Non-behavioural temporal logics

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One logic, two semantics and many sub-logics

Inclusion graph
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Inclusion graph

\[ SL_{\text{float}} \]
\[ NG_{\text{float}} \]
\[ BG \]
\[ AG \]
\[ CG \]
\[ DG \]
\[ 1 - \text{Goal} \]
\[ 2\text{Exptime complete} \]

\[ ATL^*_SC \]

\[ SL \]
\[ NG \]

\[ ATL^* \]

\[ ATL \]

\[ BISL \]

Tower complete
Let $\phi : \exists x \forall y \exists z \ [\circ/x][\square/y] \ Fq \to [\circ/z][\square/y] \ Fp$
Definition (BG-SL)

Formula are built upon the rules:

\[ \varphi := \exists x \varphi \mid \forall x \varphi \mid \phi \]

\[ \phi := [agent/strategy] \phi \mid \phi \lor \phi \mid \neg \phi \mid \land \phi \mid \psi \]

\[ \psi \in LTL \]

Normal form

\[ \varphi \bigwedge_{i \in I} \bigvee_{j \in J} B_{i,j} \psi_{i,j} \]

with \( \varphi \) a bunch of quantifications, \( B_{i,j} \) are sets of bindings and the \( \psi_{i,j} \) are LTL formulas.
One can express qualitative Nash equilibrium for $LTL$ objectives:

Let $a_1, \ldots, a_l$ be $l$ agents, each $a_i$ with their own objective $\psi_i$. Then the following express existence of a Nash equilibrium:

$$\exists x_1 \ldots x_l \forall y_1 \ldots y_l \bigwedge_{i \leq l} [a_1/x_1] \ldots [a_{i-1}/x_{i-1}] [a_i/y_i] [a_{i+1}/x_{i+1}] \ldots [a_l/x_l] \psi_i$$

$$\Rightarrow [a_1/x_1] \ldots [a_{i-1}/x_{i-1}] [a_i/x_i] [a_{i+1}/x_{i+1}] \ldots [a_l/x_l] \psi_i$$
Multi-purposes strategies are the key. In concurrent-games, a strategy can be shared by multiple players:

$$\exists x \ y \ \forall z \ \left( [a_1/x][a_2/y][a_3/z]G(p) \leftrightarrow [a_1/z][a_2/y][a_3/x]G(p) \right)$$

For a given objective, checking a player privileges against another player become possible - in contrast to usual $\omega$-objectives.
Non behavioural strategy

\[ \psi_0 := \forall w, x \exists y, z \]

\[ \left( [\circ/w][\Box/x]F(p) \right) \Leftrightarrow \left( [\circ/y][\Box/z]F(q) \right) \]
\[
\psi_0 := \forall w, x \exists y, z \\
\left( [\circ/w][\Box/x] F(p) \right) \iff \left( [\circ/y][\Box/z] F(q) \right)
\]
Non behavioural strategy

\[ \psi_0 := \forall w, x \exists y, z \]

\[
\left( [\circ/w][\Box/x]F(p) \right) \iff \left( [\circ/y][\Box/z]F(q) \right)
\]
Mathematical representation for different behaviours

Non-behavioural:

\[(\text{path} \mapsto \text{action})\forall \text{strats} \iff \text{path} \mapsto \text{action})\exists \text{strat}\]

with $x_i$ only depending on universal choices of strategies quantified before $x_i$.

Behavioural:

\[\text{path} \mapsto \left( (\text{action})\forall \text{strats} \iff (\text{action})\exists \text{strat} \right)\]

with $x_i$ only depending on universal choices of strategies quantified before $x_i$. 
Theorem

Formulas of the following types admit a 2—EXPTIME model checking:

- **Unique LTL objective over turn based game.**
- **Conjunctive goals** [MMPV12]: \( \emptyset \wedge_{i \in I} B_i \psi_i \)
- **Disjunctive goals**: \( \emptyset \vee_{j \in J} B_j \psi_j \)
- **"Linear" formulas** [MMS14]: \( \emptyset B_1 \psi_1 \lor \left( B_2 \psi_2 \land (B_3 \psi_3 \lor (B_4 \psi_4 \land \ldots)) \right) \)

On the full fragment, previous works give a non elementary algorithm and a 2—EXPTIME hardness. The completeness of the full fragment was unknown.
Conjunctive and disjunctive goals

Theorem (MMPV12)

Both the conjunctive and disjunction goals admit behavioural strategies

Let \( \phi := Q_1 x_1, \ldots, \exists x_a \ldots Q_l x_l \land \beta_i \psi_i \) with \( Q_j \in \{\exists, \forall\} \).

We look for which objectives will be realizable on \( \pi \).

We play for all objectives "realizable" on \( \pi \).

Existential strategies have interest in separating different objectives into different paths.
Alternating goal

Theorem (MMS14)

The alternating goal also admit behavioural strategies.

