

Unit-8: Algorithms for LTL

B. Srivathsan

Chennai Mathematical Institute

NPTEL-course

July - November 2015

Module 1:
**Automata-based LTL
model-checking**

Does **Transition system** satisfy **LTL formula ϕ** ?

Does **Transition system** satisfy **LTl formula** ϕ ?

Negation $\neg \phi$

Does **Transition system** satisfy **LTL formula** ϕ ?

Negation $\neg \phi$



NBA $\mathcal{A}_{\neg \phi}$

Does **Transition system** satisfy **LTL formula ϕ** ?



NBA $\mathcal{A}_{T.S}$

Negation $\neg \phi$



NBA $\mathcal{A}_{\neg\phi}$

Does **Transition system** satisfy **LTL formula ϕ** ?



NBA $\mathcal{A}_{T.S.}$

Negation $\neg \phi$



NBA $\mathcal{A}_{\neg\phi}$

Is $L(\mathcal{A}_{T.S.}) \cap L(\mathcal{A}_{\neg\phi})$ empty?

Does **Transition system** satisfy **LTL formula ϕ** ?



NBA $\mathcal{A}_{T.S.}$

Negation $\neg \phi$



NBA $\mathcal{A}_{\neg\phi}$

Is $L(\mathcal{A}_{T.S.}) \cap L(\mathcal{A}_{\neg\phi})$ empty?

Is $L(\mathcal{A}_{T.S.} \times \mathcal{A}_{\neg\phi})$ empty?

Here: Converting LTL formulas to NBA

Here: Converting LTL formulas to NBA

Coming next: Examples

Atomic propositions $\mathbf{AP} = \{ p_1, p_2 \}$

Alphabet:

$\{ \{ \}, \{ p_1 \}, \{ p_2 \}, \{ p_1, p_2 \} \}$

F p_1 Words where p_1 occurs sometime

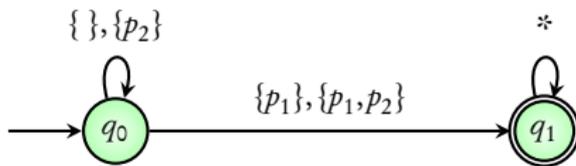
$\{p_2\} \{ \} \{ \} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{p_1, p_2\} \{ \} \{ \} \{ \} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

\vdots

F p_1 Words where p_1 occurs sometime

$\{p_2\} \{ \} \{ \} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$
 $\{p_1, p_2\} \{ \} \{ \} \{ \} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$
 \vdots



G p_1 Words where p_1 occurs always

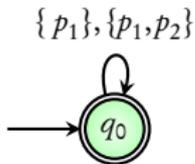
$\{p_1\} \{p_1, p_2\} \{p_1\} \{p_1, p_2\} \{p_1\} \{p_1\} \{p_1\} \dots$

$\{p_1, p_2\} \{p_1, p_2\} \{p_1\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

\vdots

G p_1 Words where p_1 occurs always

$\{p_1\} \{p_1, p_2\} \{p_1\} \{p_1, p_2\} \{p_1\} \{p_1\} \{p_1\} \dots$
 $\{p_1, p_2\} \{p_1, p_2\} \{p_1\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$
 \vdots



$p_1 \wedge \neg p_2$ Words starting with $\{p_1\}$

$\{p_1\} \{\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{p_1\} \{\} \{\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

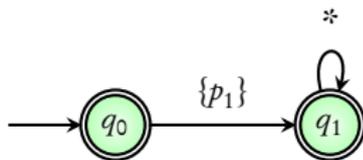
\vdots

$p_1 \wedge \neg p_2$ Words starting with $\{p_1\}$

$\{p_1\} \{\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{p_1\} \{\} \{\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

\vdots



$$p_1 \wedge \mathbf{X} \neg p_2$$

$$\{p_1\} \{\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$$

$$\{p_1, p_2\} \{p_1\} \{\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$$

