Symbolic Reachability Analysis of Lazy Linear Hybrid Automata

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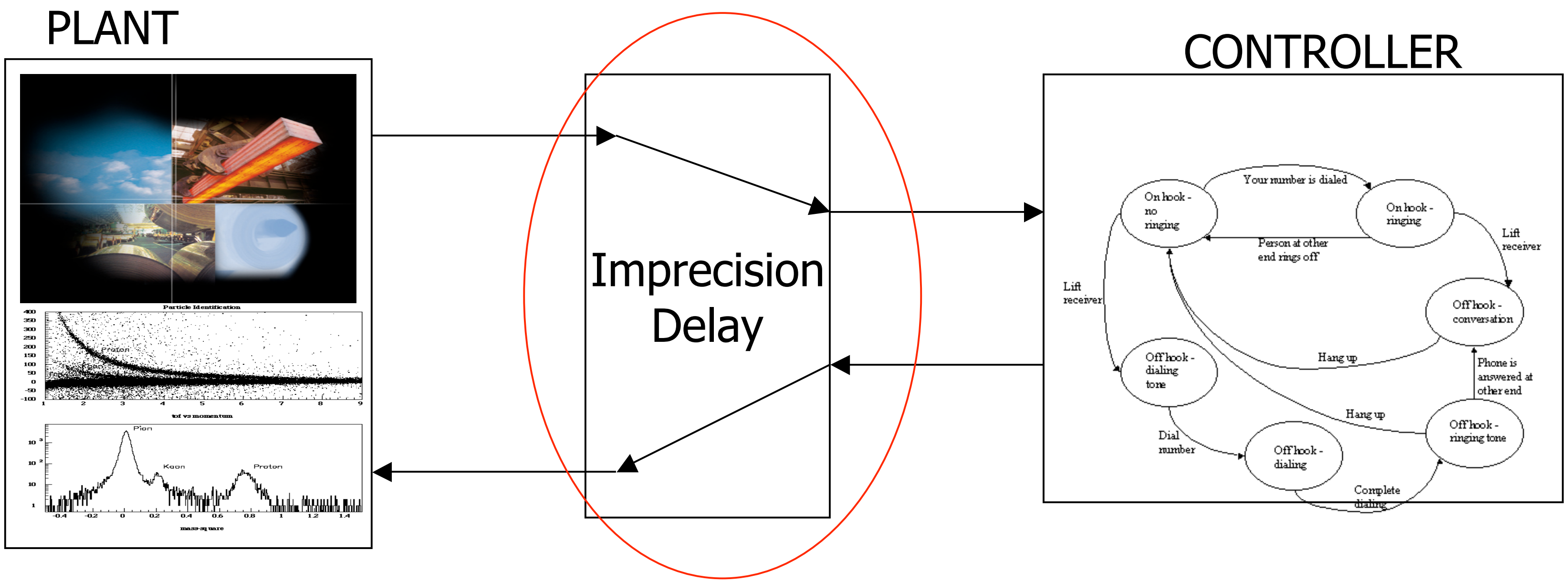
Traditional Hybrid Automata

Traditional Hybrid Automata do not model
delay and finite precision in sensing and actuation



But implementations of hybrid system have
inertial delays and imprecision in sensing and actuation

Alternative models

- Discrete Hybrid Automata (Torrisi et al) – Consists of a finite state machine communicating with a switched affine system through mode selector and event generator.

- Linear and Polynomial Hybrid Automata (Franzle et al) – Semi-decidable in most cases barring some pathological cases in which safety depends on complete absence of noise.

- Lazy Linear Hybrid Automata (LLHA) (Agrawal and Thiagarajan) – Models the inertial delays as well as finite precision of sensors and actuators. Reachability in LLHA is decidable.

Contributions

Goal: To develop a scalable technique for reachability analysis of LLHA

- New sound abstraction technique for LLHA
  - Along with a counter-example guided approach to refinement
- Symbolic Bounded Model Checking (BMC) of abstraction of LLHA, with k-induction
  - BMC extended to deal with inertial delays
- Demonstration of scalability of our approach on examples like TCAS and AHS

Talk Outline

- Background: Lazy Linear Hybrid Automata (LLHA)
- Overview of Approach
- Abstraction Hierarchy for LLHA
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Lazy Linear Hybrid Automata

LLHA is a tuple (X,V,flow,inv,init,E,jump,Σ,syn, D,ε,B,P)

X-Continuous Variables

V-Control Modes / Locations

Flow- Constant rates of change

Inv –Invariants at control modes

E - Control mode switches

Jump - Guards over switches

Σ – reset actions

Syn – synchronization labels

Lazy Linear Hybrid automata

LLHA is a tuple
$(X,V,flow,inv,init,E,jump,\Sigma,syn,D,\varepsilon,B,P)$

Corresponding to the interface

$D = \{g, \delta_g, h, \delta_h\}$ (bounded delays)

