

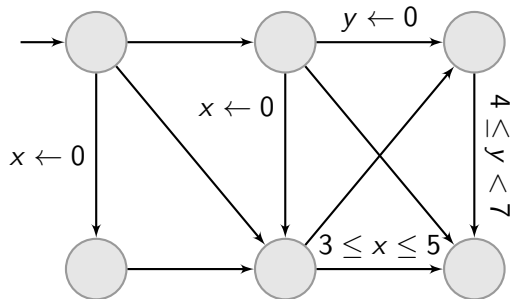
Computing Equilibria in Two-Player Timed Games *via* Turn-Based Finite Games

Patricia Bouyer Romain Brenguier Nicolas Markey

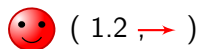
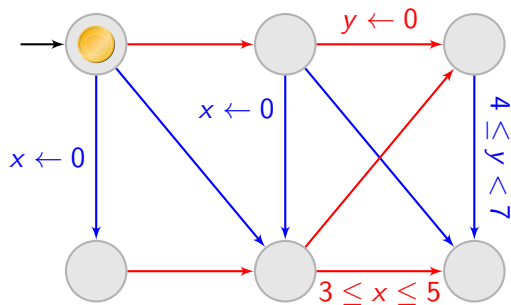
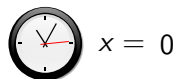
LSV, CNRS & ENS Cachan, France
{bouyer,brenguier,markey}@lsv.ens-cachan.fr

September 9, 2010

Timed Automata



Timed Games

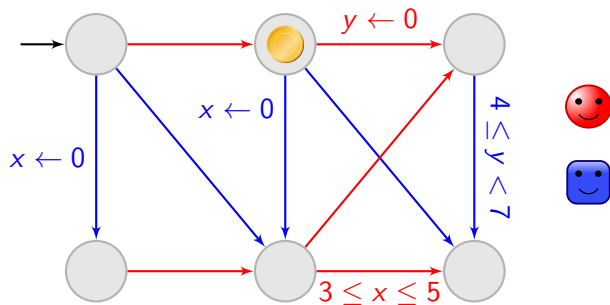
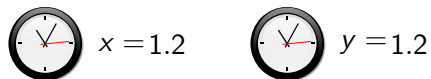


(1.2 , \rightarrow)



(3.5 , \searrow)

Timed Games



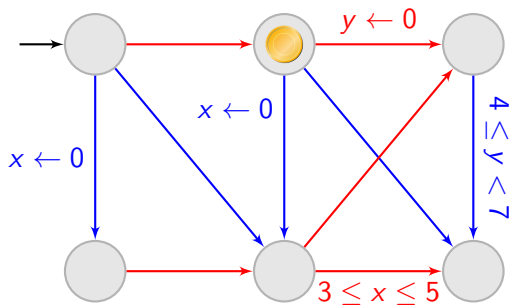
Timed Games



$x = 1.2$



$y = 1.2$

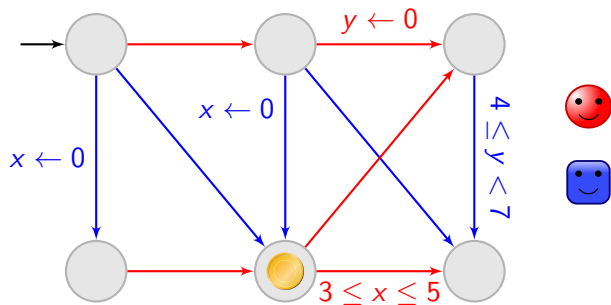
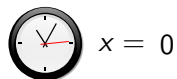


(3 , \rightarrow)



(0.8 , \downarrow)

Timed Games



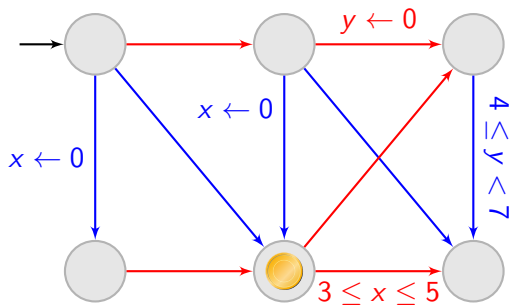
Timed Games



$x = 0$



$y = 2$



$(3, \rightarrow)$



Timed automata and timed games

Theorem

Deciding the winner in zero-sum timed games is decidable.

