

Chennai Mathematical Institute

Probability Theory: Quiz II

Name _____

Answer all questions and show your work.

- (1) Let X be a random variable with density function given by

$$f(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} \frac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}}, \quad x > 0,$$

where $\alpha > 0$ and $\beta > 0$. Here $B(\alpha, \beta)$ is the Beta function.

Find the distribution of $Y = \frac{1}{1+X}$. Specify the range of the new random variable.

- (2) The median of a continuous random variable X with cdf $F_X(x)$ is the value m such that $F_X(m) = 0.5$.

Find the median for the exponential distribution with parameter β .

- (3) Let $X \sim G(\alpha, \beta)$.

- (a) Find the mgf of X .
- (b) Use the mgf to compute $E(X)$.