
Definition 1 (signature) *A* signature *is a pair* $(\mathcal{F}, ar)$ *where :*
– $\mathcal{F}$ *is a finite set of function symbols, and*
– $ar : \mathcal{F} \to \mathbb{N}$ *is a function that associates to each* $f \in \mathcal{F}$ *an integer (*arity of f*).*
We denote by $\mathcal{F}_p = \{f \in \mathcal{F} \mid ar(f) = p\}$*. The function symbols in* $\mathcal{F}_0$ *are also called* constants*.*

Definition 2 (term) *Let* $\mathcal{F}$ *be a signature and* $\mathcal{X}$ *be a finite set of constants, called* variables*, such that* $\mathcal{X} \cap \mathcal{F} = \emptyset$*. The set of* terms*, denoted by* $\mathcal{T}(\mathcal{F}, \mathcal{X})$*, is the smallest set such that :*

1. $\mathcal{X} \subseteq \mathcal{T}(\mathcal{F}, \mathcal{X})$ *and* $\mathcal{F}_0 \subseteq \mathcal{T}(\mathcal{F}, \mathcal{X})$*,*
2. *for all* $p \geq 1$*, for all* $f \in \mathcal{F}_p$*, for all* $t_1, \ldots, t_p \in \mathcal{T}(\mathcal{F}, \mathcal{X})$*,* $f(t_1, \ldots, t_p) \in \mathcal{T}(\mathcal{F}, \mathcal{X})$*.*

Let $\mathcal{T}(\mathcal{F}) = \mathcal{T}(\mathcal{F}, \mathcal{X})$ *(*ground terms*). We denote by* $vars(t)$ *the set of variables that occur in* t*.*

Definition 3 (subterm) *Let* t *be a term. The set* $st(t)$ *of* subterms *of* t *is the smallest set such that :*

1. $t \in st(t)$*, and*
2. *for all* $u \in st(t)$ *such that* $u = f(u_1, \ldots, u_n)$ *for some* $f \in \mathcal{F}_p$*, then* $u_1, \ldots, u_n \in st(t)$*.*

This notion is extended as expected to set of terms : $st(T) = \bigcup_{t \in T} st(t)$*.*

Definition 4 (size of a term) *Let* t *be a term, the size of* t*, denote by* $|t|$*, is defined recusively as follows :*

$$
\begin{array}{ll}
|a| = 1 & \text{if } a \in \mathcal{F}_0 \cup \mathcal{X} \\
|f(t_1, \ldots, t_n)| = 1 + \sum_{i=1}^{n} |t_i| & \text{otherwise}
\end{array}
$$

Definition 5 (substitution) *A* substitution σ *is a function from a finite set* $X \subseteq \mathcal{X}$ *to* $\mathcal{T}(\mathcal{F}, \mathcal{X})$*. We denote by* $dom(\sigma)$ *the set* X*, called the* domain*, and we extend a substitition from its domain* X *to* $\mathcal{T}(\mathcal{F}, \mathcal{X})$ *as follows :*

$$
\begin{array}{rcll}
x\sigma & = & x & \text{if } x \notin dom(\sigma) \\
f(t_1, \ldots, t_n)\sigma & = & f(t_1\sigma, \ldots, t_n\sigma) & \text{otherwise}
\end{array}
$$

Definition 6 (unifier) *Let* s *and* t *be two terms. They are* unifiable *if there exists a substitution* σ *such that* $s\sigma = t\sigma$*. The substitution* σ *is a unifier of* s *and* t*. A unifier* θ *is a* most general unifier (mgu) *of* s *and* t*, denoted by* $\mathsf{mgu}(s, t)$*, if*

$$\text{for all } \sigma \text{ such that } s\sigma = t\sigma \text{ there exists } \tau \text{ such that } \sigma = \theta\tau.$$

We admit the following result : Given two terms s, t that are unifiable, there exists a unique most general unifier.

A reference about unfication :

– F. Baader and W. Snyder. *Unification theory.* In A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, chapter 8. Elsevier, 2001.