Such that $g \leq$ actuation delay $\leq g+\delta_g$

$h \leq$ sensing delay $\leq h+\delta_h$

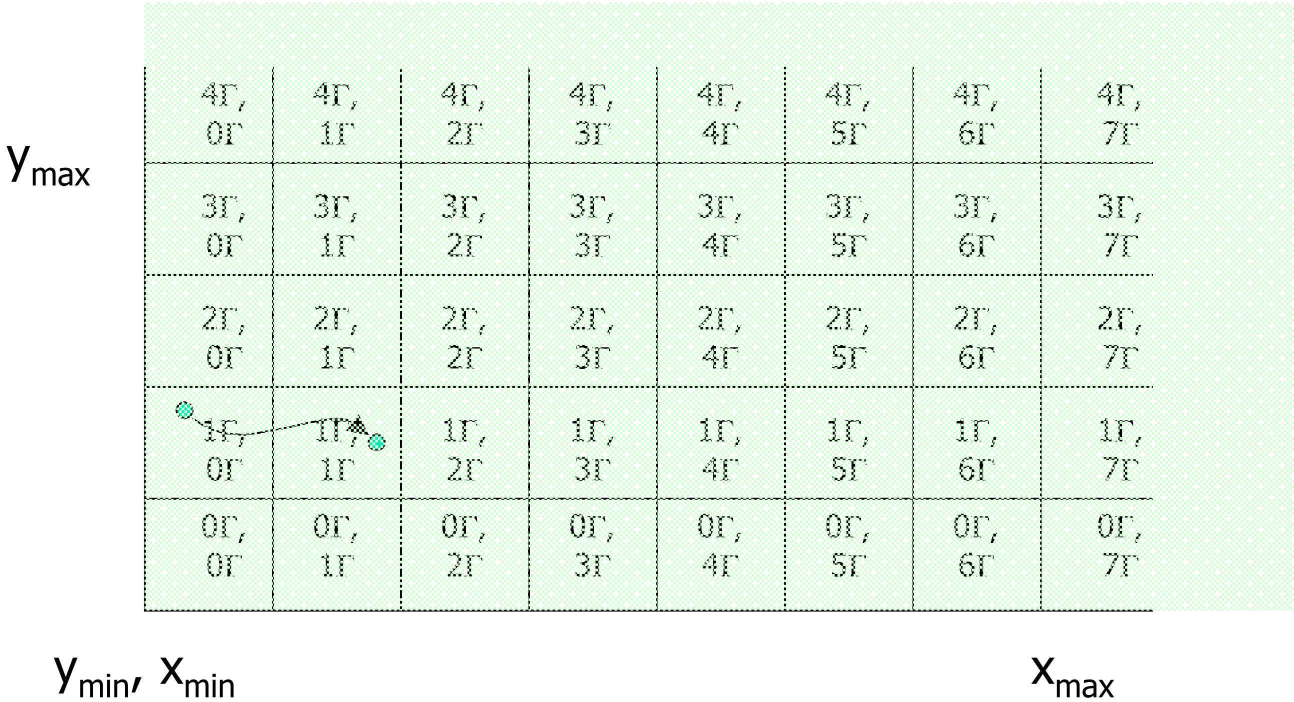
The continuous variables are observed by the controller with precision ε and are expected to be in a range $B = [B_{min}, B_{max}]$

The controller samples the values of variables at intervals of period P. For simplicity, we assume it to be 1.

Reachability in LLHA [Agrawal-Thiagarajan]

Interface defines an equivalence relation

Let Δ = GCD(P,g,δg,h,δh) and Γ = GCD(RΔ, ε, Bmax, Bmin)
Γ used to construct an equivalence class partitioning.



Equivalence classes are the interiors and line segments

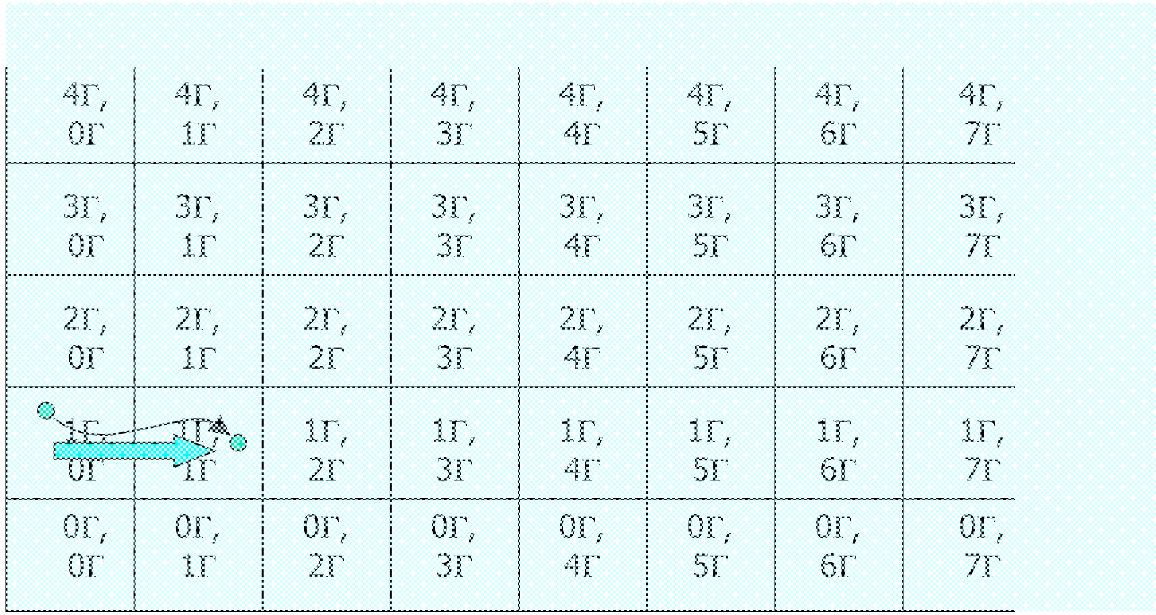
Reachability in LLHA [Agrawal-Thiagarajan]

<u>Interface defines an equivalence relation</u>

This equivalence relation is stable with respect to transitions.

[E(P1,P2) □ P1 -> Q1] => ∃ Q2 s.t. [P2 -> Q2 □ E(Q1,Q2)]

Y_{max}



Y_{min}, X_{min} X_{max}

Reachability in LLHA [Agrawal-Thiagarajan]

- Reachability of lazy linear hybrid automata is decidable. Several relaxations of LLHA like non-linear but computable guards are also decidable.

- The finite quotient space generated is finite with size

  $O(|Q|^4 2^{2n} \Sigma^{3n})$

  Where Q = number of locations

  n = number of continuous variables

  $\Sigma = B_{max}/\Gamma - B_{min}/\Gamma$

  This can be very large !

For just 4 variables, 4 control modes and Σ as 10, the above bound is 1.6777216 _ 10^{19}

Exploring Huge State Space

- Symbolic Bounded Model Checking –
  - Similar to Zone automata construction from the Region automata [Alur & Dill, 94]
  - Explicit enumeration avoided
  - Uses bit-vector decision procedure UCLID

- Abstraction Refinement –
  - Reducing the value Σ in the above formula by looking at larger quanta Γ
  - Establish a hierarchy of sound abstractions with respect to safety properties.

