CS364A: Exercise Set #2

Due by the beginning of class on Wednesday, October 9, 2013

Instructions:

(1) Turn in your solutions to all of the following exercises directly to one of the TAs (Kostas or Okke). Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page. Email your solutions to `cs364a-aut1314-submissions@cs.stanford.edu`. If you prefer to hand-write your solutions, you can give it to one of the TAs in person at the start of the lecture.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (via Piazza or office hours) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 3 Exercises

Exercise 9

Use Myerson's Lemma to prove that the Vickrey auction is the unique single-item auction that is DSIC, always awards the good to the highest bidder, and charges losers 0.

Exercise 10

Use the "payment difference sandwich" in the proof of Myerson's Lemma to prove that if an allocation rule is not monotone, then it is not implementable.

Exercise 11

We concluded the proof of Myerson's Lemma by giving a "proof by picture" that coupling a monotone and piecewise constant allocation rule $\mathbf{x}$ with the payment formula

$$p_i(b_i, \mathbf{b}_{-i}) = \sum_{j=1}^{\ell} z_j \cdot [\text{jump in } x_i(\cdot, \mathbf{b}_{-i}) \text{ at } z_j], \tag{1}$$

where $z_1, \ldots, z_\ell$ are the breakpoints of the allocation function $x_i(\cdot, \mathbf{b}_{-i})$ in the range $[0, b_i]$, yields a DSIC mechanism. Where does the proof-by-picture break down if the piecewise constant allocation rule $\mathbf{x}$ is not monotone?

Exercise 12

Give a purely algebraic proof that coupling a monotone and piecewise constant allocation rule $\mathbf{x}$ with the payment rule (1) yields a DSIC mechanism.

Lecture 4 Exercises

Exercise 13

Consider the following extension of the sponsored search setting discussed in lecture. Each bidder i now has a publicly known *quality* β_i (in addition to a private valuation v_i per click). As usual, each slot j has a CTR α_j, and $\alpha_1 \geq \alpha_2 \cdots \geq \alpha_k$. We assume that if bidder i is placed in slot j, its probability of a click is $\beta_i\alpha_j$ — thus, bidder i derives value $v_i\beta_i\alpha_j$ from this outcome.

Describe the surplus-maximizing allocation rule in this generalized sponsored search setting. Argue that this rule is monotone. Give an explicit formula for the per-click payment of each bidder that extends this allocation rule to a DSIC mechanism.

Exercise 14

Consider an arbitrary single-parameter environment, with feasible set X. The surplus-maximizing allocation rule is $\mathbf{x}(\mathbf{b}) = \arg\max_{(x_1,\ldots,x_n)\in X} \sum_{i=1}^n b_i x_i$. Prove that this allocation rule is monotone.

[You should assume that ties are broken in a deterministic and consistent way, such as lexicographically.]

Exercise 15

Continuing the previous exercise, restrict now to feasible sets X that contain only 0-1 vectors – that is, each bidder either wins or loses. We can thus identify each feasible outcome with a "feasible set" of bidders (the winners in that outcome). Assume further that the environment is "downward closed," meaning that subsets of a feasible set are again feasible.

Recall from lecture that Myerson's payment formula dictates that a winning bidder pays its "critical bid" — the lowest bid at which it would continue to win. Prove that, when S^* is the set of winning bidders and $i \in S^*$, i's critical bid equals the difference between (i) the maximum surplus of a feasible set that excludes i (you should assume there is at least one such set); and (ii) the surplus $\sum_{j\in S^*\setminus\{i\}} v_j$ of the bidders other than i in the chosen outcome S^*. Also, is this difference always nonnegative?

Remark: In the above sense, a winning bidder pays its "externality" — the surplus loss it imposes on others.

Exercise 16

Continuing the previous exercise, consider a 0-1 downward-closed single-parameter environment. Suppose you are given a "black box" that can compute the surplus-maximizing allocation rule $\mathbf{x}(\mathbf{b})$ for an arbitrary input $\mathbf{b}$. Explain how to compute the payments identified in the previous exercise by invoking this black box multiple times.

Exercise 17

[Do not hand anything in.] Review the Knapsack problem and what one learns about it in an undergraduate algorithms class. Specifically: (i) it is NP-hard; (ii) with integer values and/or item sizes, it can be solved in pseudopolynomial time via dynamic programming; (iii) a simple greedy algorithm gives a $\frac{1}{2}$-approximation in near-linear time; (iv) rounding and dynamic programming gives a $(1-\epsilon)$-approximation in time polynomial in the number n of items and in $\frac{1}{\epsilon}$. Refer to your favorite algorithms textbook or to the videos by the instructor on the course site.

Exercise 18

Prove that the Knapsack auction allocation rule induced by the greedy $\frac{1}{2}$-approximation algorithm covered in lecture is monotone.

Exercise 19

The Revelation Principle states that direct-revelation DSIC mechanisms can simulate all other mechanisms in which bidders always have dominant strategies. Critique the Revelation Principle from a practical perspective. Name at least one situation in which you might prefer a non-direct-revelation DSIC mechanism over a direct-revelation one.