# PhD Defence Soutenance de Doctorat Automata on Timed Structures

#### Samy JAZIRI

#### Supervised by Patricia Bouyer-Decitre and Nicolas Markey

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#### States of the Heater





heating

doing nothing











abnormaly heating out of order

PhD Defence





## Modeling

























#### Weakness of Times Automata



Figure: A non-determinizable timed automaton [AD94]



Figure: Timed automaton where  $\epsilon$ -transitions can't be removed [BDGP98]

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**Discrete System** 



**Dynamic System** 





Une horloge donne l'heure, nous sommes bien d'accord, elle passe même ses heures à ne faire que cela, mais elle ne montre rien de ce qu'est le temps en amont de ce processus d'actualisation. Elle dissimule plutôt le temps derrière le masque convaincant d'une mobilité parfaitement régulière. – Etienne Klein, Les tactiques de Kronos



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#### Definition

A timed domain is a couple  $\langle V, \hookrightarrow \rangle$  where V is a set called value space and  $\hookrightarrow$  is a function from  $\mathbb{R}_{\geq 0}$  to  $\mathcal{P}(V \times V)$  called *delay transition*, such that

• for all 
$$v \in V$$
,  $(v,v) \in {\hookrightarrow}(0)$ 

- for all  $v \in V$  and  $d \in \mathbb{R}_{\geq 0}$ , there exists  $v' \in V$  such that  $(v, v') \in \, \hookrightarrow (d)$
- for all  $d, d' \in \mathbb{R}_{\geq 0}$ , for all  $v, v', v'' \in V$ ,

 $\text{if }(v,v')\in {\hookrightarrow}(d) \text{ and } (v',v'')\in {\hookrightarrow}(d') \text{ then }(v,v'')\in {\hookrightarrow}(d+d')$ 

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Example: The M-bounded clock domain is

$$\langle [0; M] \cup \{\infty\}, \hookrightarrow \rangle$$

In the 5-bounded clock domain 3  $\stackrel{1}{\hookrightarrow}$  4  $\stackrel{1.2}{\longrightarrow}\infty$ 

## **Timed Structures**



5

20

## **Timed Structures**


# **Timed Structures**



# **Timed Structures**

#### Definition

Let  $D = \langle V, \hookrightarrow \rangle$  be a timed domain. An *update set* on D is a subset of  $V^V$ .

### Definition

A timed structure is a triple  $T = \langle V, \hookrightarrow, U \rangle$  where  $\langle V, \hookrightarrow \rangle$  is a timed domain and U is an update set on  $\langle V, \hookrightarrow \rangle$ .

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*Example*: The one-dimensional clock structure can be equipped with the two updates **id** and **0** to form the M-clock structure :

 $\langle [\mathbf{0}; \textit{M}] \cup \{\infty\}, \hookrightarrow, \{ \mathsf{id}, \mathbf{0} \} \rangle$ 

## $\mathsf{Timed Structure}~(\mathsf{TS})~~\langle [0;2]\cup\{\infty\}, \hookrightarrow, \{\mathsf{id}, \mathbf{0}\}\rangle$

### Definition

Given a timed structure  $S = (V, \hookrightarrow, U)$  we call *guard* on S any subset of  $V \times U$ .

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 $\mathsf{Timed Structure} \ (\mathsf{TS}) \quad \langle [0;2] \cup \{\infty\}, \hookrightarrow, \{\mathsf{id}, \mathbf{0}\} \rangle$ 

Guard TA Notations  $\begin{array}{l} [0,1[\times\{\mathbf{0}\}\cup\{\left(\frac{\pi}{2},\mathsf{id}\right)\}\\ 0\leq x<1; x:=0 \text{ or } x=\frac{\pi}{2} \end{array}$ 

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Timed Structure (TS) $\langle [0; 2] \cup \{\infty\}, \hookrightarrow, \{id, 0\} \rangle$ Guard $[0, 1[\times \{0\} \cup \{(\frac{\pi}{2}, id)\}]$ TA Notations $0 \le x < 1; x := 0$  or  $x = \frac{\pi}{2}$ Guard Basis $\mathcal{G} = \{\{0\}, ]0; 1[, \{1\}, ]1; 2[, \{2\}, \{\infty\}\} \times \{0, id\}$ TA Notationsx = 0, 0 < x < 1, x = 1, 1 < x < 2, x = 2, x > 2with a potential reset on x

Example

$$\begin{split} & [1,2[\times\{\textbf{0},\textbf{id}\}=\\ & \{(1,\textbf{0})\}\cup]1,2[\times\{\textbf{0}\}\cup\{(1,\textbf{id})\}\cup]1,2[\times\{\textbf{id}\} \end{split}$$