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Overall Tool Flow

Input

Lazy Linear Hybrid Automata and Reachability query

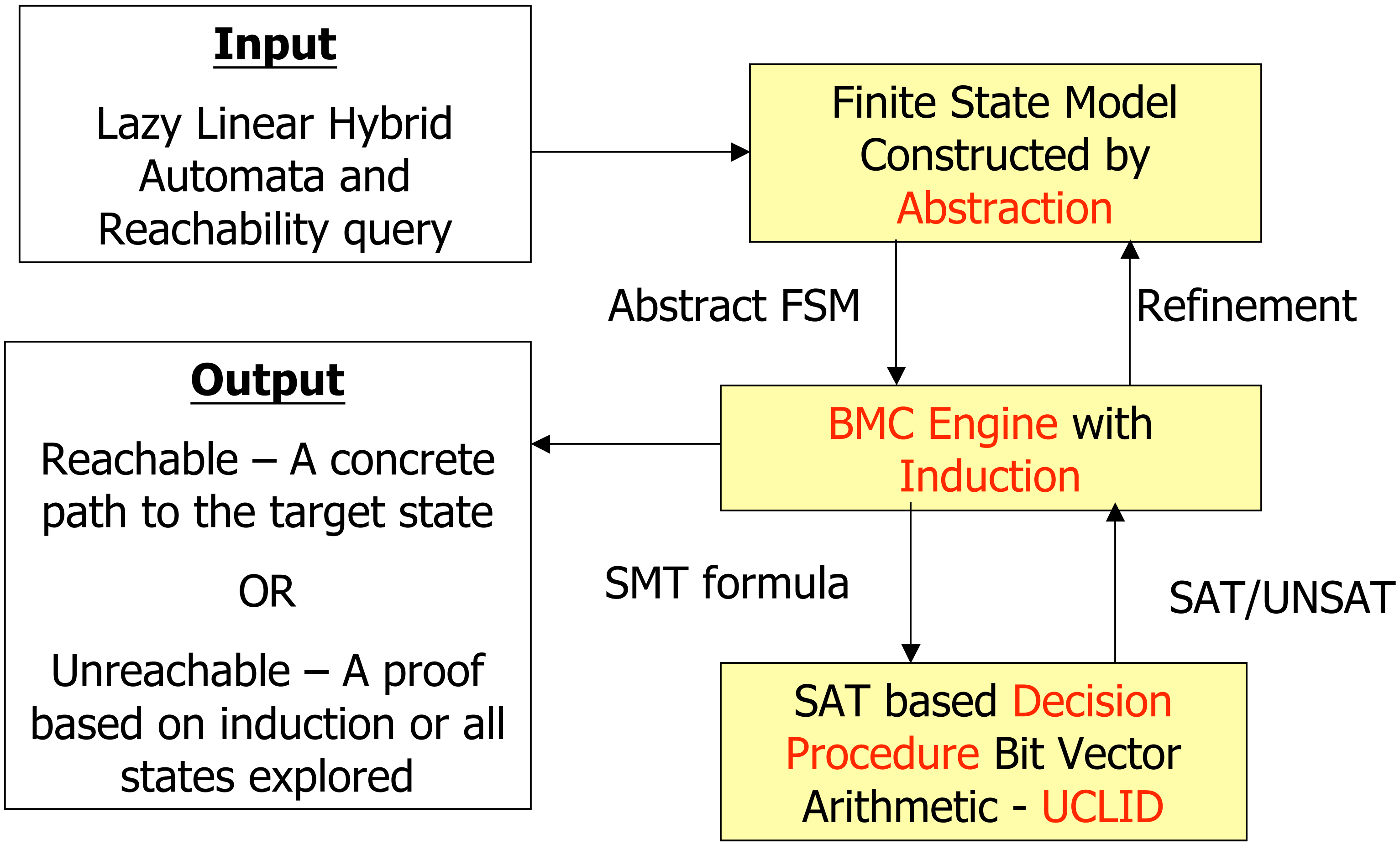
Output

Reachable – A concrete path to the target state

OR

Unreachable – A proof based on induction or all states explored

Overall Tool Flow



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Abstraction of States

Use $2^k\Gamma$ instead of Γ for abstraction. The abstraction so created is called k-abstraction

Y_{max}

4Γ, 0Γ	4Γ, 1Γ	4Γ, 2Γ	4Γ, 3Γ	4Γ, 4Γ	4Γ, 5Γ	4Γ, 6Γ	4Γ, 7Γ	4Γ, 8Γ
3Γ, 0Γ	3Γ, 1Γ	3Γ, 2Γ	3Γ, 3Γ	3Γ, 4Γ	3Γ, 5Γ	3Γ, 6Γ	3Γ, 7Γ	3Γ, 8Γ
2Γ, 0Γ	2Γ, 1Γ	2Γ, 2Γ	2Γ, 3Γ	2Γ, 4Γ	2Γ, 5Γ	2Γ, 6Γ	2Γ, 7Γ	2Γ, 8Γ
1Γ, 0Γ	1Γ, 1Γ	1Γ, 2Γ	1Γ, 3Γ	1Γ, 4Γ	1Γ, 5Γ	1Γ, 6Γ	1Γ, 7Γ	1Γ, 8Γ
0Γ, 0Γ	0Γ, 1Γ	0Γ, 2Γ	0Γ, 3Γ	0Γ, 4Γ	0Γ, 5Γ	0Γ, 6Γ	0Γ, 7Γ	0Γ, 8Γ

Y_{min}, X_{min} X_{max}

State space of k-abstraction would be

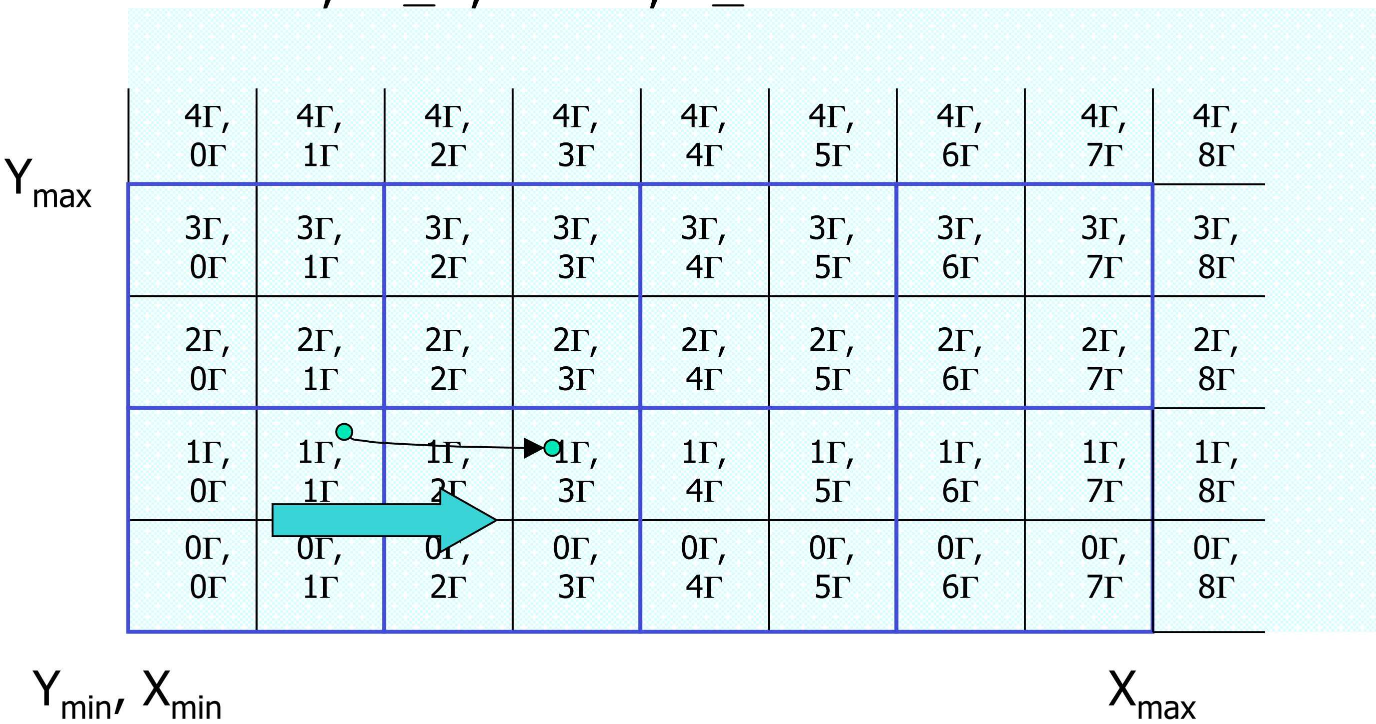
$O(|Q|^4 \, 2^{2n} \, (\Sigma/2^k)^{3n})$, i.e. decrease by 2^{3kn}

