Logiques et modèles pour la vérification de systèmes infinis

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LSV, ENS Cachan, CNRS

Soutenance de thèse de doctorat
10 novembre 2009
Cachan, France

Logics and Models for Verifying Infinite-state Systems

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PhD defense
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Context

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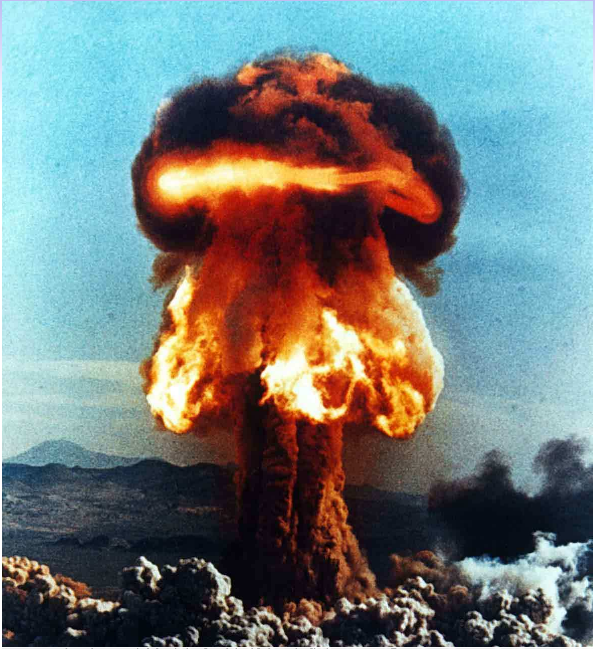
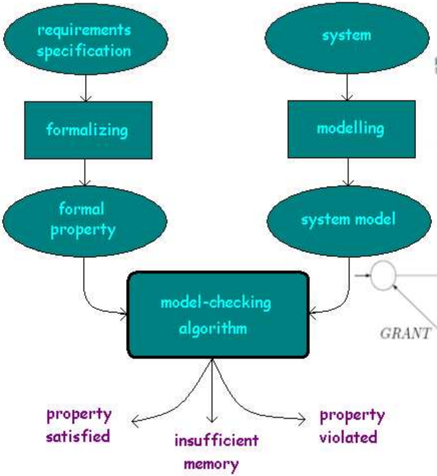


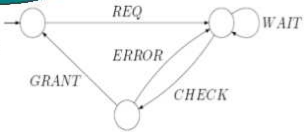
Model Checking

“Does a system achieve exactly what we want it to?”

"each request is eventually granted"

$\Box(REQ \Rightarrow \Diamond GRANT)$





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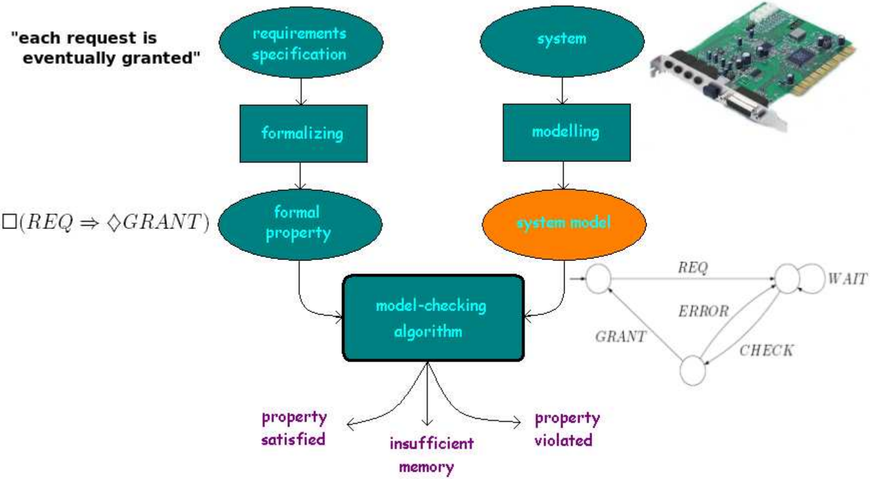

Modelling Systems with Automata

Capturing the behaviour of a system

- Control state of the automaton = general "state" of the system
  (e.g. idle, printing, moving, waiting, etc.)
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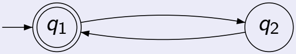
Example: an automaton



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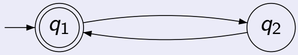

Need for richer models, closer to reality

Extend automata with:
stacks, FIFO channels, integer or real variables, trees, etc.

