (SINR) γ_i. This is a non-convex multiobjective optimization problem. In static telecommunication framework this is related to Waterfilling method.



Weighted sums of transmission power and noise power are equal to constants, for all frequencies.

Game Theory and Nash Strategy

Game theory deals with interactive decision-making, where the outcome for each participant or player depends on the actions of all. If you are a player in such a game, when choosing your course of action or strategy, you must take into account the choices of others.

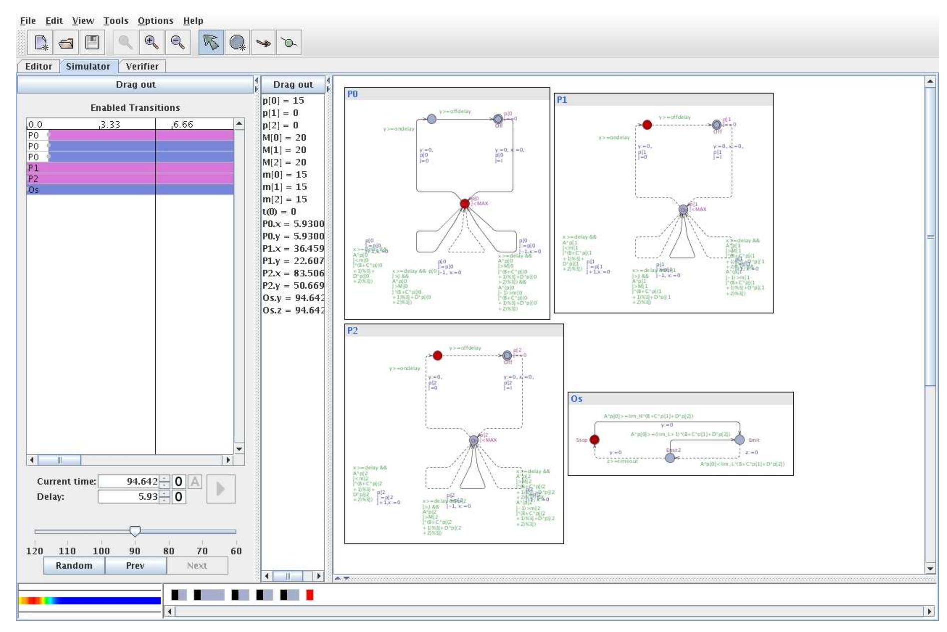
When the players have interchangeable roles, the Nash strategy is adapted to design controls. At Nash equilibrium, a user cannot improve his quality of service, if he decides not to use his predefined control, without cooperation with the other users:

$$\gamma_1(P_1^*, P_2^*) \geq \gamma_1(P_1, P_2^*), \quad \text{for any strategy } P_1,$$
$$\gamma_2(P_1^*, P_2^*) \geq \gamma_2(P_1^*, P_2), \quad \text{for any strategy } P_2.$$

The tool UPPAAL-TiGA

UPPAAL-TiGA is a state-of-the-art tool for checking quantitative properties of timed game automata, developped in Aalborg, Denmark.

`http://www.cs.aau.dk/ adavid/tiga/`



We modeled our problem as a timed game automaton with two or three players, and two modes (low and high bandwidth). In our model, the user can increase or decrease its power by a limited rate. We used UPPAAL-TiGA for checking if a user can connect and remain connected to the network for a certain amount of time.

`http://www.lsv.ens-cachan.fr/~markey/TOAST/`