

Distributed synthesis: synchronous and asynchronous semantics

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Outline

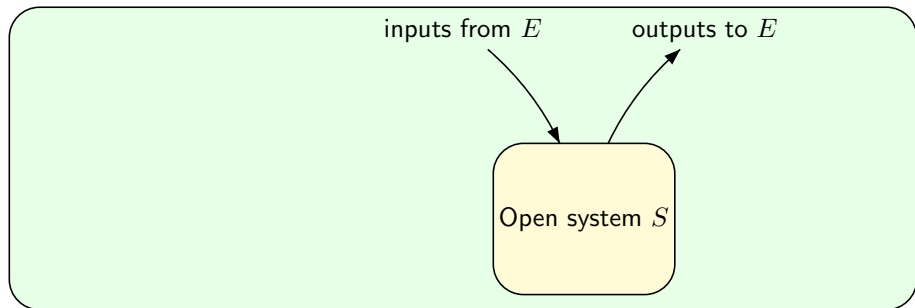
① Control for sequential systems

Control for distributed systems

Synchronous semantics

Asynchronous semantics

Open / Reactive system



Model for the open system

- ▶ Transitions system $\mathcal{A} = (Q, \Sigma, q_0, \delta)$
 - ▶ Q : finite or infinite set of states,
 - ▶ δ : deterministic or non deterministic transition function.
- ▶ $\Sigma = \Sigma_c \uplus \Sigma_{uc}$ Controllable / Uncontrollable events.
- ▶ $\Sigma = \Sigma_o \uplus \Sigma_{uo}$ Observable / Unobservable events.

Example: Elevator

Transition system

- ▶ States:
 - ▶ position of the cabin
 - ▶ flag `is_open` for each door
 - ▶ flag `is_called` for each level
 - ▶ number of persons in the cabin
- ▶ Events:

	Σ_o	Σ_{uo}
Σ_{uc}	call level i	enter/exit cabin
Σ_c	open/close door i move 1 level up/down	

- ▶ We get easily a finite and deterministic transition system.

Specification

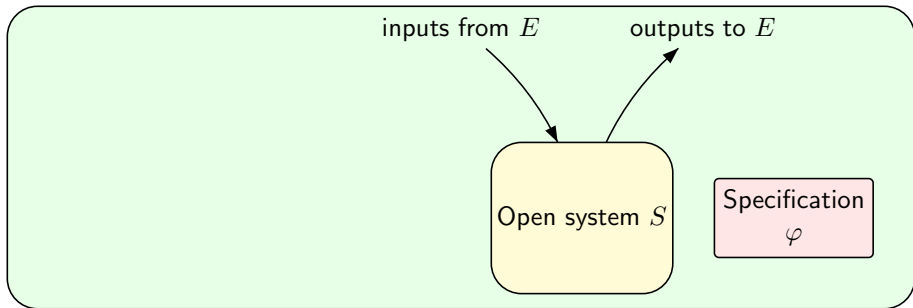
Linear time: LTL, FO, MSO, regular, ...

- ▶ Safety: $G(\text{level} \neq i \longrightarrow \text{is_closed}_i)$
- ▶ Liveness: $G(\text{is_called}_i \longrightarrow F(\text{level} = i \wedge \text{is_open}_i))$

Branching time: CTL, CTL*, μ -calculus, ...

- ▶ $AG\langle \text{call}_i \rangle T$ (call_i is uncontrollable)
- ▶ $AG EF(\text{level} = 0 \wedge \text{is_open}_0)$

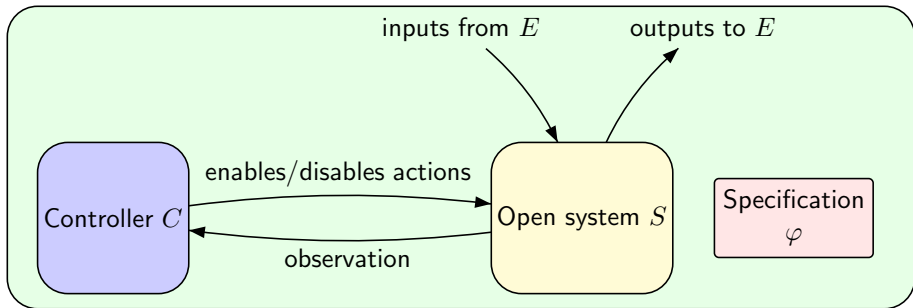
Control problem



Two problem

- ▶ Control: Given a system S and a specification φ , decide whether there exists a controller C such that $S \otimes C \models \varphi$.
- ▶ Synthesis: Given a system S and a specification φ , build a controller C (if one exists) such that $S \otimes C \models \varphi$.

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Controller

Under full state-event observation

- ▶ Controller: $f : Q(\Sigma Q)^* \rightarrow 2^\Sigma$ with $\Sigma_{uc} \subseteq f(x)$ for all $x \in Q(\Sigma Q)^*$.
- ▶ Controlled behavior: $q_0, a_1, q_1, a_2, q_2, \dots$ with $(q_{i-1}, a_i, q_i) \in \delta$ and $a_i \in f(q_0 a_1 q_1 \dots q_{i-1})$ for all $i > 0$.
- ▶ Controlled execution tree: $t : D^* \rightarrow \Sigma \times Q$ with
 - ▶ $t(\varepsilon) = (a, q_0)$ ($a \in \Sigma$ fixed arbitrarily)
 - ▶ for all $x = d_1 \dots d_n \in D^*$ with $t(d_1 \dots d_i) = (a_i, q_i)$, we have $t(\text{sons}(x)) = \{(a, q) \mid a \in f(q_0 a_1 q_1 \dots a_n q_n) \text{ and } (q_n, a, q) \in \delta\}$.

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- ▶ Controller: $f : \Sigma^* \rightarrow 2^\Sigma$ with $\Sigma_{uc} \subseteq f(x)$ for all $x \in \Sigma^*$.
Remark: same as full state-event observation if the system is deterministic.

Under partial event observation

- ▶ Controller: $f : \Sigma_o^* \rightarrow 2^\Sigma$ with $\Sigma_{uc} \subseteq f(x)$ for all $x \in \Sigma_o^*$.
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Control versus Game

Correspondance

Transition system	=	Game arena (graph).
Controllable events	=	Actions of player 1 (controller).
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Behavior	=	Play.
Controller	=	Strategy.
Specification	=	Winning condition.
Finding a controller	=	finding a winning strategy.

Control problem

Given a system S and a specification φ , does there exist a controller C such that $\mathcal{L}(C \otimes S) \subseteq \mathcal{L}(\varphi)$?

Theorem

If the system is **finite state** and the specification is **regular** then the control problem is **decidable**.

Moreover, when (S, φ) is controllable, we can synthesize a **finite state** controller.

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Ramadge - Wonham 87 →

Control problem (Exact)

Given a system S (with accepting states) and a specification $K \subseteq \Sigma^*$, does there exist a controller C such that $\mathcal{L}(C \otimes S) = K$?

Theorem

- ▶ $(S, \text{Pref}(K))$ is controllable iff $\text{Pref}(K) \cdot \Sigma_{uc} \cap \text{Pref}(\mathcal{L}(S)) \subseteq \text{Pref}(K)$.
- ▶ (S, K) is controllable without deadlock iff
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- ▶ If S is **finite state** and K **regular** then the control problem is decidable. When (S, K) is controllable, we can synthesize a **finite state** controller.

Other results

- ▶ control under partial observation
- ▶ maximal controllable sub-specification
- ▶ generalization to infinite behaviors (Thistle - Wonham)
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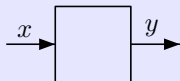
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Synthesis of reactive programs

Pnueli-Rosner 89



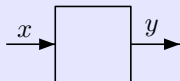
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- ▶ Program: $f : Q_x^+ \rightarrow Q_y$
- ▶ Input: $x_1 x_2 \cdots \in Q_x^\omega$.
- ▶ Behavior: $(x_1, y_1)(x_2, y_2)(x_3, y_3) \cdots$ with $y_n = f_1(x_1 \cdots x_n)$ for all $n > 0$.