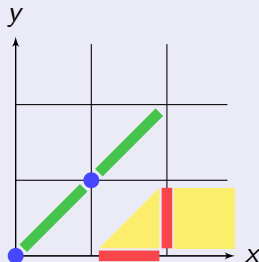
Timed automata and timed games

Theorem

Deciding the winner in zero-sum timed games is decidable.

Remarks

- Algorithm based on regions;
- The problem is EXPTIME-complete;
- Only holds if the objectives are region-definable.



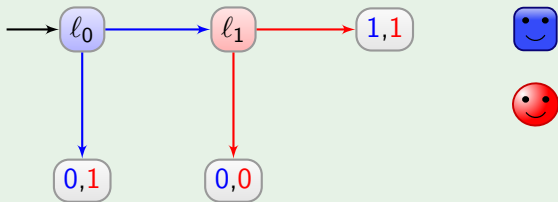
Definition

A **Nash equilibria** is a strategy profile where no player can improve his payoff by changing strategy unilaterally

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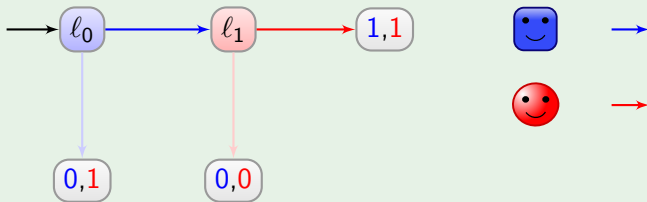
Turn-based Game



Definition

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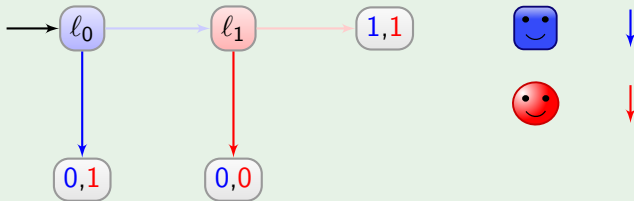
Turn-based Game



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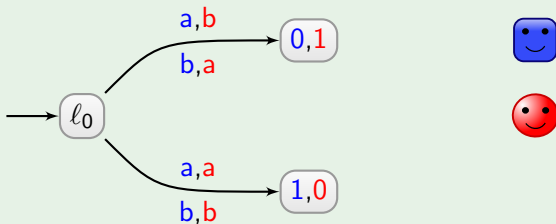
Turn-based Game



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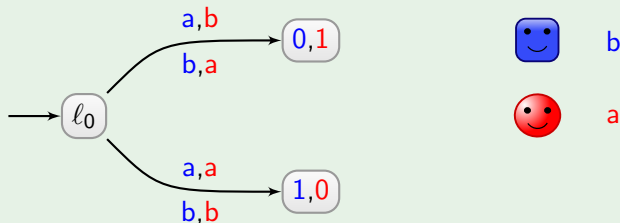
Concurrent Game



Definition

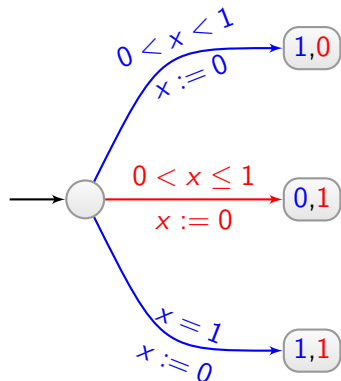
A **Nash equilibria** is a strategy profile where no player can improve his payoff by changing strategy unilaterally

