Instructions: First question carries 10 marks. Other questions carry 8 marks each. Full credit will be given only for precise answers. Make appropriate assumptions.

1. Let ϕ, ψ and χ be LTL formulas. We say two formulas are *equivalent*, written as $\phi \equiv \psi$ if they define the same language. For each of the following, prove or disprove the equivalences:
   (a) $\boldsymbol{G}(\phi \wedge \psi) \equiv (\boldsymbol{G}\phi) \wedge (\boldsymbol{G}\psi)$
   (b) $\boldsymbol{GFG}\phi \equiv \boldsymbol{FGF}\phi$
   (c) $\boldsymbol{X}(\phi \boldsymbol{U} \psi) \equiv (\boldsymbol{X}\phi) \boldsymbol{U} (\boldsymbol{X}\psi)$
   (d) $(\phi \boldsymbol{U} \psi) \boldsymbol{U} \chi \equiv \phi \boldsymbol{U} (\psi \boldsymbol{U} \chi)$

2. Let $\Sigma = \{a, b\}$. Define:

   $L_{b \geq a} := \{ \alpha \in \Sigma^\omega \mid$ in every finite prefix of α, the number of b's is greater than or equal to the number of a's $\}$

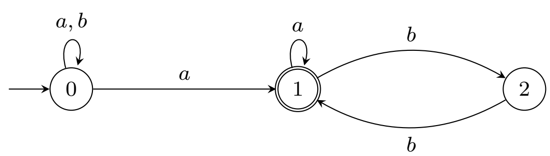
   Is $L_{b \geq a}$ ω-regular? Justify.

3. We say that an ω-automaton *co-Büchi recognizes* an ω-word α if there is a run ρ of the automaton on α in which after some point onwards, only final states are visited, i.e., there is an $n \geq 0$ such that for all $i > n$, $\rho(i)$ is a final state.
   (a) Show that L is recognizable by a deterministic co-Büchi automaton iff its complement $\overline{L}$ is recognizable by a deterministic Büchi automaton.
   (b) Is the following language recognizable by a deterministic co-Büchi automaton? Justify your answer:

   $$L := \{ \alpha \in \{a, b\} \mid \alpha \text{ contains } ab \text{ infinitely often, but } bb \text{ only finitely often} \}$$

4. Give an example of an NBA $\mathcal{A}$ on which applying the normal subset construction gives a DBA which is not equivalent to $\mathcal{A}$, but applying the marked subset construction gives an equivalent DBA (in other words, an example where subset construction is wrong, but marked subset construction is correct).

5. Use Safra's method to construct an equivalent DRA for the following NBA:



6. Prove or disprove the following statement: an ω-language is ω-regular iff it is expressible as a Boolean combination of languages of the form lim U where U is a regular $*$-language.