Implementability problem

- ▶ Given a linear time specification φ over the alphabet $\Sigma = Q_x \times Q_y$,
Does there exist a program f such that all f -behaviors satisfy φ ?
- ▶ Given a branching time specification φ over the alphabet $\Sigma = Q_x \times Q_y$,
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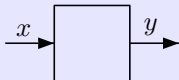
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Implementability \neq Satisfiability

- ▶ $Q_x = \{0, 1\}$ and $\varphi = F(x = 1)$
- ▶ φ is satisfiable: $(1, 0)^\omega \models \varphi$
- ▶ φ is not implementable since the input is not controllable.

Implementability \neq Validity of $\forall \vec{x} \exists \vec{y} \varphi$

- ▶ $Q_x = Q_y = \{0, 1\}$ and $\varphi = (y = 1) \iff F(x = 1)$
- ▶ $\forall \vec{x} \exists \vec{y} \varphi$ is valid.
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Theorem (Pnueli-Rosner 89)

- ▶ The specification $\varphi \in \text{LTL}$ is implementable iff the formula

$$\mathcal{A}\varphi \wedge \text{AG} \left(\bigwedge_{a \in Q_x} \text{EX}(x = a) \right)$$

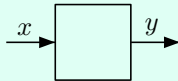
is satisfiable.

- ▶ When φ is implementable, we can construct a finite state implementation (program) in time doubly exponential in φ .

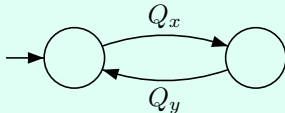
Program synthesis versus System control

Equivalence

The implementability problem for



is equivalent to the control problem for the system



Outline

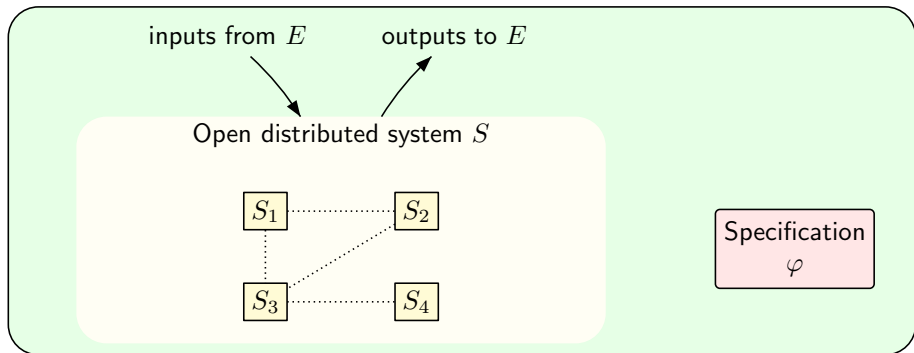
Control for sequential systems

2 Control for distributed systems

Synchronous semantics

Asynchronous semantics

Distributed control



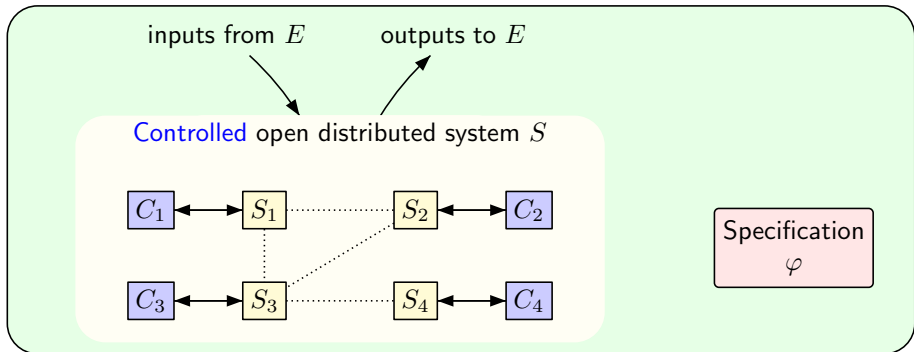
Two problems, again

- ▶ Decide whether there exists a **distributed** controller st.
 $(S_1 \otimes C_1) \parallel \dots \parallel (S_n \otimes C_n) \parallel E \models \varphi$.
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Peterson-Reif 1979, Pnueli-Rosner 1990

In general, the problems are **undecidable**.

Distributed control



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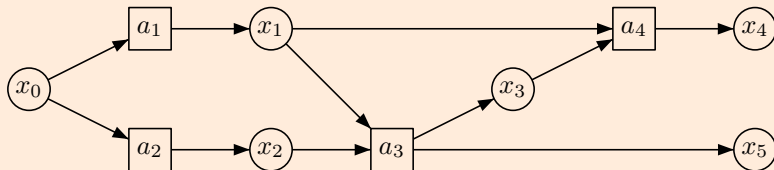
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Architectures with shared variables

Architecture $\mathcal{A} = (\mathcal{P}, \mathcal{V}, R, W)$

- ▶ \mathcal{P} finite set of **processes/agents**.
- ▶ \mathcal{V} finite set of **Variables**.
- ▶ $R \subseteq \mathcal{P} \times \mathcal{V}$: $(a, x) \in R$ iff a reads x .
 - ▶ $R(a)$ variables **read** by process $a \in \mathcal{P}$,
 - ▶ $R^{-1}(x)$ processes **reading** variable $x \in \mathcal{V}$.
- ▶ $W \subseteq \mathcal{P} \times \mathcal{V}$: $(a, x) \in W$ iff a writes to x .
 - ▶ $W(a)$ variables **written** by process $a \in \mathcal{P}$,
 - ▶ $W^{-1}(x)$ processes **writing** to variable $x \in \mathcal{V}$.

Example



Distributed systems with shared variables

Distributed system/plant/arena

- ▶ $\mathcal{A} = (\mathcal{P}, \mathcal{V}, R, W)$ architecture.
- ▶ Q_x (finite) domain for each variable $x \in \mathcal{V}$.
- ▶ $\delta_a \subseteq Q_{R(a)} \times Q_{W(a)}$ legal actions/moves for process/player $a \in \mathcal{P}$.
- ▶ $q^0 \in Q_{\mathcal{V}}$ initial state

where $Q_I = \prod_{x \in I} Q_x$ for $I \subseteq \mathcal{V}$.

Distributed Synthesis

Problem

Given a distributed system and a specification

Problem existence/synthesis of programs/strategies for the processes/players such that the system satisfies the specification (whatever the environment/opponent does).

Main parameters

▶ Which subclass of architectures?

▶ Which semantics?

synchronous (with or without delay), asynchronous

▶ What kind of specification?

LTL, CLT*, μ -calculus

Rational, Recognizable
word/tree

▶ What kind of memory for the programs?

memoryless, local memory, causal memory
finite or infinite memory

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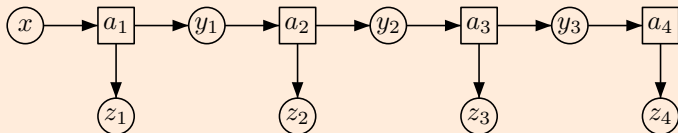
Control for distributed systems

3 Synchronous semantics

Asynchronous semantics

Pnueli-Rosner (FOCS'90)

Pipeline

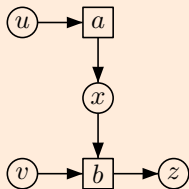


Restrictions

- ▶ Unique writer: $|W^{-1}(x)| = 1$ for all $x \in \mathcal{V}$
- ▶ Unique reader: $|R^{-1}(x)| = 1$ for all $x \in \mathcal{V}$
- ▶ Acyclic graph (0-delay)
- ▶ No restrictions on moves: $\delta_a = Q_{R(a)} \times Q_{W(a)}$ for all $a \in \mathcal{P}$.
- ▶ Synchronous behaviors: $q^0 q^1 q^2 \dots$ where $q^n \in Q_{\mathcal{V}}$ are global states.
- ▶ program with **local memory**: $f_a : Q_{R(a)}^* \rightarrow Q_{W(a)}$ for all $a \in \mathcal{P}$.
- ▶ Specification: LTL over input and output variables only.
 - ▶ Input variables: $\text{In} = W(\text{environment})$
 - ▶ output variables: $\text{Out} = R(\text{environment})$

0-delay synchronous semantics

Example



Programs: $f_x : Q_u^* \rightarrow Q_x$ and $f_z : (Q_x \times Q_v)^* \rightarrow Q_z$.

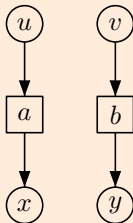
▶ Input: $\begin{pmatrix} u_1 & u_2 & u_3 & \cdots \\ v_1 & v_2 & v_3 & \cdots \end{pmatrix} \in (Q_u \times Q_v)^\omega$.