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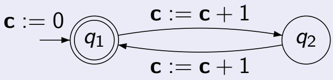
Example: an automaton with one counter



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Reprensenting Sets with Arithmetical Logics

Extending automata with variables...

...generates an infinite set of configurations!

☞ How to check all these configurations?

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Definition

Two logics L, L' (resp. formulas F, F') are equivalent iff they define the same sets. We denote it by $L \equiv L'$ (resp. $F \equiv F'$).

Outline

1. **Decomposing Mixed/Real Logics**
   - Principle of the Decomposition
   - Application over MLA and Extended MLA
2. **Timed Counter Systems**
   - Definitions
   - Region Graph
   - Decidable Subclasses
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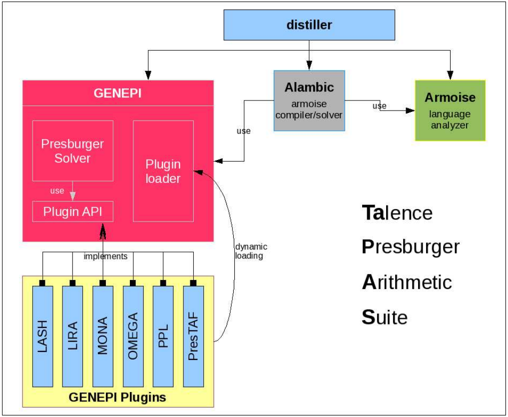
Outline

Motivation

Represent vectors of real values by using the modular framework of the GENEPI tool *[Leroux & Point, 06]*:



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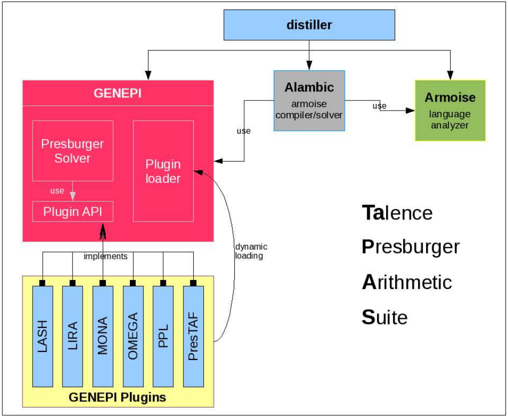

☞ Extract the integer component from reals, to use periodicity

Outline

Finite Unions of Sums "Integer+Decimal"

For any $A, B \subseteq \mathbb{R}$, let $A + B = \{a + b \mid a \in A, b \in B\}$.

Example: a real formula decomposed as "$\bigcup(int + dec)$"...

$$\Big(\{(x,y) \mid 0 \leq x < 10^6 \wedge y = 0\} + \{(x,y) \in [0,1]^2 \mid x = y\}\Big)$$
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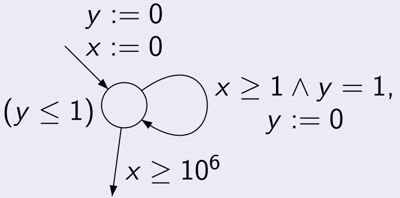

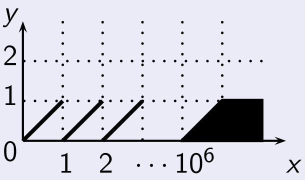

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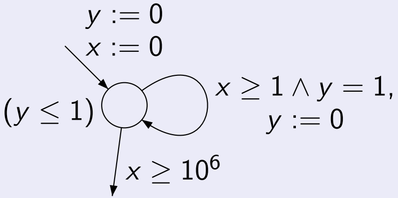

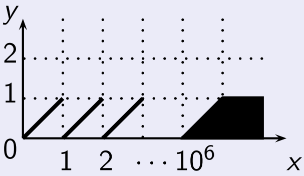

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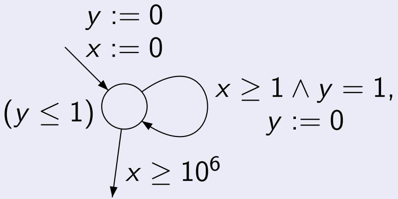