Concurrent Game



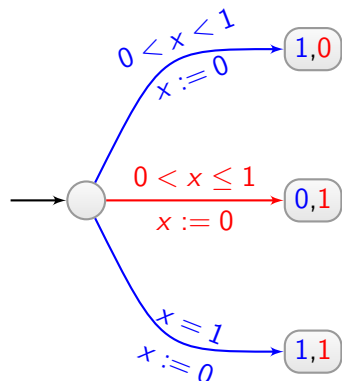
- 1 Preliminaries
- 2 From timed games to concurrent games...
- 3 ... next to two twin concurrent games...
- 4 ... next to concurrent deterministic games...
- 5 ... and finally to two turn-based games

From timed games to concurrent games...





Timed Game \mathcal{G}

From timed games to concurrent games...

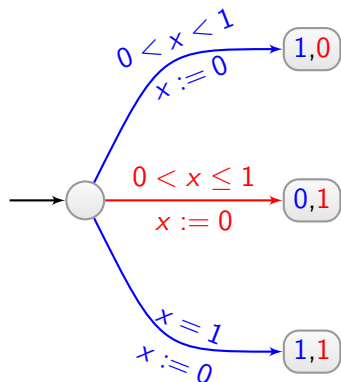


Timed Game \mathcal{G}

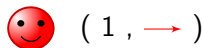
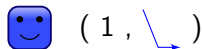
 $(0.7, \nearrow)$

 $(0.4, \rightarrow)$

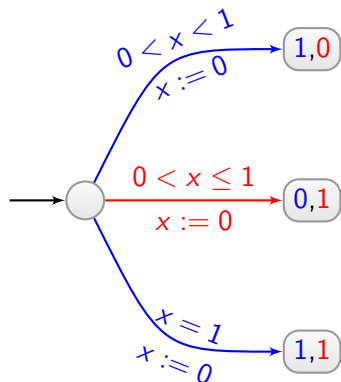
From timed games to concurrent games...



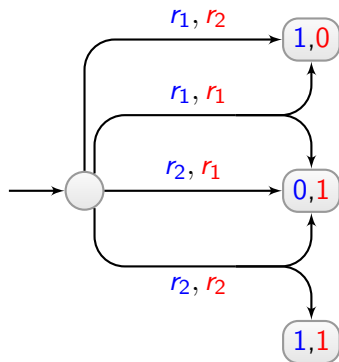
Timed Game \mathcal{G}



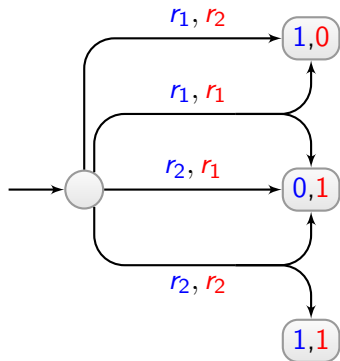
From timed games to concurrent games...



Timed Game \mathcal{G}



Region Game \mathcal{R}



Region Game \mathcal{R}

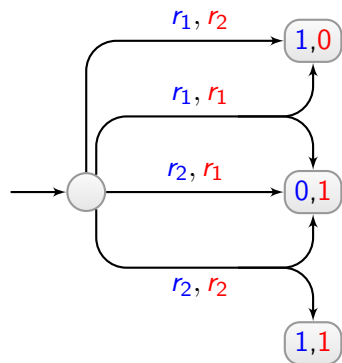


r_2



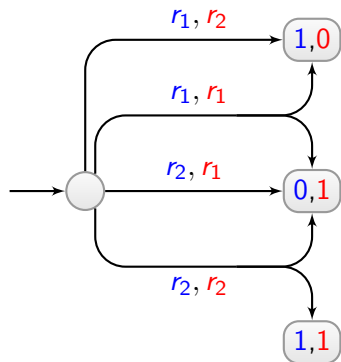
r_2

... next to two twin concurrent games...