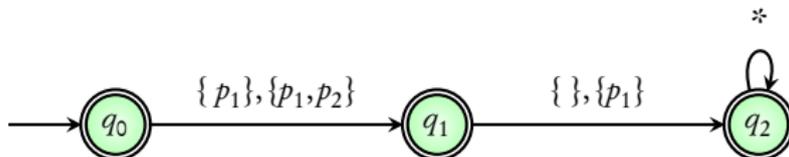
⋮

$$p_1 \wedge \mathbf{X} \neg p_2$$

$\{p_1\} \{\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{p_1, p_2\} \{p_1\} \{\} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

\vdots

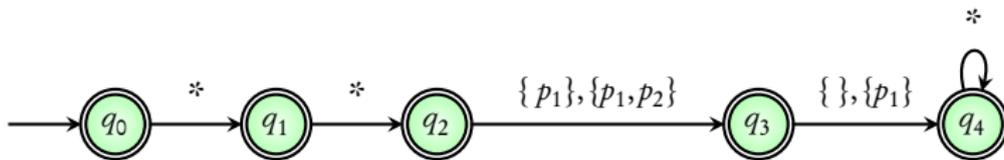


$$\mathbf{XX}(p_1 \wedge \mathbf{X}\neg p_2)$$

$$\begin{aligned} & \{ \} \{ \} \{ p_1 \} \{ \} \{ p_2 \} \{ p_1, p_2 \} \{ p_2 \} \{ p_2 \} \{ p_2 \} \dots \\ & \{ p_2 \} \{ p_1 \} \{ p_1, p_2 \} \{ p_1 \} \{ \} \{ p_1 \} \{ p_1 \} \{ p_1, p_2 \} \dots \\ & \quad \vdots \end{aligned}$$

$$\mathbf{XX} (p_1 \wedge \mathbf{X} \neg p_2)$$

$\{\}\{\}\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\dots$
 $\{p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{\}\{p_1\}\{p_1\}\{p_1,p_2\}\dots$
 \vdots



$$p_1 \cup p_2$$

$$\{p_1\} \{p_1\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$$

$$\{p_1, p_2\} \{ \} \{ \} \{ \} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$$

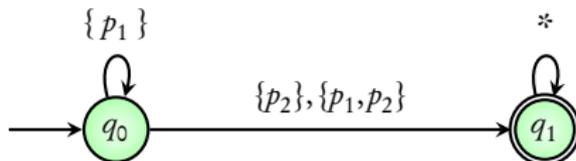
⋮

$p_1 \cup p_2$

$\{p_1\} \{p_1\} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{p_1, p_2\} \{ \} \{ \} \{p_1\} \{p_1\} \{p_1, p_2\} \dots$