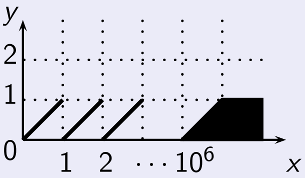

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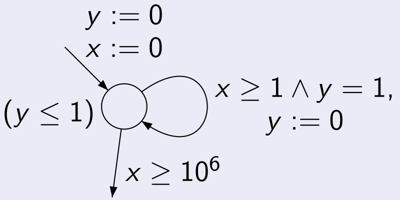

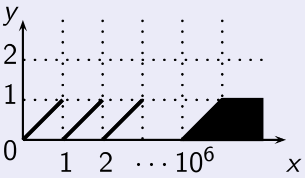

Our representation

- Let $\mathfrak{Z} \subseteq \mathrm{P}(\mathbb{Z}^n)$ and $\mathfrak{D} \subseteq \mathrm{P}(\mathbb{D}^n)$, where $\mathbb{D} = [0, 1[$

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- Let $\mathfrak{Z} \uplus \mathfrak{D}$ be the class of every $R \subseteq \mathbb{R}^n$ such that $R = \bigcup_{i=1}^{p} (Z_i + D_i)$, where $(Z_i, D_i) \in \mathfrak{Z} \times \mathfrak{D}$ and $p \geq 1$

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- Note that some R are not representable !

$$\text{Counter-example: } R = \bigcup_{j=1}^{\infty} \left(\{j\} + \left\{ \frac{1}{j+1} \right\} \right)$$

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A class $\mathfrak{R} \subseteq \bigcup_{n\in\mathbb{N}} P(\mathbb{R}^n)$ is effectively representable if it is stable and if there exist algorithms computing the above operations.

Our representation (continued)

Theorem ($\uplus$-Decidability)

The SATisfiability problem is decidable for any logic encoding $\mathfrak{Z} \uplus \mathfrak{D}$ if $\mathfrak{Z}$ and $\mathfrak{D}$ are effectively representable.

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Proof sketch

- $(Z_1 + D_1) \cap (Z_2 + D_2) = (Z_1 \cap Z_2) + (D_1 \cap D_2)$
- $(Z_1 + D_1) \backslash (Z_2 + D_2) = ((Z_1 \backslash Z_2) + D_1) \cup (Z_1 + (D_1 \backslash D_2))$
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- similarly, projection and permutation distribute easily over $\uplus$
- $\mathbb{R}^n$ can be written $\mathbb{Z}^n + \mathbb{D}^n$
- $R_+ = \{\mathbf{r} \in \mathbb{R}^3 \mid r_1 + r_2 = r_3\}$ can be written
  $$\bigcup_{c \in \{0,1\}} \{\mathbf{z} \in \mathbb{Z}^3 \mid z_1 + z_2 + c = z_3\} + \{\mathbf{d} \in \mathbb{D}^3 \mid d_1 + d_2 = d_3 + c\}$$
- similarly, $\emptyset$, $\leq$, $\mathbb{Z}^n$ are easily definable $\square$

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A quick reminder about logics...

Presburger logic: $\mathbf{FO}\,(\mathbb{Z}, +, \leq)$

$$\begin{aligned}
\text{expression } E &::= 0 \mid 1 \mid x \mid E + E \mid E - E \\
\text{atomic formula } C &::= E = E \mid E < E \\
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Example: a non-Presburger-definable set

There is no Presburger formula defining the set $\{2^n \mid n \in \mathbb{N}\}$.