Abstraction of Transitions

Transition due to switches – Guards and invariants are relaxed.

For example,

- $267(x-35)/x \leq 150$, that is, $x \leq 35 \times 267/117$.
- Let Γ be 1 and the abstraction be taken $2^5\Gamma$, $8((m-2)/m) \leq 5$, that is, $m \leq 6$, that is, $x \leq 6 \times 2^5$



Abstraction of Flows

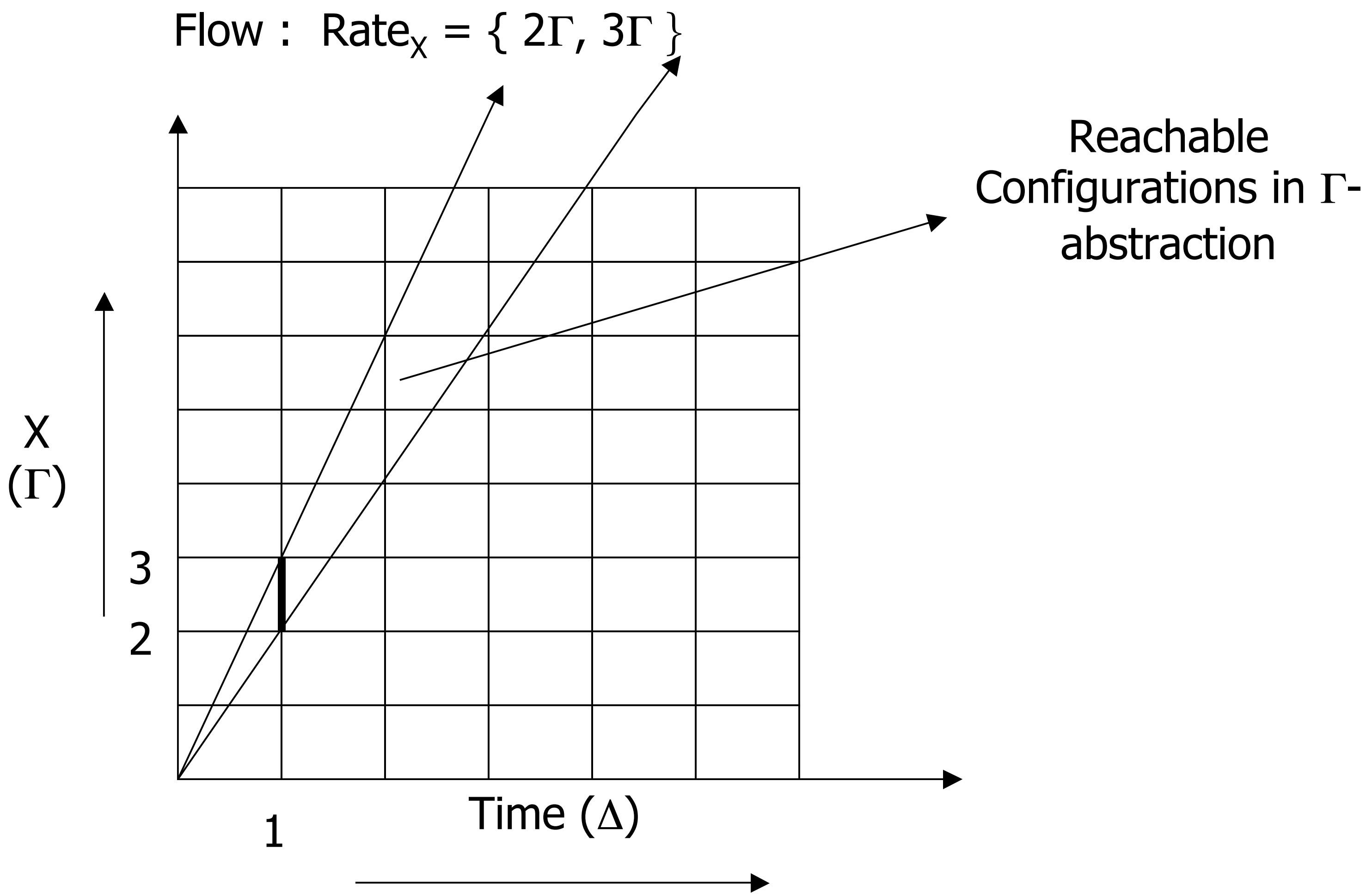
- Key Idea: Adding more flows to preserve simulation

- If rates of change of a variable X is given as the discrete set $R_x = \{r_i\}$

- The rates of change of the variable in k-abstraction is given by
  $R'_x = \cup_i \{ \lfloor r_i/2^k\Gamma \rfloor 2^k\Gamma , \lceil r_i/2^k\Gamma \rceil 2^k\Gamma \}$