▶ Behavior: $\begin{pmatrix} u_1 & u_2 & u_3 & \cdots \\ v_1 & v_2 & v_3 & \cdots \\ x_1 & x_2 & x_3 & \cdots \\ z_1 & z_2 & z_3 & \cdots \end{pmatrix}$

with $\begin{cases} x_n = f_x(u_1 \cdots u_n) \\ z_n = f_z((x_1, v_1) \cdots (x_n, v_n)) \end{cases}$ for all $n > 0$.

Undecidability

Architecture \mathcal{A}_0



Theorem (Pnueli-Rosner FOCS'90)

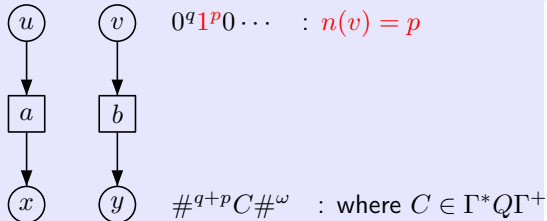
The synthesis problem for architecture \mathcal{A}_0 and LTL (or CTL) specifications is undecidable.

Proof

Reduction from the halting problem on the empty tape.

Undecidability proof 1

SPEC₁: processes a and b must output configurations



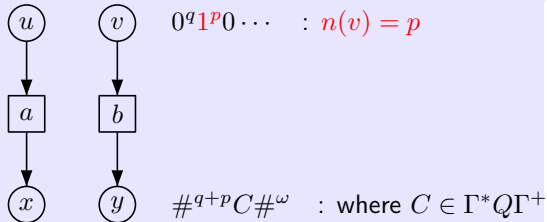
$$(v = 0 \wedge y = \#) W \left(v = 1 \wedge (v = 1 \wedge y = \#) W (v = 0 \wedge y \in \Gamma^* Q \Gamma^+ \#^\omega) \right)$$

where

$$y \in \Gamma^* Q \Gamma^+ \#^\omega \stackrel{\text{def}}{=} y \in \Gamma \cup \left(y \in Q \wedge \exists (y \in \Gamma \cup (y \in \Gamma \wedge \exists G y = \#)) \right)$$

Undecidability proof 1

SPEC₁: processes a and b must output configurations



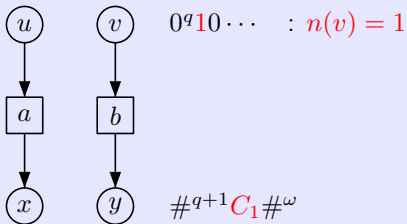
$$(v = 0 \wedge y = \#) \text{W} \left(v = 1 \wedge (v = 1 \wedge y = \#) \text{W} (v = 0 \wedge y \in \Gamma^* Q \Gamma^+ \#^\omega) \right)$$

where

$$y \in \Gamma^* Q \Gamma^+ \#^\omega \stackrel{\text{def}}{=} y \in \Gamma \cup \left(y \in Q \wedge \text{X} (y \in \Gamma \cup (y \in \Gamma \wedge \text{X} G y = \#)) \right)$$

Undecidability proof 2

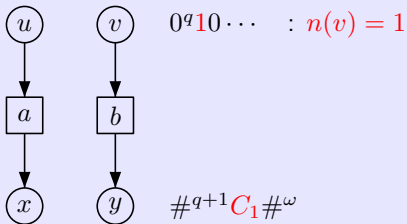
SPEC₂: processes a and b must start with the first configuration



$$v = 0 \text{ W } (v = 1 \wedge \text{X}(v = 0 \longrightarrow y \in C_1 \#^\omega))$$

Undecidability proof 2

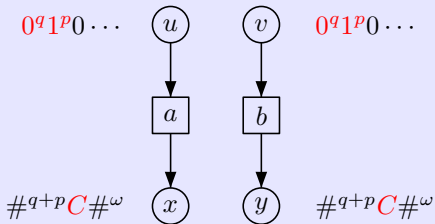
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Undecidability proof 3

SPEC₃: if $n(u) = n(v)$ are synchronized then $x = y$



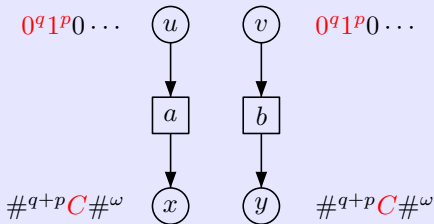
$$n(u) = n(v) \longrightarrow G(x = y)$$

where

$$n(u) = n(v) \stackrel{\text{def}}{=} (u = v = 0) \cup (u = v = 1 \wedge (u = v = 1 \cup u = v = 0))$$

Undecidability proof 3

SPEC₃: if $n(u) = n(v)$ are synchronized then $x = y$



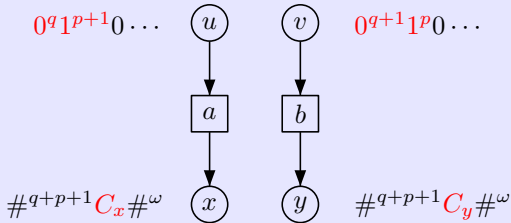
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Undecidability proof 4

SPEC₄: if $n(u) = n(v) + 1$ are synchronized then $C_y \vdash C_x$



$$n(u) = n(v) + 1 \quad \longrightarrow \quad x = y \cup \left(\text{Trans}(y, x) \wedge X^3 G x = y \right)$$

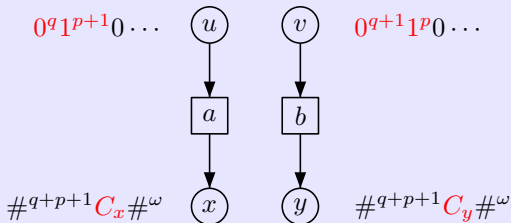
where $\text{Trans}(y, x)$ is defined by

$$\bigvee_{(p,a,q,b,\leftarrow) \in T, c \in \Gamma} (y = cpa \wedge x = qcb) \quad \vee \quad \bigvee_{(p,a,q,b,\rightarrow) \in T, c \in \Gamma} (y = pac \wedge x = bqc)$$

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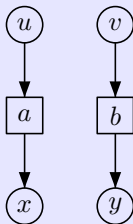
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Undecidability proof 5

Lemma: winning strategies must simulate the Turing machine

For each $p \geq 1$, if $n(u) = p$ then $C_x = C_p$ is the p -th configuration of the Turing machine starting from the empty tape.

Proof



Corollary

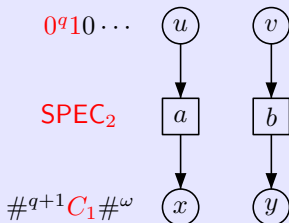
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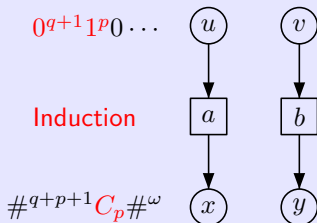
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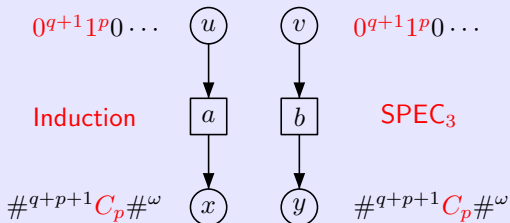
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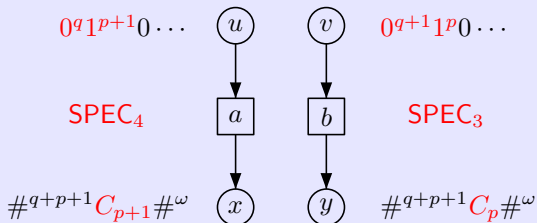
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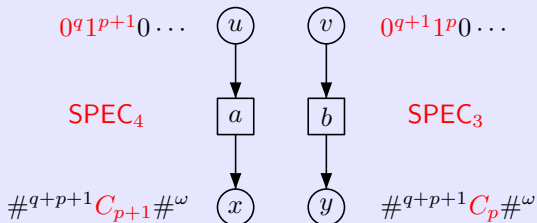
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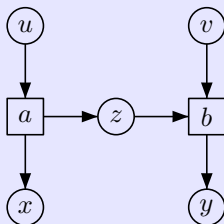


Corollary

Specifications 1-4 and 5: $Gx \neq \text{stop}$ are implementable iff the Turing machine does not halt starting from the empty tape.