A quick reminder about logics... (continued)

Mixed Linear Arithmetic (MLA): $\mathrm{FO}(\mathbb{R}, \mathbb{Z}, +, \leq)$

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Decomposing MLA and Extended MLA

Theorem *[TIME'08]*

$$\mathrm{FO}\left(\mathbb{R},\mathbb{Z},+,\leq\right) \equiv \mathrm{FO}\left(\mathbb{Z},+,\leq\right) \uplus \mathrm{FO}\left(\mathbb{D},+,\leq\right)$$

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Proof idea

Application of the $\uplus$-Decidability theorem

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$\mathrm{FO}(\mathbb{R}, \mathbb{Z}, +, \leq, X_b) \equiv \mathrm{FO}(\mathbb{Z}, +, \leq, V_b) \uplus \mathrm{FO}(\mathbb{D}, +, \leq, W_b)$

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Proof idea

- Application of the $\uplus$-Decidability theorem
- Mutual encodings of X_b, V_b, W_b

Extended MLA: X_b, V_b, W_b

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- "$W_b(x) = u$: in x's b-ary writing, the leftmost non-null decimal digit is at position u"

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Example illustrating X_b, V_b, W_b

$$x = (42.0625)_{10} = (0^*101010 \star 00010^\omega)_2$$

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$$X_2(x, 2^4, 0)$$

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$$X_2(x, 2^4, 0)$$

$$x = (42.0625)_{10} = (0^* 101010 \star 00010^\omega)_2$$

Extended MLA: X_b, V_b, W_b

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$$\mathrm{FO}\left(\mathbb{R}, \mathbb{Z}, +, \leq, X_b\right) \equiv \mathrm{FO}\left(\mathbb{Z}, +, \leq, V_b\right) \uplus \mathrm{FO}\left(\mathbb{D}, +, \leq, W_b\right)$$

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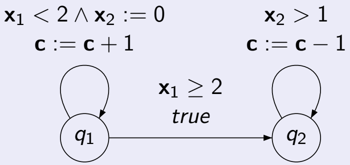
Definitions

Example: a Timed Counter System (TCS)



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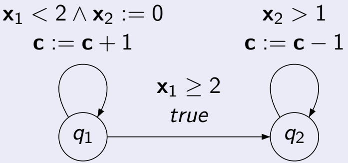

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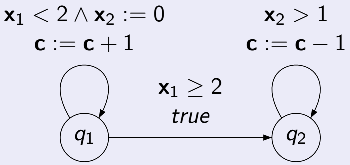

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C = a set of n integer-valued variables, called counters.
$\mathbf{c}$ = a valuation of the counters, in $\mathbb{Z}^n$.
R_C = Presburger-definable binary relations over counters

Definitions (continued)

Definition *[INFINITY'08]*

A Timed Counter System is a tuple $\langle Q, X, C, E \rangle$ where:

- Q is a finite set of control states
- $E \subseteq Q \times R_X \times R_C \times Q$ is a finite set of transitions

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Definition

A *Timed Automaton* is a TCS where $C = \emptyset$.
A *Counter System* is a TCS where $X = \emptyset$.

Semantics of a TCS

For any TCS $\mathcal{S}$, define its:

- Counting Transition System $[\![\mathcal{S}]\!]_C$
- Timed Transition System $[\![\mathcal{S}]\!]_T$
- (full) Transition System $[\![\mathcal{S}]\!]$

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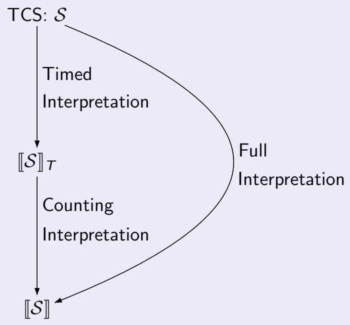

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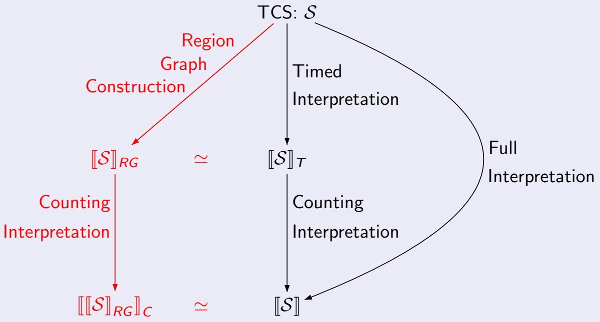
For any TCS $\mathcal{S}$, define its:

- Counting Transition System $[\![\mathcal{S}]\!]_C$
- Timed Transition System $[\![\mathcal{S}]\!]_T$ $\simeq$ Region Graph $[\![\mathcal{S}]\!]_{RG}$
- (full) Transition System $[\![\mathcal{S}]\!]$ $\simeq$ $[\![[\![\mathcal{S}]\!]_{RG}]\!]_C$



