Verification of temporal logics on infinite-state systems

Exercises on Day 3

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ESSLLI 2007, Dublin, July 2007

If you are not familiar with the concepts referred in a particular exercise, just ignore it.

1 Büchi automata

1. Verify the necessary and sufficient condition for non-emptiness of a Büchi automaton: some accepting state must be reachable from the initial state and from itself.

2. Show that the automaton on infinite words $A = \langle \{a, b\}, \{s_0, s_1\}, \rho, \{s_0\}, \{s_1\} \rangle$, where $\rho(s_0, a) = \{s_0\}, \rho(s_0, b) = \{s_1\}, \rho(s_1, a) = \{s_0\}, \rho(s_1, b) = \{s_1\}$ accepts those words in which $b$ occurs infinitely often.

3. Construct Büchi automata on infinite words in the alphabet $\{a, b\}$ which accept the following languages:

   (a) the set of words which contain $a$ at least once;
   (b) the set of words which contain $a$ exactly once;
   (c) the set of words which eventually contain only $a$.
   (d) the set of words which contain at least one $a$ between every occurrences of $b$.
   (e) the set of words which in which both $a$ and $b$ occur infinitely often.
4. Construct a Büchi automaton accepting the computations on which the LTL-formula $G(p \rightarrow F q)$ is true.

5. Show that the language accepted by a Büchi automaton is non-empty precisely when there is an accepting state which is reachable from an initial state and from itself.

6. Construct automata on infinite words in $\{a, b\}$ which accept the following languages:

   (a) the set of words which contain $a$ at least once;
   (b) the set of words which contain $a$ exactly once;
   (c) the set of words which eventually contain only $a$.
   (d) the set of words which contain at least one $a$ between every occurrences of $b$.

2 Model checking pushdown systems with simple valuations

Read through the slides for lecture 3.1 posted on the website and:

1. understand the definition of Büchi pushdown system and how the local model checking from initial configuration reduces to non-emptiness of such systems.

2. understand how non-emptiness of the Büchi pushdown system $\mathcal{PB}_{\neg \varphi}$ is reduced to computing $\text{pre}^* (\text{Rep}\Gamma^*)$.

3. prove the Lemma: A head is repeating if and only if it belongs to a strongly connected component of the head reachability graph that contains at least one marked edge.

4. put all these together to solve the local model checking from initial configuration $(p_0, \omega)$ of the formula $A U B$ in the example of a PDS considered in lecture 3.1, with added transition $(p_0, \omega) \rightarrow (p_0, \gamma_1)$, where $A$ is true on all configurations with control location $p_0$ or $p_1$ and $B$ is true on all configurations with control location $p_2$ and top stack symbol $\gamma_2$. 