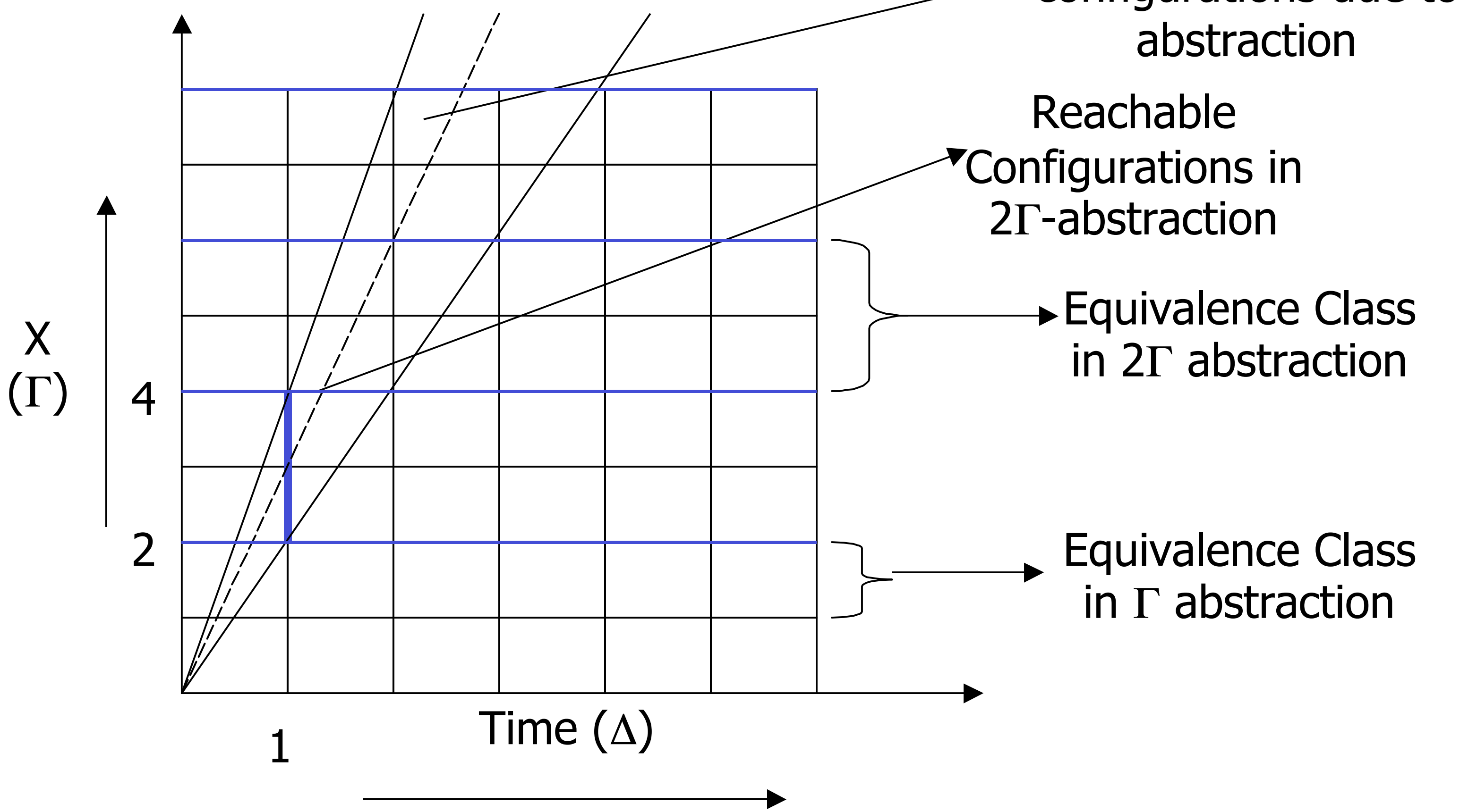
- So if the rates of change were [a,a+1……b], then the abstract rates of change is given by
  $[\lfloor a/2^k\Gamma \rfloor 2^k\Gamma$ □□□ $\lceil b/2^k\Gamma \rceil 2^k\Gamma]$

Abstraction of Flows



Abstraction of Flows

Abstract Flow : $Rate_X = \{ 2\Gamma, 3\Gamma, 4\Gamma \}$



Key Results

- Simulation Result:

The k-abstraction defined above simulates the lazy linear hybrid automata.

- Hierarchy Result:

For any k>m, k-abstraction simulates the m-abstraction.

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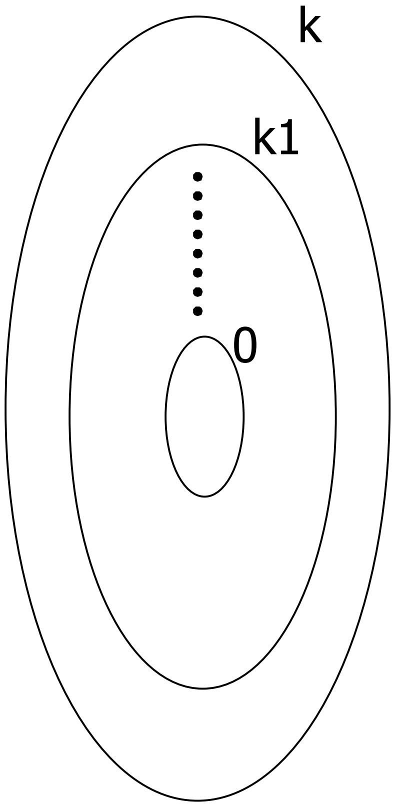
The k-abstraction defined above simulates the lazy linear hybrid automata.

- Hierarchy Result:

For any k>m, k-abstraction simulates the m-abstraction.