Reachability

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Counter Reachability Problem (CRP)

Inputs: A TCS $\mathcal{S}$, an initial configuration $(q_0, \mathbf{x}_0, \mathbf{c}_0)$ of $[\![\mathcal{S}]\!]$, and a configuration $(q, \mathbf{c})$ of $[\![\mathcal{S}]\!]_C$.
Question: Is there a clock valuation $\mathbf{x}$ such that $(q, \mathbf{x}, \mathbf{c})$ is reachable from $(q_0, \mathbf{x}_0, \mathbf{c}_0)$ in $[\![\mathcal{S}]\!]$?

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CRP extends reachability problem of CS, known to be undecidable *[Minsky, 61]*; therefore CRP is undecidable for TCS.

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Counter+region Reachability Problem (CRP)

Inputs: A TCS $\mathcal{S}$, an initial configuration $(q_0, \rho_0, \mathbf{c}_0)$ of $[\![\mathcal{S}]\!]$, a region ρ, and a configuration $(q, \mathbf{c})$ of $[\![\mathcal{S}]\!]_C$.
Question: Is $(q, \rho, \mathbf{c})$ reachable from $(q_0, \rho_0, \mathbf{c}_0)$ in $[\![[\![\mathcal{S}]\!]_{RG}]\!]_C$?

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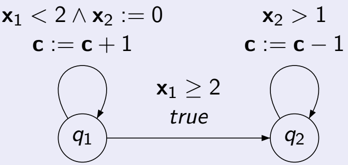
Outline

Regions

Timed Counter System $\mathcal{S}$



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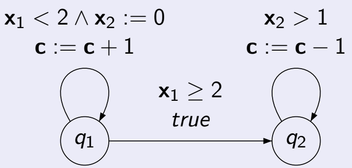
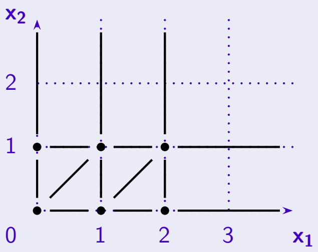


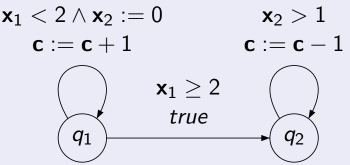
Clock regions of $\mathcal{S}$



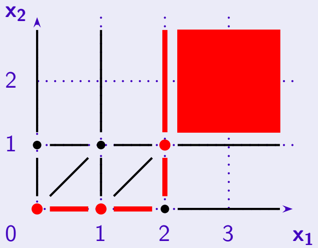
28 regions in total:
6 points, 9 line segments, 5 half-lines, 4 triangular closed areas, and 4 open areas

Regions

Timed Counter System $\mathcal{S}$



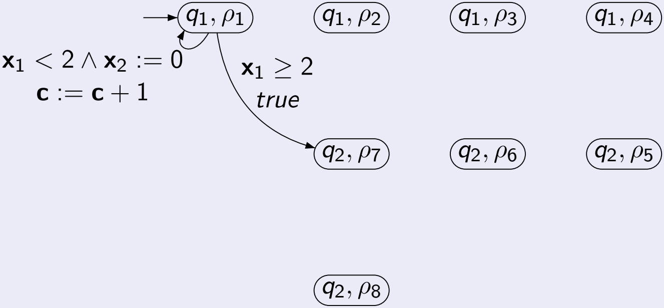
Reachable clock regions of $\mathcal{S}$



8 reachable regions (out of 28), considering the initial configuration $\left(q_1, \binom{0}{0}, 0\right)$

Region Graph

Region Graph $[\![\mathcal{S}]\!]_{RG}$



N.B.: ρ_i are the 8 reachable regions

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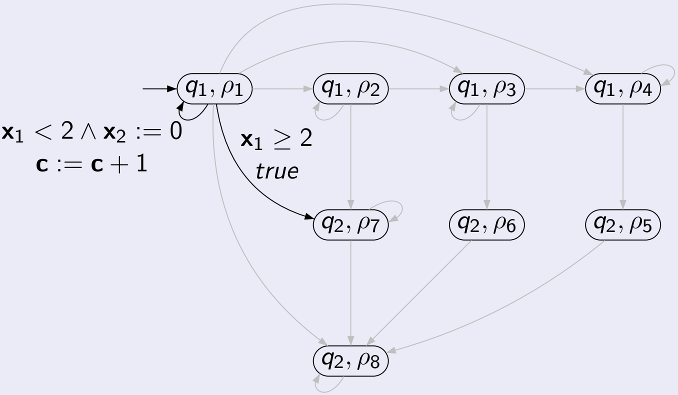