\vdots



$(X \ p_1) \ U \ p_2$

$\{p_2\} \{\}\{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

$\{\}\{p_1\} \{p_1\} \{p_1\} \{p_1, p_2\} \{p_1, p_2\} \dots$

$\{\}\{p_1, p_2\} \{\}\{\}\{p_2\} \{p_1, p_2\} \dots$

\vdots

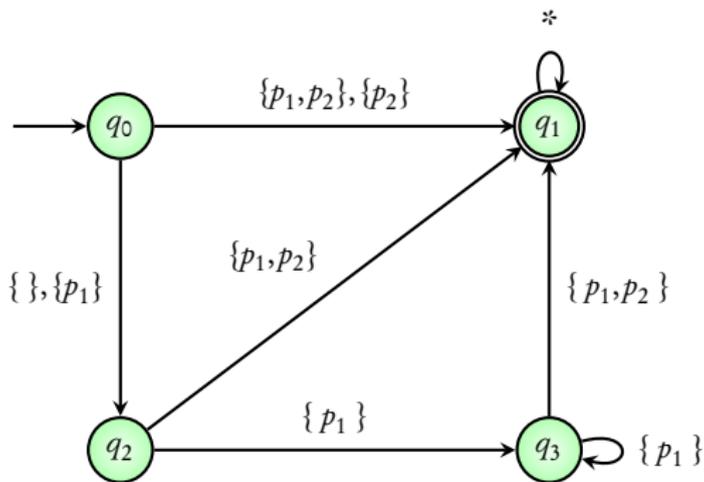
$(X p_1) U p_2$

$\{p_2\} \{ \} \{p_2\} \{p_1, p_2\} \{p_2\} \{p_2\} \{p_2\} \dots$

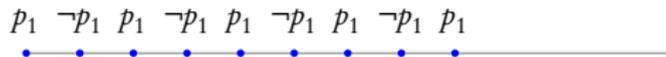
$\{ \} \{p_1\} \{p_1\} \{p_1\} \{p_1, p_2\} \{p_1, p_2\} \dots$

$\{ \} \{p_1, p_2\} \{ \} \{ \} \{ \} \{p_2\} \{p_1, p_2\} \dots$

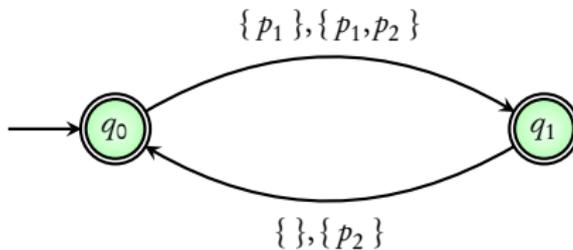
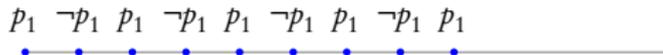
\vdots



$$p_1 \wedge \mathbf{X} \neg p_1 \wedge \mathbf{G} (p_1 \leftrightarrow \mathbf{X} \mathbf{X} p_1)$$



$$p_1 \wedge \mathbf{X} \neg p_1 \wedge \mathbf{G} (p_1 \leftrightarrow \mathbf{X} \mathbf{X} p_1)$$



GF p_1 Words where p_1 occurs infinitely often

$\{ \} \{ p_1 \} \{ p_2 \} \{ p_1, p_2 \} \{ p_2 \} \{ p_1 \} \{ p_2 \} \dots$

$\{ \} \{ \} \{ \} \{ \} \{ p_1 \} \{ p_1 \} \{ p_1 \} \dots$

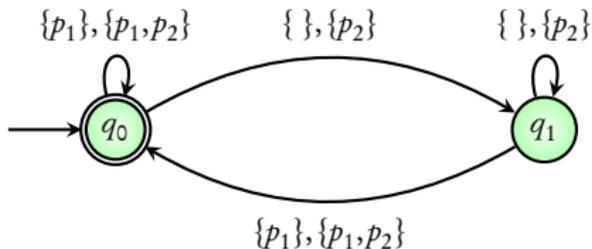
\vdots

GF p_1 Words where p_1 occurs infinitely often

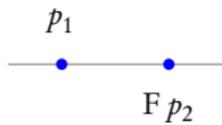
$\{ \} \{ p_1 \} \{ p_2 \} \{ p_1, p_2 \} \{ p_2 \} \{ p_1 \} \{ p_2 \} \dots$

$\{ \} \{ \} \{ \} \{ \} \{ p_1 \} \{ p_1 \} \{ p_1 \} \dots$

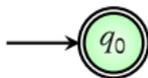
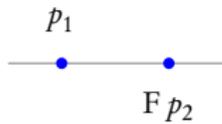
\vdots



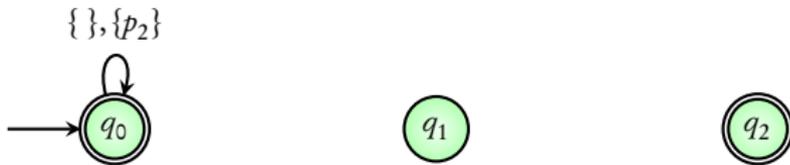
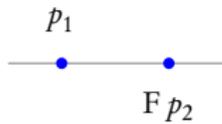
$G(p_1 \rightarrow \mathbf{XF} p_2)$