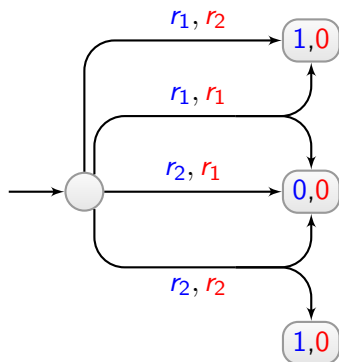


Region Game \mathcal{R}

... next to two twin concurrent games...

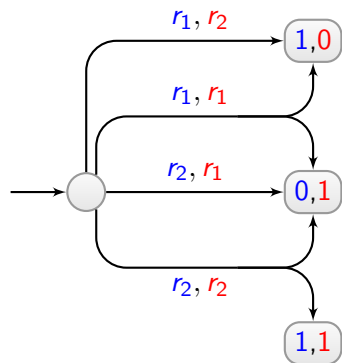


Region Game \mathcal{R}

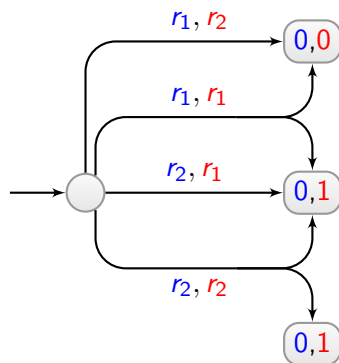


Concurrent Game \mathcal{R}_1

... next to two twin concurrent games...

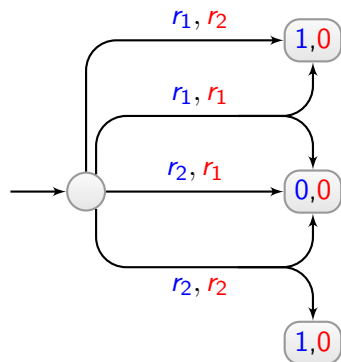


Region Game \mathcal{R}



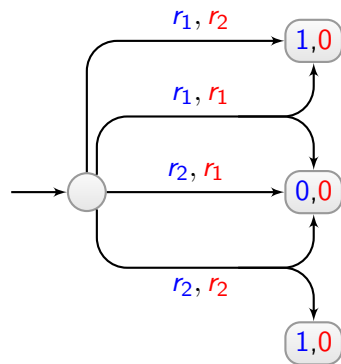
Concurrent Game \mathcal{R}_2

... next to concurrent deterministic games...

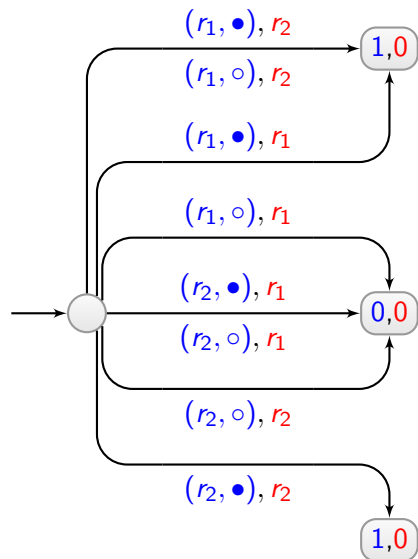


Concurrent Game \mathcal{R}_1

... next to concurrent deterministic games...