Communication allows to cheat

Architecture with communication



► Strategy for a :

► copy u to z

► if $u = 0^q 1^p 0 \dots$ then $x = \begin{cases} \#^{p+q} C_1 \#^\omega & \text{if } p = 1 \text{ (for SPEC}_2\text{)} \\ \#^{p+q} C_2 \#^\omega & \text{otherwise (for SPEC}_4\text{)}. \end{cases}$

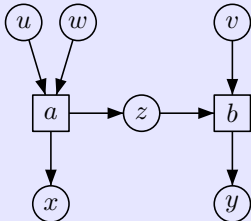
► Strategy for b : if $z = 0^{q'} 1^{p'} 0 \dots$ and $v = 0^q 1^p 0 \dots$ then

$$y = \begin{cases} \#^{p+q} C_1 \#^\omega & \text{if } p = 1 \text{ (for SPEC}_2\text{)} \\ \#^{p+q} C_2 \#^\omega & \text{if } p = p' > 1 \text{ and } q = q' \text{ (for SPEC}_3\text{)} \\ \#^{p+q} C_1 \#^\omega & \text{otherwise (for SPEC}_4\text{)}. \end{cases}$$

More undecidable architectures

Exercises

1. Show that the architecture below is undecidable.



2. Show that the undecidability results also hold for CTL specifications

Uncomparable information

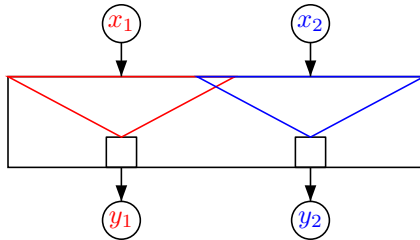
Definition

For an output variable y , $\text{View}(y)$ is the set of input variables x such that there is a path from x to y .

Definition

An architecture has **uncomparable information** if there exist y_1, y_2 output variables such that $\text{View}(y_2) \setminus \text{View}(y_1) \neq \emptyset$ and $\text{View}(y_1) \setminus \text{View}(y_2) \neq \emptyset$.

Otherwise it is said to have **preordered information**.



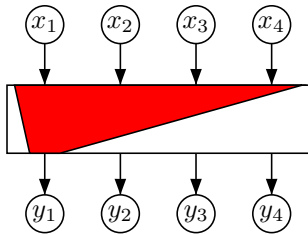
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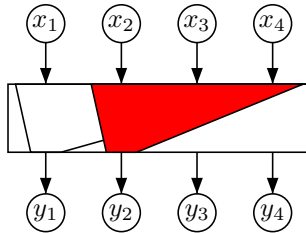
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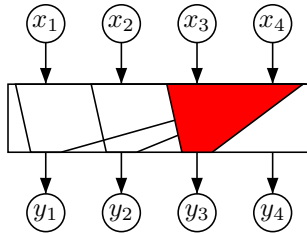
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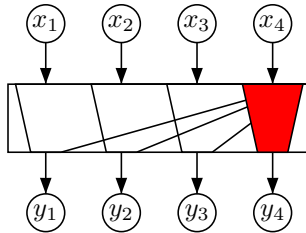
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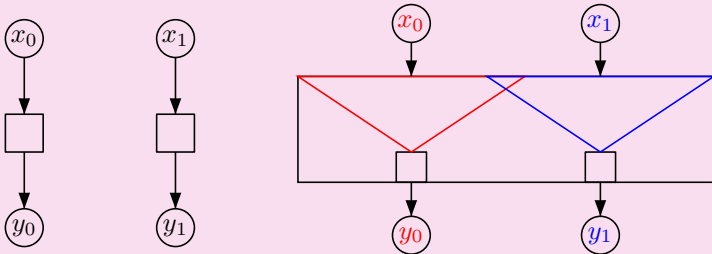


Uncomparable information yields undecidability

Theorem

Architectures with uncomparable information are undecidable for LTL or CTL input-output specifications.

Proof for LTL specifications

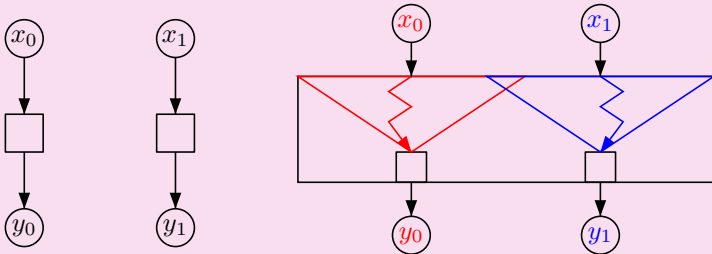


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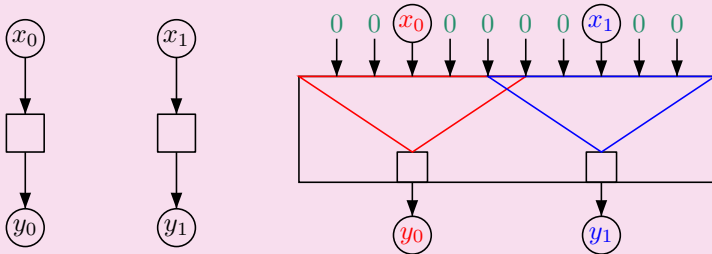


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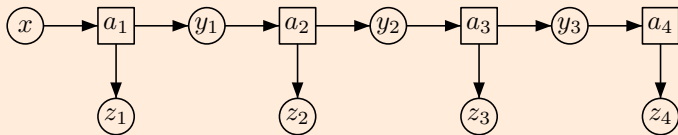
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Decidability

Pipeline

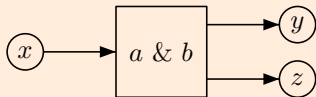
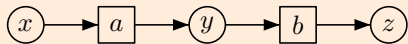


Pnueli-Rosner (FOCS'90)

The synthesis problem for pipeline architectures and LTL specifications is non elementary decidable.

Decidability proof 1

Pipeline



From distributed to global

If $f_y : Q_x^+ \rightarrow Q_y$ and $f_z : Q_y^+ \rightarrow Q_z$ are **local (distributed)** strategies then we can define an equivalent **global** strategy $h = f_y \otimes f_z : Q_x^+ \rightarrow Q_y \times Q_z$ by

$$h(x_1 \cdots x_n) = (y_n, f_z(y_1 \cdots y_n)) \quad \text{where} \quad y_i = f_y(x_1, \cdots, x_i).$$

From global to distributed

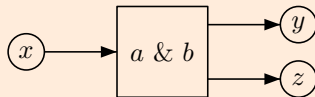
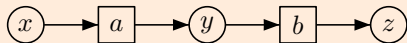
z should only depend on y .

We cannot transmit x to y if $|Q_y| < |Q_x|$.

We have to check whether **there exists** a global strategy that **can be distributed**.

Decidability proof 1

Pipeline



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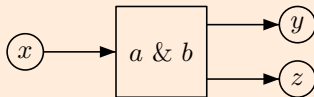
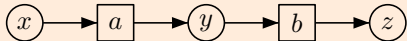
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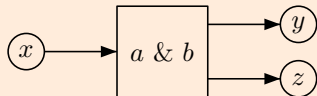
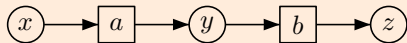
z should only depend on y .

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We have to check whether **there exists** a global strategy that **can be distributed**.