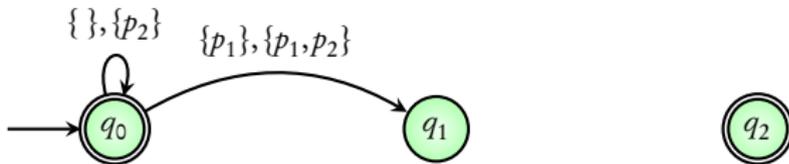
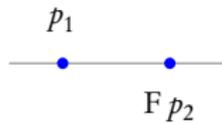
$G(p_1 \rightarrow \text{XF } p_2)$



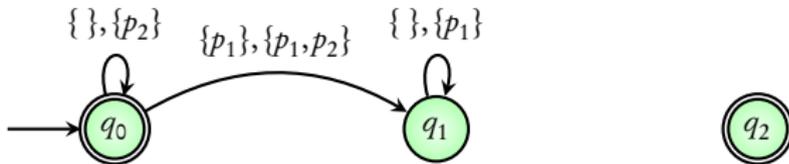
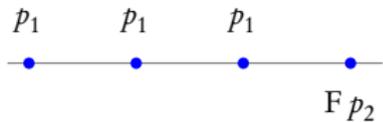
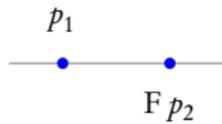
$G(p_1 \rightarrow \text{XF } p_2)$



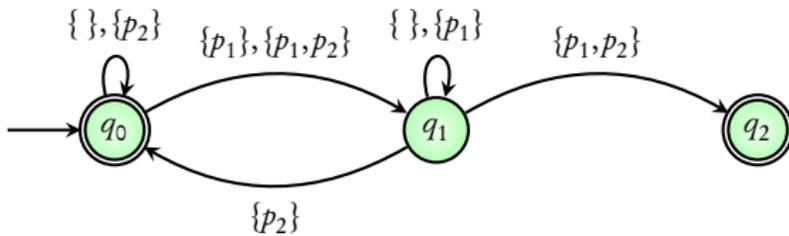
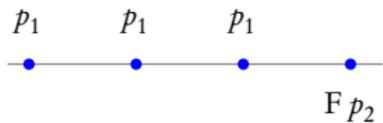
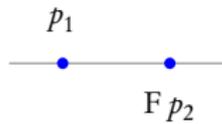
$G(p_1 \rightarrow \text{XF } p_2)$



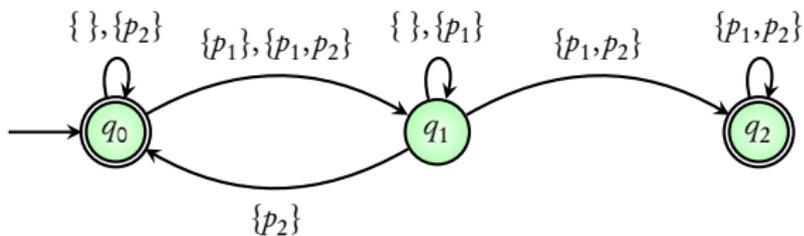
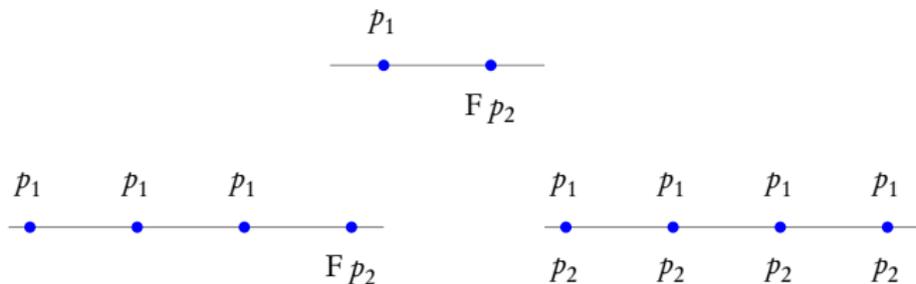
$G(p_1 \rightarrow \text{XF } p_2)$