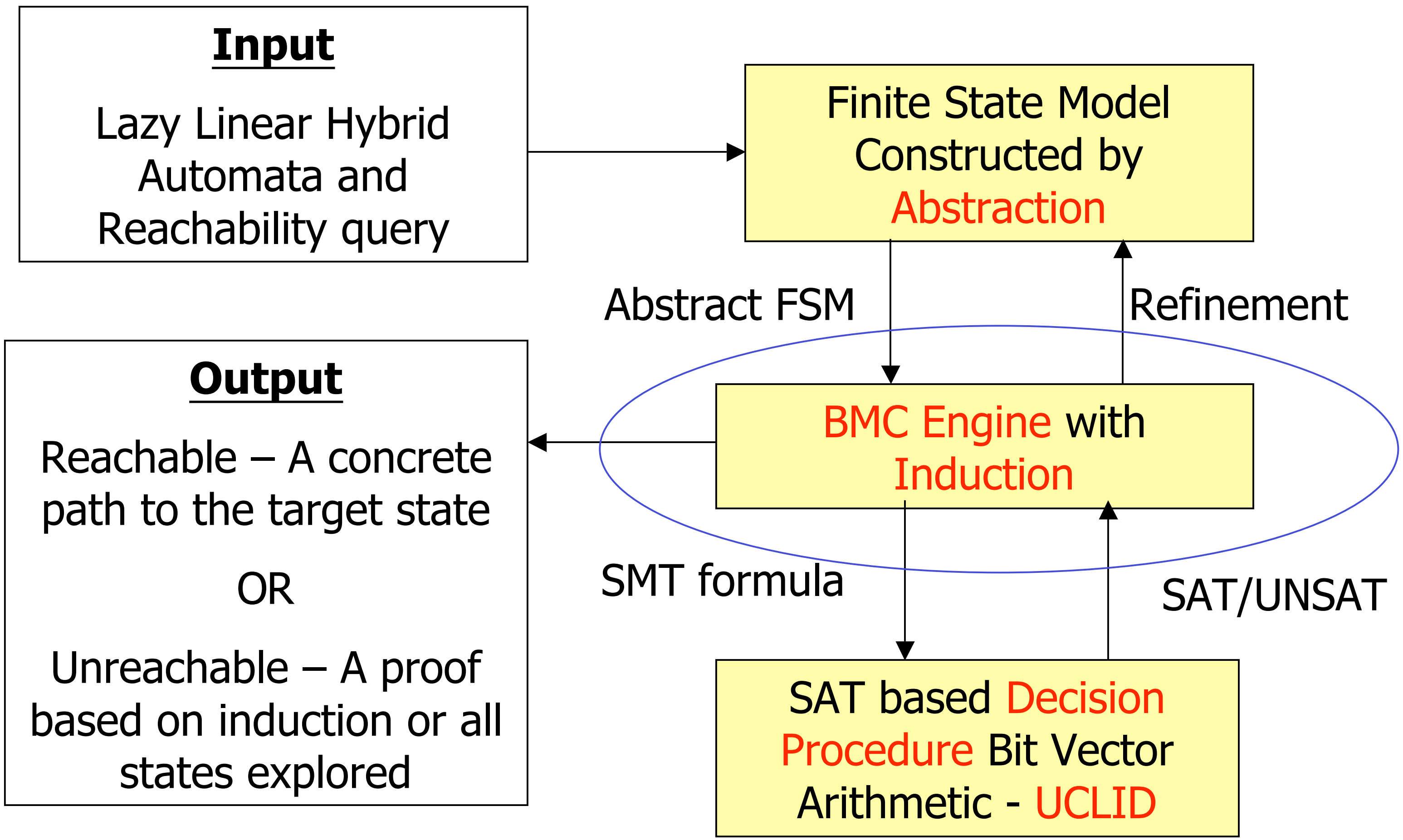
Corollary: If a configuration is not reachable in k-abstraction for some k, it is not reachable in any k'-abstraction for k' < k and is also not reachable in the lazy linear hybrid automata.

Abstraction-Refinement



- Given an LLHA, chose a “suitable” k, to construct a k-abstraction with tractable state space.
- If the target state is not reachable, then declare safe.
- If the target state is reachable, do counter-example guided refinement.
- So, sequence of considered abstraction would be k,k1,k2,…… where k>k1>k2… So, at most k iterations.
- Repeat till 0-abstraction. If target state is still reachable, then it is also reachable in LLHA since 0-abstraction bisimulates LLHA.

Overall Tool Flow



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BMC Formulation

Initial State:

$\text{Init}(F_0) := (l = v_{start}) \;\Box\; \phi_0(X),$

where l denoted the control mode and

ϕ_0 is the initial predicate over the continuous variables.

Transition Predicate:

$T(F_{k-1}, F_k) := \Box_{(i,j) \in E} \, G_{ij}(F_{k-1}, F_k) \;\Box\; \Box_{i \in V} \, E_i(F_{k-1}, F_k),$

where G_{ij} corresponds to switches and E_i corresponds to evolutions.

Is $\text{Init}(F_0) \;\Box\; \Box_{0 \leq i \leq d} \, T(F_i, F_{i+1}) \;\Box\; !\text{safe}(F_d)$ satisfiable ?

(Is !safe reachable in d-steps)

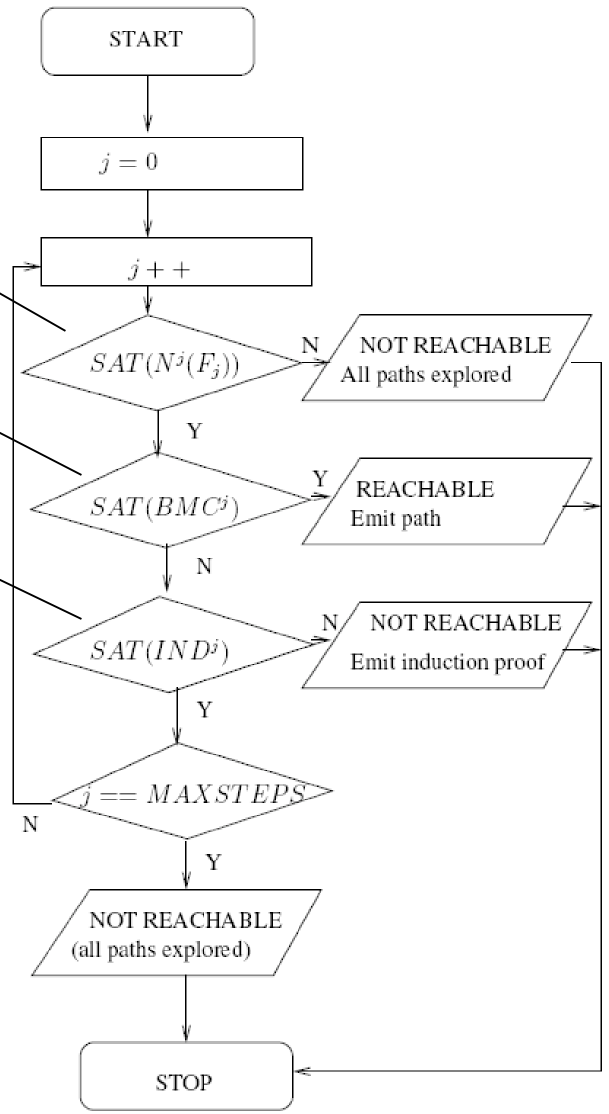
Complete IND-BMC

Check if there exists a simple path unexplored ?