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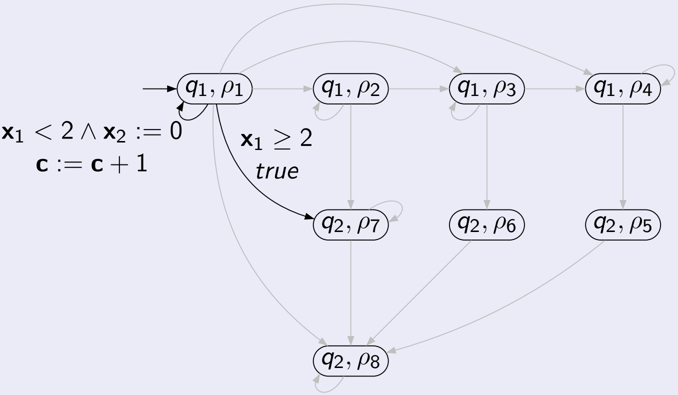
Region Graph

Region Graph $[\![\mathcal{S}]\!]_{RG}$ is a Counter System!



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The Region Graph as a Counter System

Key idea of the analysis of TCS:

For a TCS $\mathcal{S}$, its region graph $[\![\mathcal{S}]\!]_{RG}$ is also a Counter System.

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Let $\mathfrak{C}$ be a class of TCS such that there is an algorithm solving the reachability problem for $[\![\mathcal{S}]\!]_{RG}$, for any $\mathcal{S} \in \mathfrak{C}$.
(e.g. $\mathfrak{C}$ can be VASS/Petri Nets, bounded CS, etc.)

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Proof sketch ▸ time-abstract bisimulation

- By definition, $[\![[\![\mathcal{S}]\!]_T]\!]_C = [\![[\![\mathcal{S}]\!]_C]\!]_T = [\![\mathcal{S}]\!]$.
- $[\![\mathcal{S}]\!]_{RG} \simeq [\![\mathcal{S}]\!]_T$ *[Alur & Dill, 90]*.
- Therefore $[\![[\![\mathcal{S}]\!]_{RG}]\!]_C \simeq [\![\mathcal{S}]\!]$. □

Outline

Subclasses of TCS

Decidability results

Model	Region Graph	Counter Reach. Prob.
TCS	CS	Undecidable
TVASS	VASS	Decidable
Reversal-bounded TCM	Reversal-bounded CM	Decidable
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- Timed Counter Machine (TCM) = TCS whose relations on counters are translations with guards of the form $\mathbf{c} \geq k$ or $\mathbf{c} = k$, with $k \in \mathbb{N}^n$
- Timed VASS (TVASS) = TCM with only $\mathbf{c} \geq k$ guards
- Bounded TCS = TCS whose counter values are bounded
- Reversal-Bounded TCM = TCM whose counters do a bounded number of alternations between increasing and decreasing modes

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1. **Decomposing Mixed/Real Logics**
   - Principle of the Decomposition
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   - Definitions
   - Region Graph
   - Decidable Subclasses
3. **Dense-choice Counter Machines**
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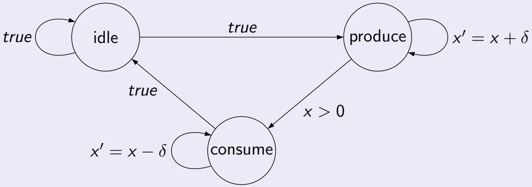

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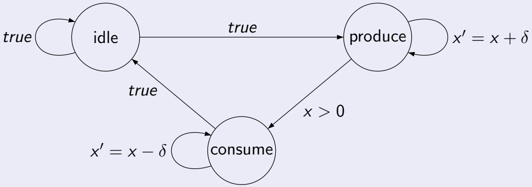

A new value δ is chosen in $]0, 1[$ at each step!

