Exercise 1:
Let $\Sigma = \{a, b, c\}$. Define a NFHA $A$ such that $L(A)$ is the set of all trees such that there exist two nodes labelled by $b$ whose greatest common ancestor is labelled by $c$.

Exercise 2:
Let $\Sigma = \{a, b\}$. Define a DFHA $A$ such that $L(A)$ is the set of all trees such that "for every leaf labeled with $a$, there is an ancestor from which there is a path whose nodes are labeled with $b$". Here "ancestor" means strict ancestor and "from which there is a path" means that there is a path from a son of this ancestor to a leaf.

Definition (extension encoding).
Let $t$ be an unranked tree on $\Sigma$. Let $F_{ext}^{\Sigma} = \{\@ (2)\} \cup \{a(0) \mid a \in \Sigma\}$. We define the ranked tree $ext(t)$ by induction on the size of $t$ by:
- for $a \in \Sigma$, $ext(a) = a$
- if $t = a(t_1, ..., t_n)$ with $n \geq 1$, $ext(t) = \@ (ext(a(t_1, ..., t_{n-1})), ext(t_n))$

that is $ext(a(t_1, ..., t_n))$ is equal to:

Exercise 3:
Give the extension encoding of:

Exercise 4:
Let $L$ be a language of unranked trees. Prove that $L$ is recognizable by a NFHA iff $ext(L)$ is recognizable by a NFTA.