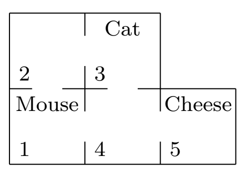
Probabilistic Aspects of Computer Science: Homework 1

Due on October 21, 2011

(1) A mouse is wandering in a maze partitioned into 5 rooms . We suppose that it is changing rooms at each time period $t = 0, 1, 2, 3, \ldots$ and that, if at time n, it is in a room with k doors, then it chooses one of them with probability $1/k$ and is at time $n + 1$ in the next room. All its choices are supposed to be independent.



Suppose that the mouse is initially in room 1. When the mouse reaches room 5, it eats the cheese and when the mouse reaches room 3, the cat eats the mouse.

What is the probability that the mouse eats the cheese before the cat eats the mouse? (2 points)

(2) M stones marked $S_1, \ldots, S_M$ are placed one besides the other. At step n, one of them is chosen randomly and its position is exchanged with the preceding stone, except if the selected stone is in first position, in which case no exchange is done.

A configuration is then a sequence of the markings read from left to right. For example, suppose $M = 5$ and the configuration just before step n is $S_2S_3S_5S_1S_4$. If S_5 is selected at step n, then the new configuration is $S_2S_5S_3S_1S_4$. On the other hand, if S_2 is selected, the configuration would not be modified. At each step, S_i has probability $\alpha_i > 0$ of being selected. Denote by $X_n = S_{i_1} \ldots S_{i_M}$ the configuration at step n. X_n can be modeled as a Markov Chain $M = (Q, \mu, \delta)$ where Q is the set of configurations.

- Give the transition matrix δ. (1 point)
- Is M irreducible? Aperiodic? (2 point)
- Compute all the stationary distributions of M. (3 points)

(3) Given an alphabet Σ and PFAs A and B, show that the problem of checking if $\mathcal{L}_{=1}(\mathcal{A}) \subseteq \mathcal{L}_{=1}(\mathcal{B})$? is decidable in exponential time. (2 points)