Check if the new paths found (with length=j) can reach bad state ?

Check if j-depth induction can be applied ?

SAT function used in decision boxes correspond to calls to underlying decision procedure - UCLID



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Case study 1: AHS

Normal cruise speed – [a,f]

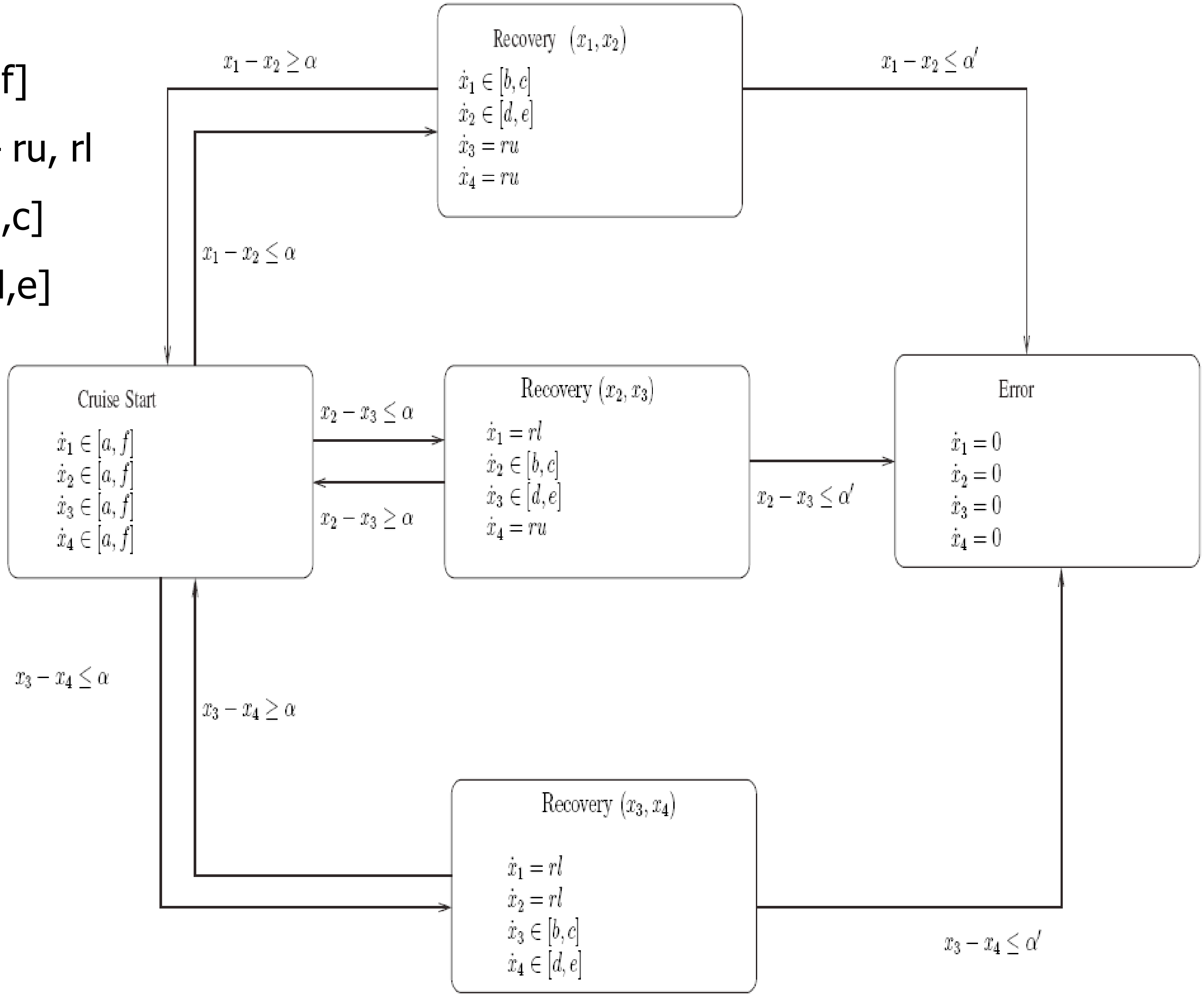
Recovery cruise speed – ru, rl

Recovery speed – slow[b,c]

fast [d,e]