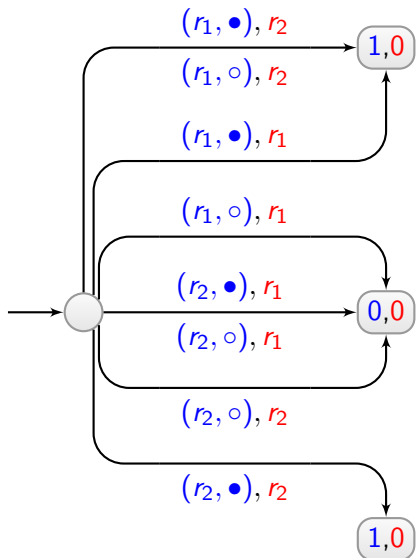


Concurrent Game \mathcal{R}_1

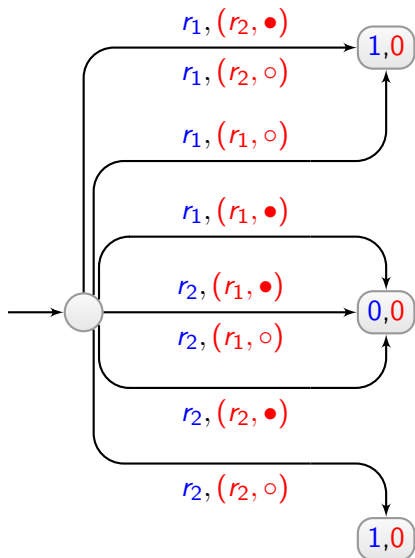


Concurrent Deterministic Game \mathcal{C}_1

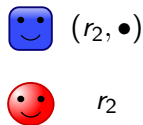
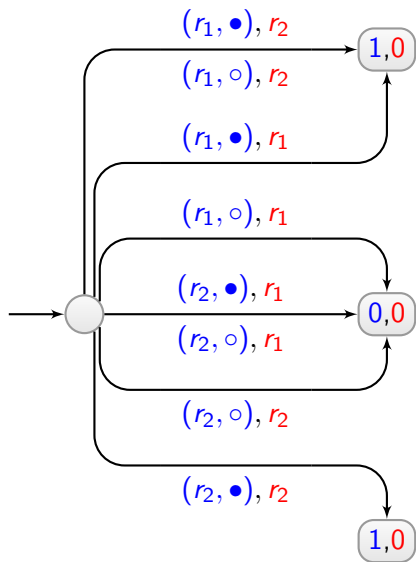
... next to concurrent deterministic games...



Concurrent Deterministic Game \mathcal{C}_1

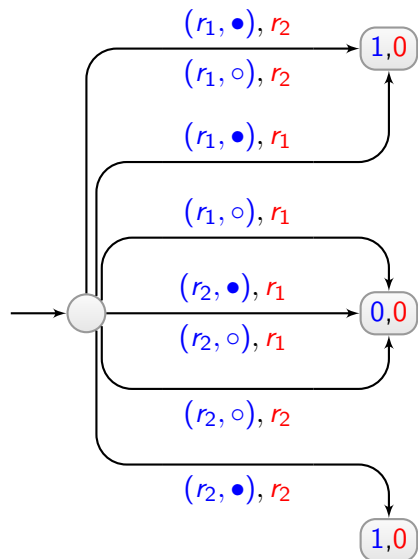


Concurrent Deterministic Game \mathcal{C}_2



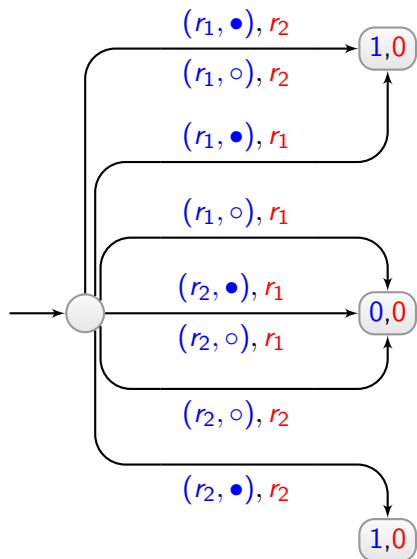
Concurrent Deterministic Game \mathcal{C}_1

