Verification of temporal logics on infinite-state systems

Exercises on Day 1

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If you are not familiar with the concepts referred in a particular exercise, just ignore it.

1 Transition systems

1. Show that the set of states in an unfolding of a finite transition system \( T = (S, R) \) is represented by a regular language over the alphabet \( S \).

2. Construct a finite transition system that unfolds into a tree, where every non-leaf node has one immediate successor leaf and either one or three other immediate successors.

2 Verification of the basic tense logic on rational Kripke models

1. Show that:
   
   (a) the infinite grid \( N \times N \) is a configuration graph of a word-rewriting system.
   
   (b) the complete binary tree \( T_2 \) is a configuration graph of a word-rewriting system.
   
   (c) every rational graph is a configuration graph of a word-rewriting system.
(d) $(\mathbb{N}, <)$ is an automatic graph.
(e) $(\mathbb{N}, +)$ is an automatic graph.
(f) the configuration graph of every pushdown automaton is a rational graph. Then show that it is an automatic graph.
(g) the configuration graph of every Turing machine is a rational graph. Then show that it is an automatic graph.
(h) every Presburger counter system is a rational graph. Then show that it is an automatic graph.
(i) the configuration graph of every Petri net is a rational graph. Then show that it is an automatic graph.

2. Reduce the Post Correspondence Problem to testing an input rational relation for having a reflexive point (i.e., testing of the truth of $\exists x Rxx$).

3. In the example of rational Kripke model $\mathcal{G} = (S, R, V)$ from p. 15 on the slides, construct finite automata computing the set of states each of the following formulae is true:

(a) $\neg p$, $p \land q$, $p \lor q$.
   (These are just exercises on finite automata; if you haven’t done such, consult any standard textbook on finite automata.)
(b) $(R)p$, $[R](p \lor q)$, $[R](R)p$.  

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