Decidability proof 2

Pipeline



Proof

1. We first solve the global game: We obtain an **ND** tree-automaton \mathcal{A} accepting the **global** strategies $h : Q_x^+ \rightarrow Q_y \times Q_z$ that implement the specification.
Easily obtained from a ND tree automaton for the specification.
2. We build from \mathcal{A} an **alternating** tree automaton \mathcal{A}' accepting a local strategy $f_z : Q_y^+ \rightarrow Q_z$ iff there exists a local strategy $f_y : Q_x^+ \rightarrow Q_y$ such that $h = f_y \otimes f_z : Q_x^+ \rightarrow Q_y \times Q_z$ is accepted by \mathcal{A}

Decidability proof 2

Pipeline

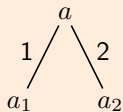


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Tree automata

non deterministic transitions

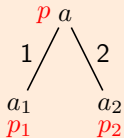


Alternating transitions

or

Tree automata

non deterministic transitions

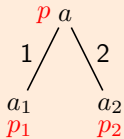


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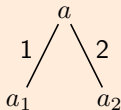
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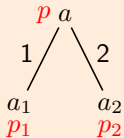
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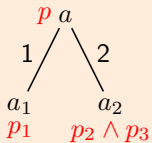
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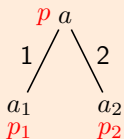
Alternating transitions



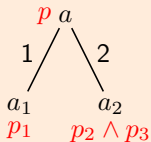
or

Tree automata

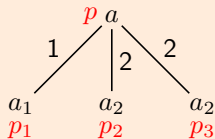
non deterministic transitions



Alternating transitions

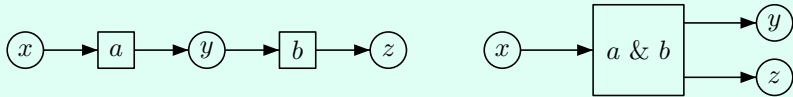


or



Decidability proof 3

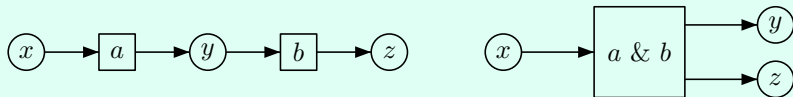
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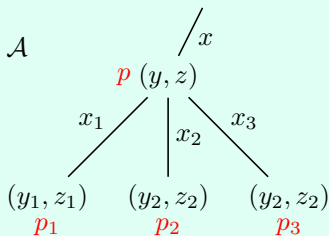
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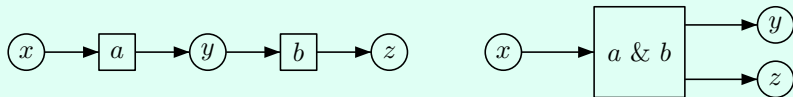


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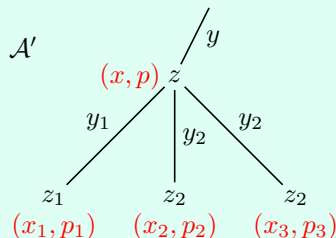
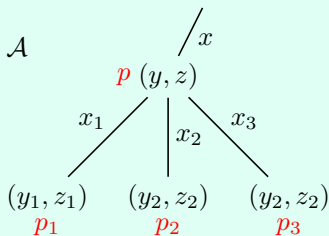


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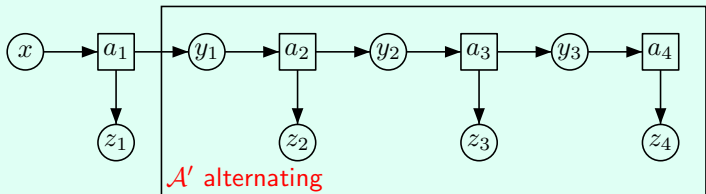


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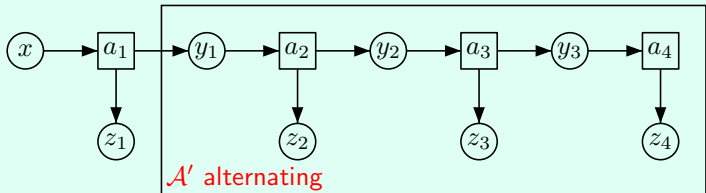
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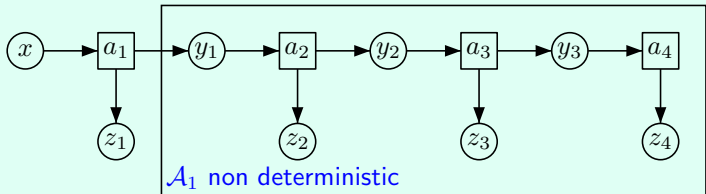
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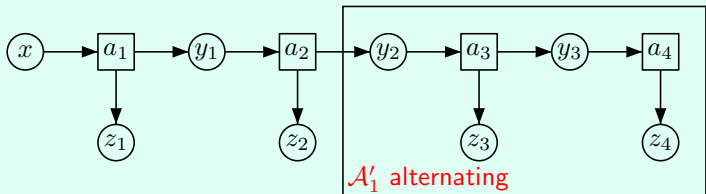
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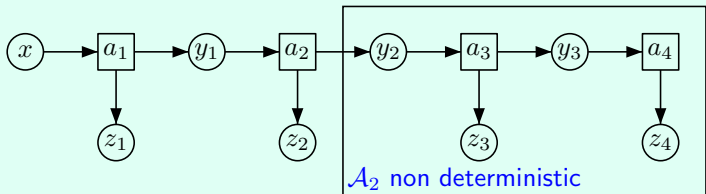
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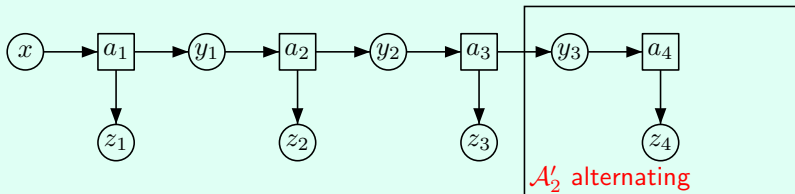
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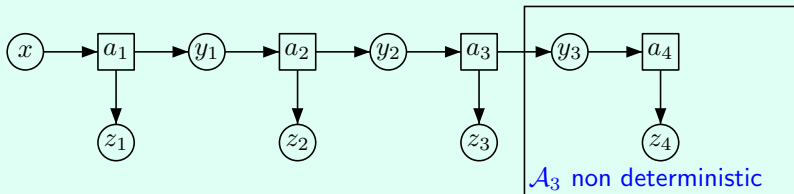
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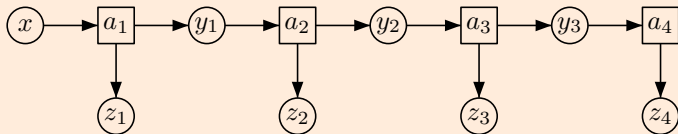
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Decidability

Pipeline



Pnueli-Rosner (FOCS'90)

The synthesis problem for pipeline architectures and LTL specifications is non elementary decidable.

Peterson-Reif (FOCS'79)

multi-person games with incomplete information.

⇒ non-elementary lower bound for the synthesis problem.

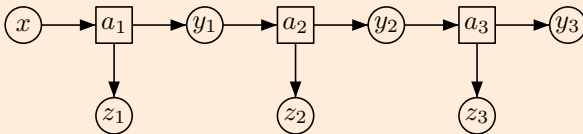
Decidability

Kupferman-Vardi (LICS'01)

The synthesis problem is non elementary decidable for

- ▶ one-way chain, one-way ring, two-way chain and two-way ring,
- ▶ CTL* specifications (or tree-automata specifications) **on all variables**,
- ▶ synchronous, 1-delay semantics,
- ▶ **local** strategies.

one-way chain



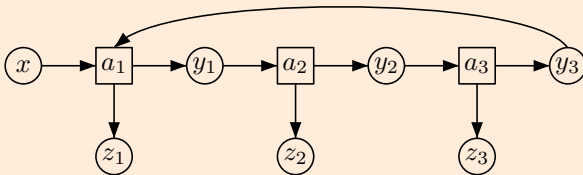
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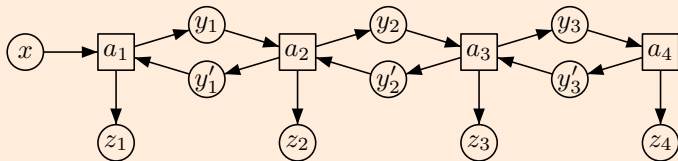
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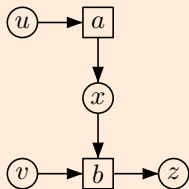
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two-way chain



1-delay synchronous semantics

Example



Programs: $f_x : Q_u^* \rightarrow Q_x$ and $f_z : (Q_x \times Q_v)^* \rightarrow Q_z$.

▶ Input: $\begin{pmatrix} u_1 & u_2 & u_3 & \cdots \\ v_1 & v_2 & v_3 & \cdots \end{pmatrix} \in (Q_u \times Q_v)^\omega$.