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Guard Basis TA Notations	$\begin{split} \mathcal{G} &= \{\{0\}, ]0; 1[, \{1\}, ]1; 2[, \{2\}, \{\infty\}\} \times \{\textbf{0}, \textbf{id}\} \\ &x = 0, 0 < x < 1, x = 1, 1 < x < 2, x = 2, x > 2 \\ &\text{with a potential reset on } x \end{split}$
Example	$ \begin{split} & [1,2[\times\{\textbf{0},\textbf{id}\} = \\ & \{(1,\textbf{0})\}\cup]1,2[\times\{\textbf{0}\}\cup\{(1,\textbf{id})\}\cup]1,2[\times\{\textbf{id}\} \end{split} $
Guarded TS	$\langle [0;2]\cup\{\infty\}, \hookrightarrow, \{\text{id},0\}, \mathcal{G}\rangle$

### Definition

Let  $S = \langle V, \hookrightarrow, U \rangle$  be a timed structure and let G be a guard basis on S. The tuple  $S_G = \langle V, \hookrightarrow, U, G \rangle$ , is named guarded timed structure.

#### Definition

Fix  $\mathcal{S} = \langle V, \hookrightarrow, U \rangle$ , a timed structure. An automaton on  $\mathcal{S}$  is a tuple

 $A = \langle Q, I, T, F \rangle$ 

where Q is a finite set of states,  $I \subseteq Q \times V$  is the set of initial configurations,  $T \subseteq Q \times V \times U \times Q$  is the transition relation, and  $F \subseteq Q$  is the set of final states.

We write  $\mathbb{A}(S)$  for the set of automata over the timed structure S.

If  $S_{\mathcal{G}}$  is a guarded timed structure and the guards of an automaton A are decomposable on the guard basis  $\mathcal{G}$  we say that A is compatible with  $\mathcal{G}$  and write  $A \in \mathbb{A}(S_{\mathcal{G}})$ .



An automaton  $A = \langle Q, I, T, F \rangle$  induces an labeled infinite-state transition system on  $Q \times V$ .

$$(q, v) \xrightarrow{d} (q', v') \quad \Leftrightarrow \quad q = q' \text{ and } v \xrightarrow{d} v'$$
  
 $(q, v) \xrightarrow{u} (q', v') \quad \Leftrightarrow \quad (q, v, u, q') \in T \text{ and } v' = u(v).$ 

 $\textit{Example:} A \text{ trace of an automaton in } \mathbb{A}(\langle [0;2] \cup \{\infty\}, \hookrightarrow, \{\text{id}, 0\} \rangle).$ 

$$(q_1,0) \stackrel{1.2}{\longrightarrow} (q_1,1.2) \stackrel{\mathbf{0}}{\rightarrow} (q_2,0) \stackrel{3}{\rightarrow} (q_2,\infty) \stackrel{\mathsf{id}}{\rightarrow} (q_3,\infty)$$





All runs recognizing  $(a, id) \cdot (a, 0) \cdot 1 \cdot (a, id)$ :

$$(q_1,0) \xrightarrow{(a,\mathsf{id})} (q_1,0) \xrightarrow{(a,0)} (q_2,0) \xrightarrow{1} (q_2,1) \xrightarrow{(a,\mathsf{id})} \begin{cases} (q_3,1) \\ (q_2,1) \end{cases}$$



All runs recognizing  $a \cdot a \cdot 1 \cdot a$ :

$$(q_{1},0) \xrightarrow{a} \begin{cases} (q_{1},0) \xrightarrow{a} \\ (q_{1},0) \xrightarrow{a} \\ (q_{2},0) \xrightarrow{1} (q_{2},1) \xrightarrow{a} \\ (q_{2},0) \xrightarrow{1} (q_{2},1) \xrightarrow{a} \\ (q_{2},1) \\ (q_{2},0) \xrightarrow{a} (q_{2},0) \xrightarrow{1} (q_{2},1) \xrightarrow{a} \\ (q_{3},1) \\ (q_{3},1) \end{cases}$$

#### Theorem

Let  $\Sigma$  a finite alphabet, S a timed structure and A an automaton on Sequipped with a timed control  $\kappa$  over  $\Sigma$  and without silent transitions. A is  $\Sigma$ -determinizable ; i.e. there exists A' an automaton on  $\mathbf{D}_{A,\kappa}S$ equipped with a timed control  $\mathbf{D}\kappa$  such that A' is deterministic w.r.t  $\mathbf{D}\kappa$ and A' recognize the same language as A. Moreover, if A is finitely representable, we can construct a determinized automaton of A with a finite representation.