... and finally to two turn-based games

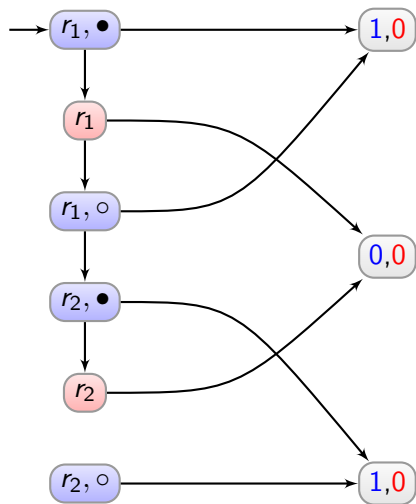


Concurrent Deterministic Game \mathcal{C}_1

... and finally to two turn-based games

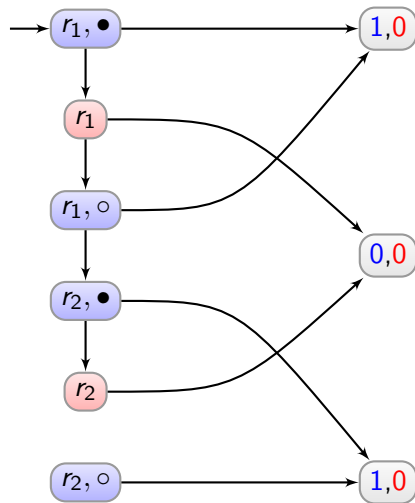


Concurrent Deterministic Game \mathcal{C}_1

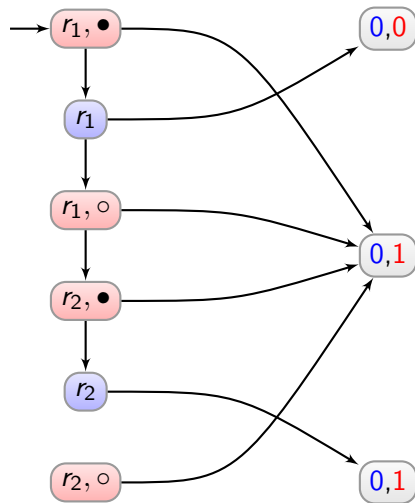


Turn-based Game \mathcal{T}_1

... and finally to two turn-based games



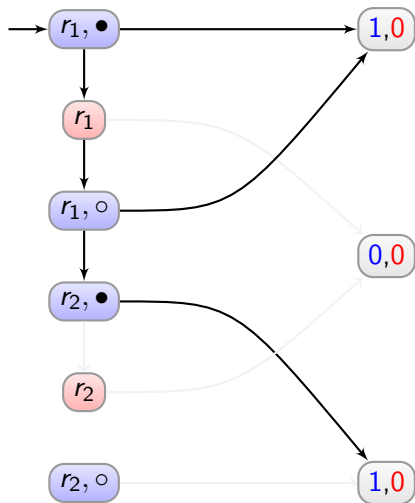
Turn-based Game \mathcal{T}_1



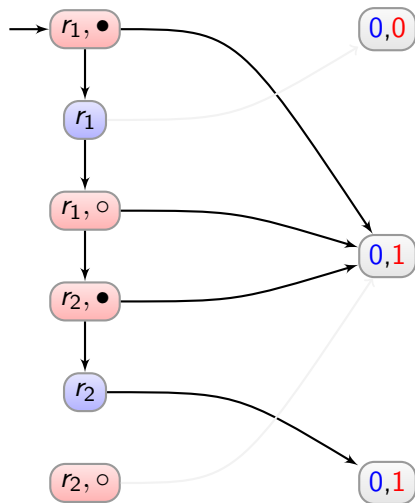
Turn-based Game \mathcal{T}_2

Theorem

There exists a Nash equilibrium in the timed game if and only if there exists a “twin” Nash equilibrium in the two turn based games



Turn-based Game \mathcal{T}_1



Turn-based Game \mathcal{T}_2

Perspectives

- N-players games
- Quantitatives objectives
- Other kinds of equilibria (subgame perfect equilibria, ...)