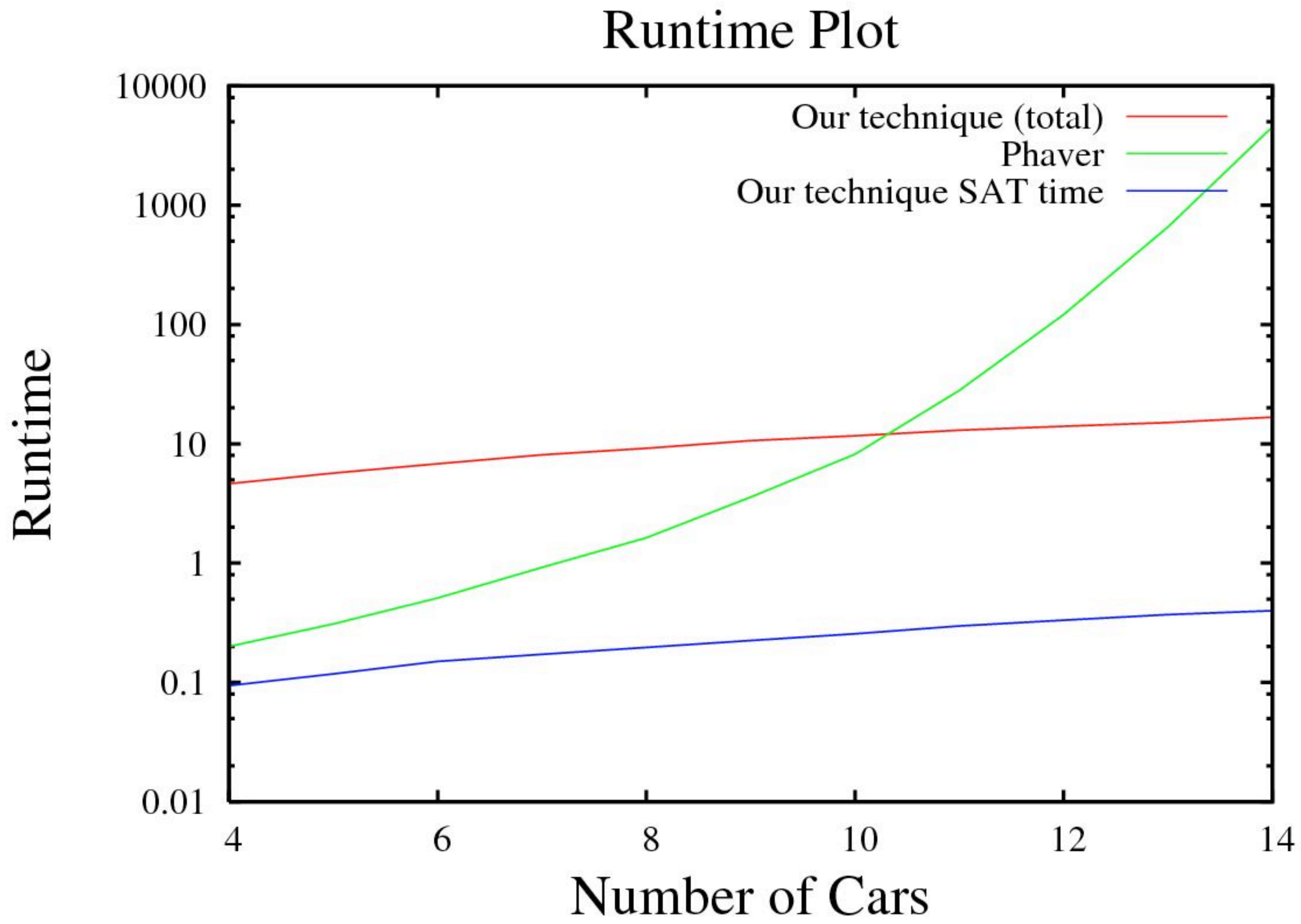
Possible collision α

Actual collision α'



[Model]

Case Study 1: AHS

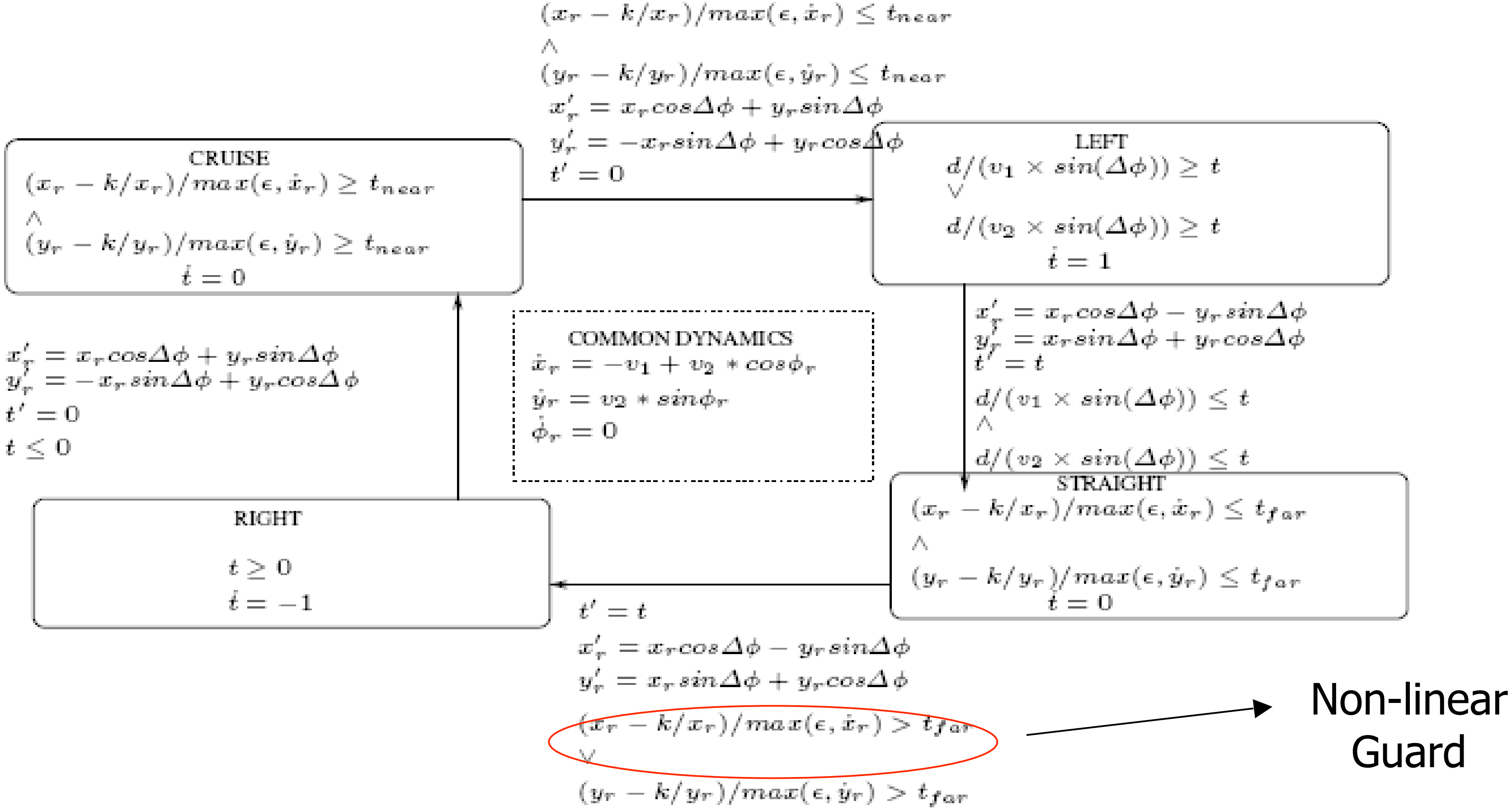


Phaver times out (>10 hours for 15 cars), our technique took less than 2 minutes for 150 cars.

Case Study 2: Simplified TCAS

Model similar to those considered by Tomlin-Pappas

The parameter values obtained from TCAS document by Avionics



Case Study 2: Simplified TCAS

16-abstraction is 10 times faster than 0-abstraction

k	Runtime (sec) for different angles		
	$\Delta\phi = 30^\circ$	$\Delta\phi = 45^\circ$	$\Delta\phi = 60^\circ$
0	400.50	181.47	177.49
2	300.01	732.24	253.96
4	904.76	136.39	544.97
8	117.25	101.01	55.45
16	27.45	18.21	17.64

Runtimes of our model checker for TCAS with varying angles.

Conclusion

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- Demonstration of scalability of our approach on examples like TCAS and AHS