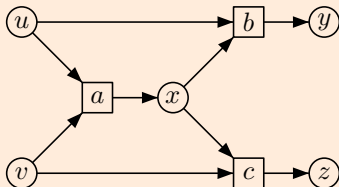
▶ Behavior: $\begin{pmatrix} u_1 & u_2 & u_3 & \cdots \\ v_1 & v_2 & v_3 & \cdots \\ x_1 & x_2 & x_3 & \cdots \\ z_1 & z_2 & z_3 & \cdots \end{pmatrix}$

with $\begin{cases} x_{n+1} = f_x(u_1 \cdots u_n) \\ z_{n+1} = f_z((x_1, v_1) \cdots (x_n, v_n)) \end{cases}$ for all $n > 0$.

Decidability

Adequately connected sub-architecture

$Q_x = Q$ for all $x \in \mathcal{V}$



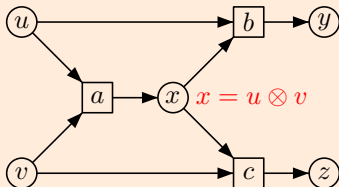
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- ▶ An adequately connected architecture is equivalent to a singleton architecture.
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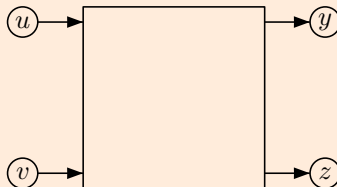
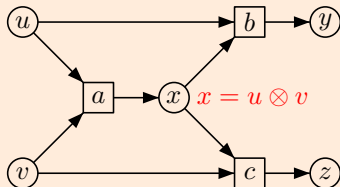
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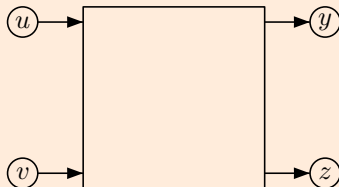
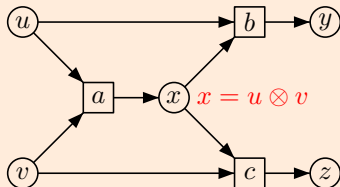
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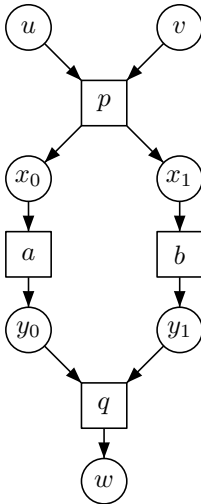
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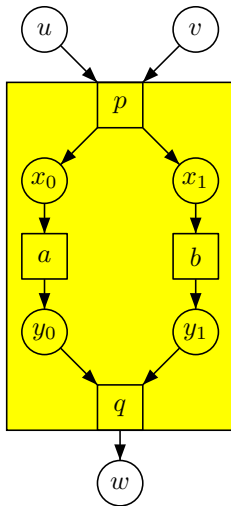
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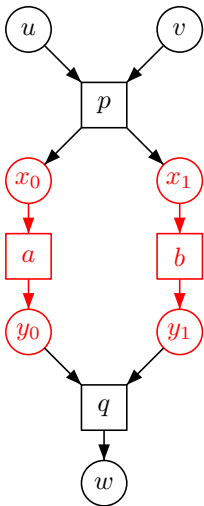
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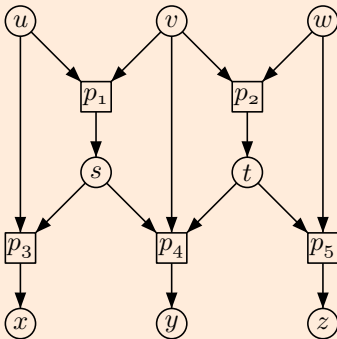


Uniformly well connected architectures

Definition

An architecture is uniformly well connected if there is a uniform way to route variables in $\text{View}(y)$ to y for each output variable y .

Example

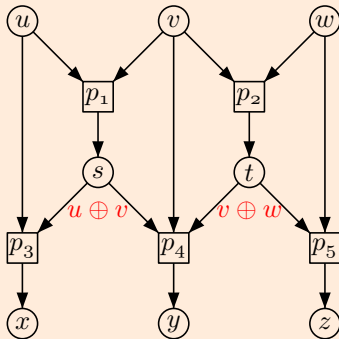


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- ▶ If the architecture is **uniformly well-connected** then we can use **causal strategies** instead of **local** ones.

Proposition

Checking whether a given architecture is uniformly well connected is NP-complete.

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Reduction to the multicast problem in Network Information Flow.
The multicast problem is NP-complete (Rasala Lehman-Lehman 2004).

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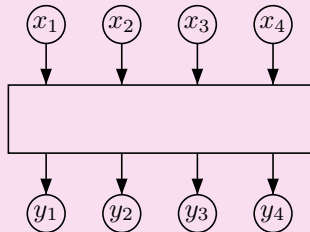
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Theorem (PG, Nathalie Sznajder, Marc Zeitoun)

Uniformly well connected architectures with preordered information are decidable for CTL* external specifications.

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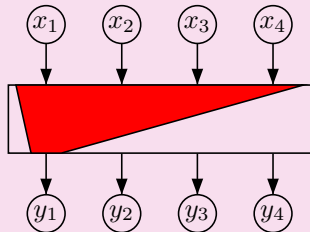
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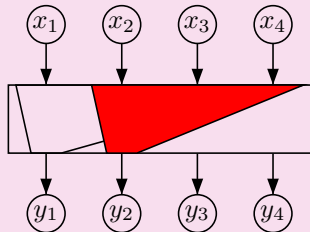
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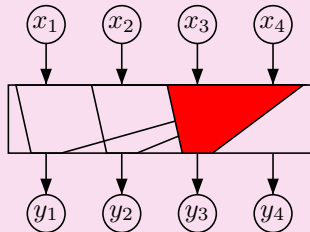
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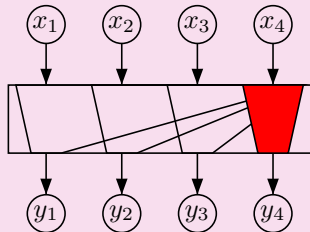
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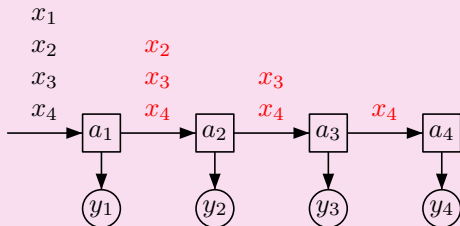
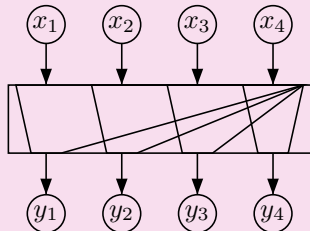
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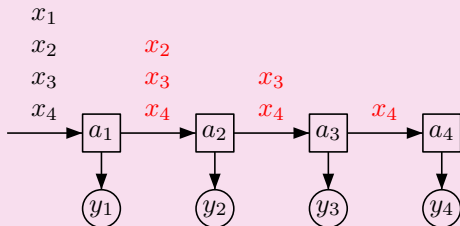
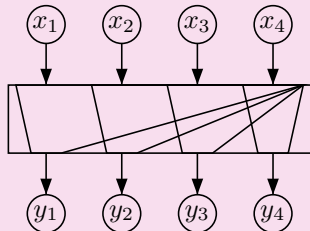
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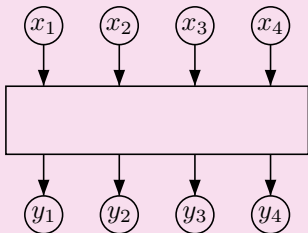
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A specification φ is **robust** if it can be written $\varphi = \bigvee \bigwedge_{z \in \text{Out}} \varphi_z$ where φ_z depends only on $\text{View}(z) \cup \{z\}$.

Theorem

The synthesis problem for **uniformly well-connected** architectures and **external and robust** CTL* specifications is decidable.

Proof.