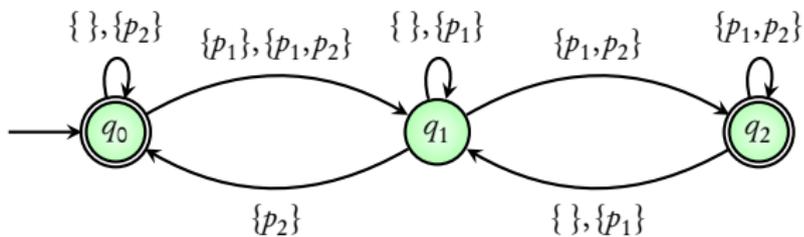
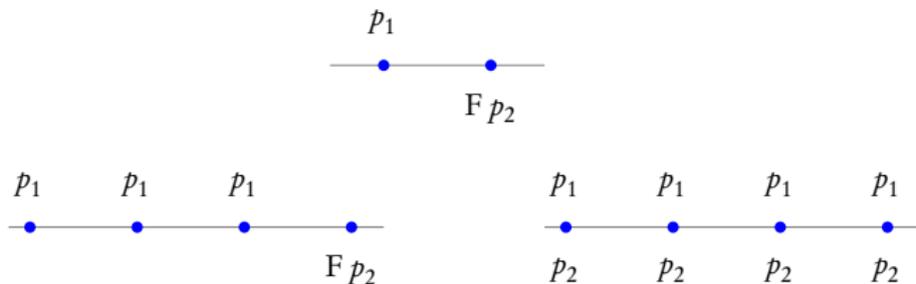
$G(p_1 \rightarrow \text{XF } p_2)$



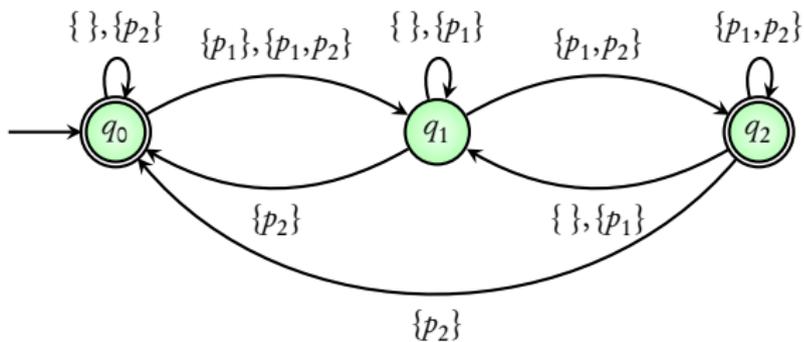
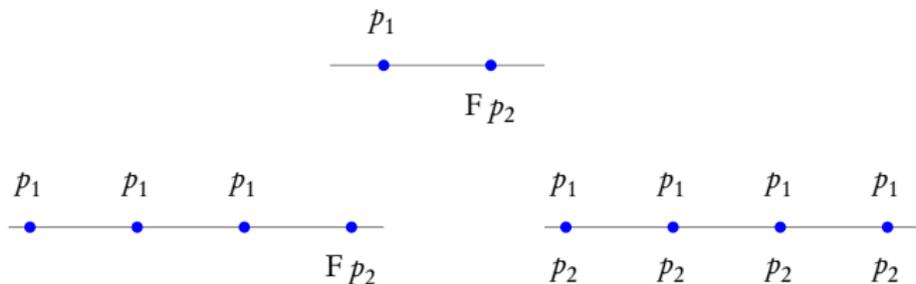
$G(p_1 \rightarrow \text{XF } p_2)$



$G(p_1 \rightarrow \text{XF } p_2)$



$G(p_1 \rightarrow \text{XF } p_2)$



Summary

LTL model-checking

Method

LTL to NBA examples