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Actions $A = \{1, \Delta\}$ (i.e. increments/decrements)
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DCM

A Dense-choice Counter Machine with $n > 0$ counters is a tuple $\mathcal{M} = \langle S, T \rangle$ where:

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Example

A transition $x > 0 \wedge x := x - 3\delta$ is written $(x = 0, -3, \Delta)$.

Definitions (continued)

Semantics of a DCM $\mathcal{M} = \langle S, T \rangle$

The transition system $TS(\mathcal{M}) = \langle C, \rightarrow \rangle$ is defined by:

- $C = S \times \mathbb{R}_+^n$ is the set of configurations
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  $(s, \mathbf{x}) \xrightarrow{\mathbf{g}, \boldsymbol{\lambda}, \mathbf{a}} (s', \mathbf{x}') \iff (s, (\mathbf{g}, \boldsymbol{\lambda}, \mathbf{a}), s') \in T \wedge \exists \delta \in \mathbb{R}$ such that:
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Subclasses of DCM

- purely-DCM: $a_i = \Delta$ for each counter $i \leq n$
- (discrete) CM: $a_i = 1$ for each counter $i \leq n$ (i.e. Minsky machine)
- reversal-bounded DCM: $\exists r \in \mathbb{N}$ s.t. on every run, each counter switches between increments and decrements at most r times
- bounded DCM: $\exists b \in \mathbb{N}$ s.t. on every run, no counter exceeds b

Encodings of macro-operations in DCM

$$s \xrightarrow{\textbf{reset(x)}} s' : \quad \begin{tikzcd} s \arrow[loop above, "x := x - \delta"] \arrow[r, "x = 0"] & s' \end{tikzcd}$$

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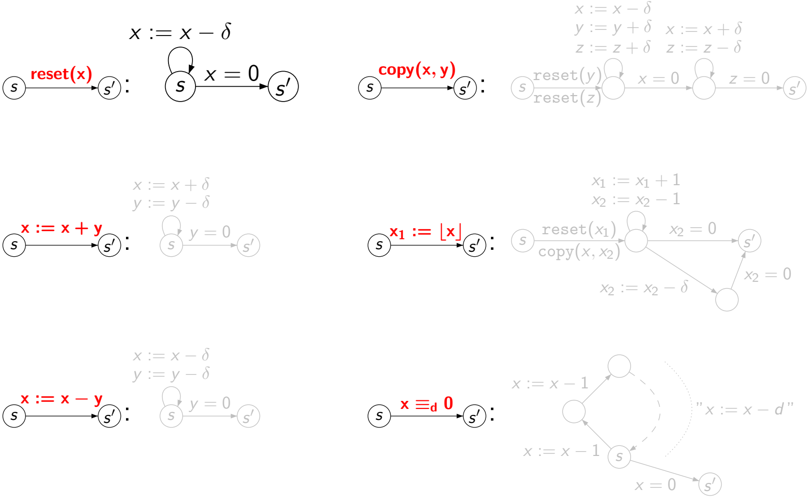

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Theorem *[Ibarra, 78]*

The reachability relation of a reversal-bounded CM is definable in Presburger logic.

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Theorem *[INFINITY'09]*

Each MLA formula defines the reachability relation of a reversal-bounded DCM.

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(De)composition of logics

- Characterization of logics
- Decidability proofs for SATisfiability of logics

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DCM

- Full logical characterization of reversal-bounded DCM with MLA

Future Work

Theory

- Study new logics through decomposition (for charac. & decidability)
- Extend decidable subclasses (flat TCS)
- Generalize TCS analysis technique to Timed *Heterogeneous* Systems
- Solve open decidability problems for subclasses of DCM
- Study complexity of decision procedures in DCM

Future Work

Theory

- Study new logics through decomposition (for charac. & decidability)
- Extend decidable subclasses (flat TCS)
- Generalize TCS analysis technique to Timed *Heterogeneous* Systems
- Solve open decidability problems for subclasses of DCM
- Study complexity of decision procedures in DCM

Implementation

- Enhance GENEPI prototype decomposing logics
- Implement region construction in FAST
- Design a tool for reversal-bounded DCM