Robust specifications

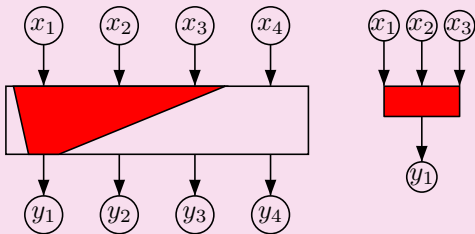
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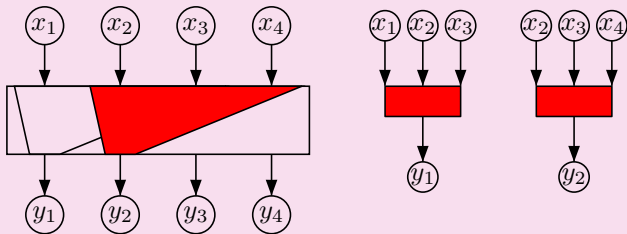
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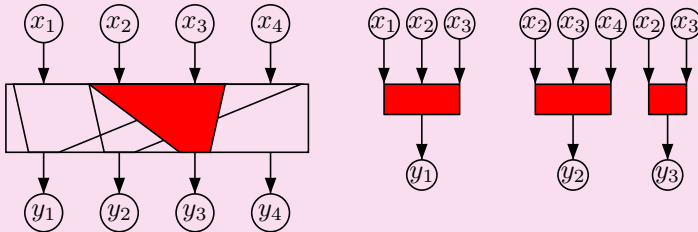
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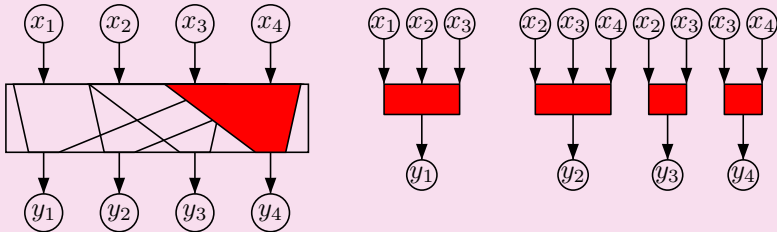
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Open problem

- ▶ Decidability of the distributed control/synthesis problem for **robust** and **external** specifications.

Outline

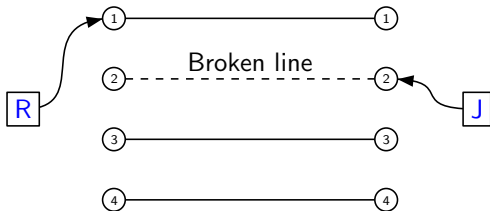
Control for sequential systems

Control for distributed systems

Synchronous semantics

4 Asynchronous semantics

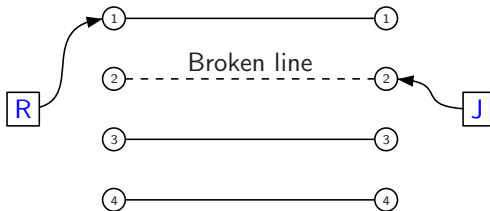
An example: Romeo and Juliet



Romeo and Juliet against the environment

- ▶ Want to communicate through the same communication line.
- ▶ At any time, one line is broken.
- ▶ Environment looks where R&J are connected, and then, atomically, changes (possibly) the broken line.
- ▶ Romeo/Juliet looks status of lines and, atomically, chooses where to connect.

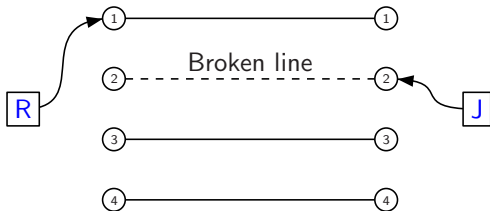
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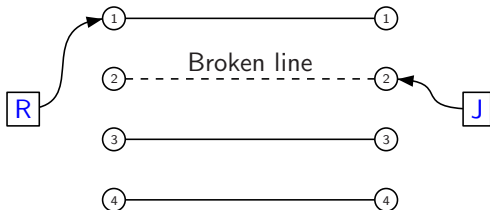
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Romeo and Juliet (continued)

Architecture

Variables:

- ▶ x_1 : Romeo's current line.
- ▶ x_2 : broken line
- ▶ x_3 : Juliet's current line.

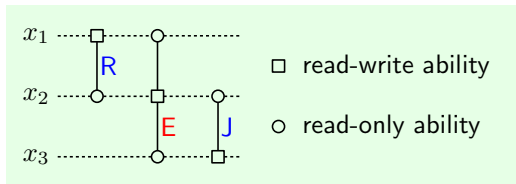
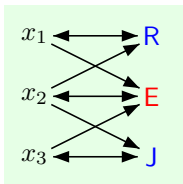
$$Q_1 = \{1, 2, 3, 4\}$$

$$Q_2 = \{1, 2, 3, 4\}$$

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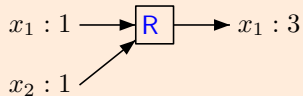
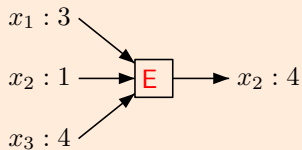
- ▶ Agents: **R**omeo, **J**uliet and **E**nvironment.
- ▶ Read/Write table

	Romeo	Juliet	Environment
Read	$\{x_1, x_2\}$	$\{x_2, x_3\}$	$\{x_1, x_2, x_3\}$
Write	$\{x_1\}$	$\{x_3\}$	$\{x_2\}$



Romeo and Juliet (continued)

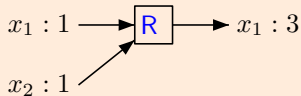
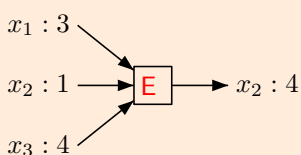
Legal moves: $\delta_a \subseteq Q_{R(a)} \times Q_{W(a)}$



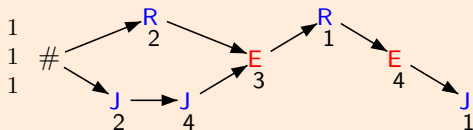
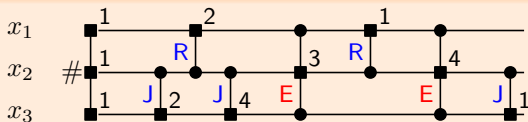
A distributed play of the asynchronous system, R & J against E

Romeo and Juliet (continued)

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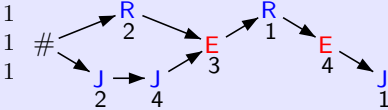


A distributed play of the asynchronous system, R & J against E



Distributed Behaviors

A play is a Mazurkiewicz (real) trace

- ▶ A finite play: 
- ▶ Move: extension of the current Mazurkiewicz trace following the rules.
- ▶ The game is not “position based”, nor “turn based”.
- ▶ Winning condition: set of finite or infinite Mazurkiewicz traces $\mathcal{W} \subseteq \mathbb{R}(\Sigma, D)$. Team 0 wins plays of \mathcal{W} and loses plays of $\mathbb{R}(\Sigma, D) \setminus \mathcal{W}$.

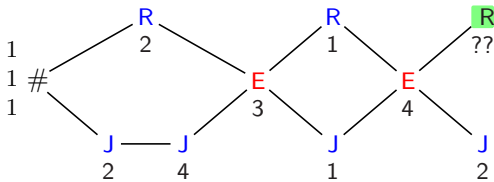
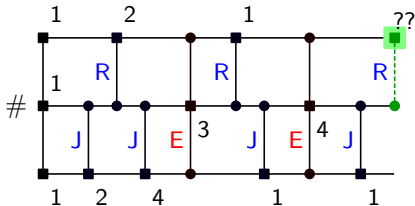
Romeo and Juliet

\mathcal{W} imposes fairness conditions to the environment.

Memory for strategies

Memory

- ▶ Each player only has a partial view of the global history.
- ▶ Memoryless: move can depend only on the current state.
- ▶ Local memory: a player can remember its **read** history.



Causal memory (intuitively, maximal history a player can observe)

- ▶ Players gather and forward as much information as possible.
- ▶ but **no global view**, the choice for an action cannot depend on a concurrent event.

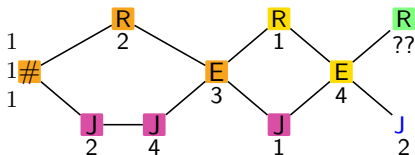
Winning strategies

Tuple $(f_a)_{a \in \mathcal{P}_0}$ where f_a tells player $a \in \mathcal{P}_0$ how to play.