Let \( \phi := \diamond B_1 \psi_1 \lor (B_2 \psi_2 \land (B_3 \psi_3 \lor (B_4 \psi_4 \land \ldots))) \).

There is an implicit order on objective \( \psi_1 > \psi_2 \cdots > \psi_n \). Such ordering doesn’t exist in non-linear fragments (ex : \( \beta_1 \psi_1 \leftrightarrow \beta_2 \psi_2 \)).

A strategy can make its choice by following the order and not looking at counter-factual plays. You do not have to explore the full possibilities.
Hardness Results

**Theorem**

$BG - SL$ is Tower-hard ($k$-EXPTIME hard for all $k \in \mathbb{N}$) over concurrent games.
QLTL

QLTL : \( \exists p_1 \; \forall p_2 \ldots \; \exists p_l \phi \) with \( \phi \in LTL \) over the atomic set \( \{p_1, \ldots p_l\} \).

Semantics :

- \( \exists p_i \mapsto \), we can extend previous colouring of \( \mathbb{N} \) with a new color \( p_i \).
- \( \forall p_i \mapsto \), for all extensions of the previous colouring with a new color \( p_i \).
- \( \phi \in LTL \mapsto \mathbb{N} \) with previous colouring satisfy \( \phi \).

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & \cdots \\
p_1 & p_1 & p_1 & p_1 & p_1 & p_1 & \cdots \\
p_2 & p_2 & p_2 & p_2 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_l & p_l & p_l & p_l & p_l & \cdots \\
\end{array}
\]
Theorem

QLTL satisfiability is $k$–hard for formulas with $k$ quantifiers alternations.

We will use the *Buchi* automaton associate to $\phi$.

Remainder: *States of the Buchi automaton are labelled by sub-formulas of $\phi$*
Sketch of proof

States of Buchi automaton for $\phi$(LTL part)

$p_1, p_2, \ldots \in q_1$

$p_1, \neg p_2, \ldots \in q_2$

$\neg p_1, p_2, \ldots \in q_3$

$\neg p_1, \neg p_2, \ldots \in q_4$

Define the path that must satisfy $\phi$
States of Buchi automaton for $\phi$(LTL part)

**How it works**

1. Check that the corresponding atomic propositions or negation are in $q_i$.
2. Check that $q_{i+1}$ is accessible from $q_i$.
3. Check the sequence $(q_i)_{i \in \mathbb{N}}$ see infinitely often an accepting state.

Define the path that must satisfy $\phi$
Sketch of proof: \( q_i \) follows the choices among the \( p_j \)’s

\[ \forall s_\alpha \text{ mark the state before leaving the main path} \]
\[ : \forall s_\alpha \exists s_{blue}, s_1, \ldots, s_l, s_{black} \ldots \bigwedge_{q \in Buchi, i \leq l} \]
\[ [s_\alpha, s_i, s_{black}] G(\bigwedge_{j \leq l} a_i \leftrightarrow \beta_j) \]
\[ [s_\alpha, s_{blue}, s_{black}] G(\bigwedge_{j \leq l} \beta_j \leftrightarrow b) \]

\[ [s_\alpha, s_i, s_{black}] F(p_i) \]
\[ \rightarrow [s_\alpha, s_{blue}, s_{black}] \bigvee_{q | p_i \in q} F(q) \]
Checking for an infinite number of accepting state

We use concurrency to create a strategy who loops around $s \times \alpha \times \gamma$ more time than $s_\alpha$

Then ask that when $[s_{blue}^F, s_\alpha^F, s_\gamma^F]$ give the hand to player Black, we see an accepting state.
Checking a state is accessible from its predecessor

We ask that there is no strategy $s^G_\gamma$ such that:
1. $[s_{\text{blue}}, s_\alpha, s_\gamma]$ sees one of the $\beta_i$'s and delta, this implies that $s^G_\gamma$ stop looping on $\alpha$ after more than $n$ steps
2. $[s_{\text{blue}}, s^+_\alpha, s^+_\gamma]$ sees $\alpha \wedge \gamma$, this implies that $s^G_\gamma$ loops less than $s^+_\alpha$.
Aftermath

• "Linear built" formulas
  \[ [\text{MMS14}] \quad \varnothing B_1 \psi_1 \lor \left( B_2 \psi_2 \land (B_3 \psi_3 \lor (B_4 \psi_4 \land \ldots)) \right) \] model checking is 2-EXPTIME complete

• \[ \varnothing \land _{i \in I} \lor _{j \in J} B_{i,j} \psi_{i,j} \text{ is Tower – hard complete.} \]

Question

Is an order on the objectives absolutely necessary for a 2 – EXPTIME model checking ?

Do non-behavioural strategies (strategies depending on counter-factual play) always imply Tower – hard model checking ?

Is concurrency necessary for the hardness proof ?
Is concurrency essential to the hardness result?

One cannot use the same technique in turn-based:

Commutable, hence we have a set and not a sequence.
Thank you!