# General Powerset Construction



Simulation of  $a \cdot 2 \cdot a \cdot 0.3 \cdot a \cdot 0.4 \cdot a \cdot 0.6 \cdot a$ 

## General Powerset Construction





Samy JAZIR

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### Example of Powerset Construction



# **Classical Determinization**

- Bounded Sets Case  $\longrightarrow$  [BBBB09] <sup>1</sup>
- Strongly non-Zeno Timed Automata
- 0-Bounded Timed Automata [OW04]<sup>2</sup>
- Integer Reset Timed Automata [SPKM08] <sup>3</sup>
- Finally Imprecise Timed Automata

<sup>&</sup>lt;sup>1</sup>Baier, Bertrand, Bouyer, and Brihaye. When Are Timed Automata Determinizable? *ICALP09* 

 $<sup>^{2}</sup>$ Ouaknine and Worrell. On the language inclusion problem for timed automata: Closing a decidability gap. *LICS04* 

#### Theorem

Let  $\Sigma$  a finite alphabet,  $S_G$  a guarded deterministic timed structure where U is finite and  $A \in \mathbb{A}(S_G)$  equipped with a compatible  $\Sigma$ -full control. Then A can be  $\Sigma$ -determinized in the same guarded timed structure  $S_G$ .

<sup>4</sup>Alur, Fix, and Henzinger. A determinizable class of timed automata. CAV94
<sup>5</sup>Abdulla, Atig, and Stenman. Dense-timed pushdown automata LICS12
<sup>6</sup>Bhave and Guha. Adding dense-timed stack to integer reset timed automata. RP17

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### **Applications.**

- Event-clock automata [AFH94] <sup>4</sup>
- (Timed) Visibly Pushdown Automata [AAS] <sup>5</sup>
- Strict Integer Reset Timed Automata [BG17] <sup>6</sup>

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## **Toward Diagnosis**



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### Timed Automaton Powerset construction

$\xrightarrow{a}$	U <sub>a</sub>	
$\xrightarrow{d}$	$\mathbf{U}_d$	

### Timed Automaton Powerset construction

$\xrightarrow{a}$	<b>U</b> <sub>a</sub>	easy to compute
$\xrightarrow{d}$	$\mathbf{U}_d$	

### Timed Automaton Powerset construction

$\xrightarrow{a}$	<b>U</b> <sub>a</sub>	easy to compute
$\xrightarrow{d}$	$\mathbf{U}_d$	difficult to compute

# What's left to be done

Timed Automaton	Powerset construction	
$\xrightarrow{a}$	<b>U</b> <sub>a</sub>	easy to compute
$\xrightarrow{d}$	$U_d$	difficult to compute
	U.a	
	$\nu \longrightarrow \nu'$	
	¥ ¥ ¥	
	$I \longrightarrow I'$	
	$\epsilon, +a$	



### $\boldsymbol{U}_0$

 $q_0: \{0\}$ 

 $q_1$ : Ø



### $U_0$ $U_2$

 $q_0: \{0\}$  {2}

 $q_1: \emptyset \qquad \{2\}$ 



 $U_0 U_2 U_{3.5}$ 

 $q_0: \{0\}$  {2} {3.5}  $\cup$  [0; 0.5[

 $q_1: \quad \emptyset \qquad \{2\} \qquad \qquad \{3.5\}$ 
















$U_5$  $q_0: \{5\} \cup [1,2[$ 

#### $q_1: \{5\} \cup [1,2[$





 $U_6$  $q_0: \{6\} \cup [2,3[$ 







 $U_7$  $q_0:\{7\} \cup [3,4[ \cup [0,1[$ 











 $q_1:\{8\} \cup [4,5[\cup[1,2[$ 







 A delay transition on the powerset automaton is equivalent to an addition on regular timed intervals once ε have been applied

• The operator  $\epsilon$  is computable by sequential set theoretic operations

• Most of interval part of the computation can be precomputed

• Computing a delay can be done in constant time once the  $\epsilon$  have been applied

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• pprox 5500 lines of Python code

• Implementation of both methods

• Library for Regular Timed Intervals

• Several ways of using the diagnoser using channels

DOTA



	Example2	Example3	Example4	Example6	Example8
#State/#Silent Trans	3/6	4/6	4/7	7/10	7/5
Precomputation Time	173.25s	0.38s	791.06s	11.01s	4.96s
Actions DiagOTA	0.014s	0.019s	0.029s	0.17s	0.15s
Actions TripakisDOTA	0.020s	0.078s	0.049s	0.26s	0.042
Ratio (actions)	0.73	0.25	0.59	0.64	3.71
Delays DiagOTA	0.000012s	0.0000011s	0.000011s	0.000011s	0.000012s
Delays TripakisDOTA	0.032s	0.057 s	0.049s	0.30s	0.033s
Ratio (delays)	0.0004	0.0002	0.0002	0.00003	0.0004

Bench for 5 examples over 400 runs with 10 to 20 actions

- New quantitative model : Automata on Timed Structures
- General powerset construction
- Comparison with existing work
- Exploitation of the powerset construction in the context of diagnosis
- Comparison of our diagnoser with the diagnoser constructed by Tripakis

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#### Perspectives

- Improve DOTA and the benchmark
- Extend the diagnoser construction to *n*-clock timed automata
- Export classical theoretical result in the framework of automata on timed structures.

### Thank You