Memoryless $f_a : Q_{R(a)} \rightarrow Q_{W(a)} \cup \text{Stop}$

Local memory $f_a : (Q_{R(a)})^* Q_{R(a)} \rightarrow Q_{W(a)} \cup \text{Stop}$

Causal memory $f_a : \mathbb{M}(\Sigma, D) \times Q_{R(a)} \rightarrow Q_{W(a)} \cup \text{Stop}$



f -maximal f -plays

Given a strategy $f = (f_a)_{a \in \mathcal{P}_0}$, one looks at plays t which are

- ▶ **consistent** with f : all a -moves played according to f_a (f -play).
- ▶ **maximal**: f predicts to **Stop** for all a -moves enabled at t with $a \in \mathcal{P}_0$.

Winning strategies

A strategy f is winning in G if all f -maximal f -plays in G are in \mathcal{W} .

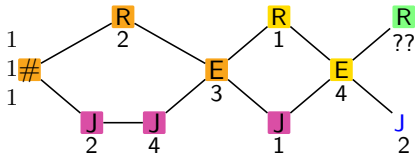
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Finite abstraction of the causal memory

Distributed memory

A **distributed memory** is a mapping $\mu : \mathbb{M}(\Sigma, D) \rightarrow M$ satisfying the following equivalent properties:

1. $\mu^{-1}(m)$ is recognizable for each $m \in M$,
2. μ is an abstraction of an asynchronous mapping (cf. Zielonka),
3. μ can be computed in a distributed way (allowing additional contents inside existing communications (piggy-backing), but no extra communications).

Strategy with memory μ

$$f_a : M \times Q_{R(a)} \rightarrow Q_{W(a)} \cup \text{Stop}$$

the associated strategy is defined by

$$f_a^\mu(t, q) = f_a(\mu(t), q)$$

If M is finite then f_a^μ is a distributed strategy with finite memory.

If $|M| = 1$ then f_a^μ is memoryless.

Embedding causal memory inside games

Proposition: PG-Lerman-Zeitoun (LATIN'04)

For a distributed game G and a distributed memory μ , one can build a game G^μ such that

team 0 has a WDS in G with memory μ

iff

team 0 has a memoryless WDS in G^μ .

Proof.

$$G^\mu = G \times \mu$$

From distributed to sequential games

Theorem: PG-Lerman-Zeitoun (LATIN'04)

Given a finite distributed game (G, \mathcal{W}) , we can effectively build a finite sequential 2-players game $(\tilde{G}, \tilde{\mathcal{W}})$ st. the following are equivalent:

- ▶ There exists a **memoryless distributed** WS for team 0 in (G, \mathcal{W}) .
- ▶ There exists a memoryless WS for player 0 in $(\tilde{G}, \tilde{\mathcal{W}})$.
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Moreover, if \mathcal{W} is recognizable then so is $\tilde{\mathcal{W}}$

Naive idea Consider the game on the global transition system.

Main problem The controller has more information than its causal memory.

Solution

- ▶ The opponent controls the linearization to be played.
- ▶ Using reset moves, he can replay different linearizations for the same play.
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Proposition: (Folklore)

Deciding whether team 0 has a distributed WS with causal memory is undecidable for rational winning conditions.

Proof. Simple reduction of the universality problem for rational trace languages.

Peterson-Reif Madhusudan–Thiagarajan Bernet–Janin–Walukiewicz

Deciding whether team 0 has a distributed WS with local memory is undecidable even:

- ▶ for reachability or safety winning conditions.
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Series-parallel architectures

Theorem: PG-Lerman-Zeitoun (FSTTCS'04)

Distributed games with **recognizable** winning conditions are decidable for **series-parallel** systems and **causal** memory strategies.

Definition : let $\mathcal{A} = (\mathcal{P}, \mathcal{V}, R, W)$ be some architecture.

- ▶ \mathcal{A} is a **parallel product** if
 $\mathcal{P} = A \uplus B$ with $R(a) \cap W(b) = \emptyset$ for all $(a, b) \in A \times B$.
- ▶ \mathcal{A} is a **serial product** if
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- ▶ \mathcal{A} is **series-parallel** if it can be obtained from singletons ($|\mathcal{P}| = 1$) using serial and parallel compositions.
- ▶ \mathcal{A} is series-parallel iff the associated dependence relation does not contain a P_4 : $a - b - c - d$ as induced subgraph.
- ▶ Behaviors of series parallel architectures are series-parallel posets.

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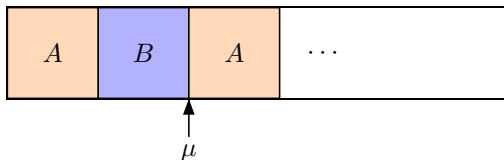
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Proof outline

Team 0 has a WDS \Rightarrow it has a WDS with a "small" distributed memory.

Induction on Σ .

Difficult case: serial product.



1. A WS on $A \uplus B$ induces WS on the restrictions of the game to A and B .
2. Replace the WS on A, B by WS with small memory (induction).
3. Finally, glue together these WS on A and B to obtain a WS on $A \cup B$ using small memory.

Main problem

- ▶ Team 0 must know on which small game it is playing.
- ▶ Team 0 has to compute this information in a distributed way.

Madhusudan and Thiagarajan (Concur'02)

Setting

- ▶ Architecture: $\mathcal{A} = (\mathcal{P}, \mathcal{V}, R, W)$ with $R(a) = W(a)$ for all $a \in \mathcal{P}$.
- ▶ Moves: δ_a are built from local moves for variables $\delta_{a,x} \subseteq Q_x \times Q_x$:

$$\delta_a = \prod_{x \in R(a)} \delta_{a,x}$$

- ▶ Strategies with **local** memory: associated with **variables** and not with agents, and **only predict the next actions** and not the next state:

$$f_x : Q_x^* \rightarrow 2^{R^{-1}(x)}$$

action a is enabled by $(f_x)_{x \in \mathcal{V}}$ at some finite play t if

$$\forall x \in R(a), \quad a \in f_x(\pi_{Q_x}(t))$$

- ▶ The environment decides which a -transition should be taken among the actions a enabled by the strategies.

Madhusudan and Thiagarajan (Concur'02)

Restricted control synthesis problem

Given a distributed system and a **recognizable** specification,

Question existence of a **clocked** and **com-rigid** non-blocking winning distributed strategy with local memory.

- ▶ **clocked**: $f_x(w)$ only depends on $|w|$.
- ▶ **com-rigid**: $a, b \in f_x(w)$ implies $R(a) = R(b)$.

Theorem

1. The **restricted** control synthesis problem is decidable.
2. It becomes undecidable if one of the **red** condition is dropped.

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Mohalik and Walukiewicz (FSTTCS'03)

Restrictions

- ▶ Controllable actions: $R(a) = W(a)$ is a singleton for all $a \in \mathcal{P}_0$.
- ▶ Environment actions: $R(e) = W(e) = \mathcal{V}$ and $\mathcal{P}_1 = \{e\}$.
- ▶ Moves: $\delta_e \subseteq Q_{\mathcal{V}} \times Q_{\mathcal{V}}$.
- ▶ Strategies: local memory with stuttering reduction so that a player $a \in \mathcal{P}_0$ cannot see how long it has been idle.

Theorem

- ▶ Previous settings with local memory can be encoded.
- ▶ two constructions to solve the distributed control problem subsuming previously known decidable cases with local memory.

Open problems

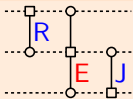
- ▶ Generalization to arbitrary symmetric architectures.
- ▶ Generalization to non-symmetric architectures.
- ▶ Reasonable upper bounds for synthesis?

Symmetric architecture

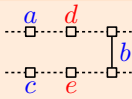
Architecture $\mathcal{A} = (\mathcal{P}, \mathcal{V}, R, W)$

- Restrictions:
$$\begin{cases} \forall a \in \mathcal{P} & \emptyset \neq W(a) \subseteq R(a) \\ \forall a, b \in \mathcal{P} & R(a) \cap W(b) \neq \emptyset \iff R(b) \cap W(a) \neq \emptyset \end{cases}$$
- Dependence: $a D b \iff R(a) \cap W(b) \neq \emptyset \iff R(b) \cap W(a) \neq \emptyset$

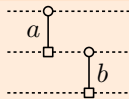
Legal and forbidden architectures



OK



OK



Forbidden (not symmetric)