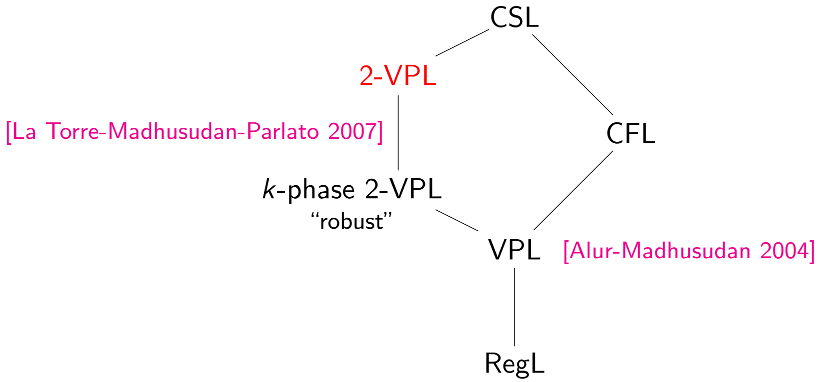
On the Expressive Power of 2-Stack Visibly Pushdown Automata

Benedikt Bollig

LSV, ENS Cachan, CNRS
France

Theorietag 2007, Leipzig

2-Stack Visibly Pushdown Automata (2-VPA)



2-Stack Visibly Pushdown Automata (2-VPA)

Input alphabet is partitioned into push, pop, and internal actions:

$$\Sigma = (Push_1, Pop_1, Push_2, Pop_2, Int)$$

A 2-VPA

- pushes a symbol onto i'th stack when reading a letter from $Push_i$
- pops a symbol from i'th stack when reading a letter from Pop_i
- does not touch the stacks when reading a letter from Int

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \qquad\qquad \delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3)$$

$$(q_1, a, A, q_2) \qquad\qquad (q_3, \overline{a}, \perp, q_4)$$

$$(q_2, b, B, q_1) \qquad\qquad (q_4, \overline{b}, B, q_4)$$

$$(q_2, b, B, q_3) \qquad\qquad (q_4, \overline{b}, \perp, q_5)$$

$\boxed{\perp}$ $\boxed{\perp}$

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\begin{array}{llcll}
\delta_{\mathsf{push}}: & (q_0, a, A, q_2) & \qquad & \delta_{\mathsf{pop}}: & (q_3, \overline{a}, A, q_3) \\
 & (q_1, a, A, q_2) & & & (q_3, \overline{a}, \perp, q_4) \\
 & (q_2, b, B, q_1) & & & (q_4, \overline{b}, B, q_4) \\
 & (q_2, b, B, q_3) & & & (q_4, \overline{b}, \perp, q_5)
\end{array}$$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_0

$\perp$ $\perp$

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

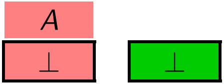
Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$

$$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$$

q_2



Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

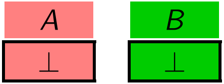
Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$

$$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$$

q_1



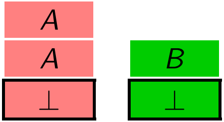
Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$



$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

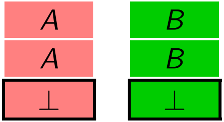
q_2

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\begin{array}{llcll}
\delta_{\text{push}}: & (q_0, a, A, q_2) & \quad & \delta_{\text{pop}}: & (q_3, \overline{a}, A, q_3) \\
& (q_1, a, A, q_2) & & & (q_3, \overline{a}, \perp, q_4) \\
& (q_2, b, B, q_1) & & & (q_4, \overline{b}, B, q_4) \\
& (q_2, b, B, q_3) & & & (q_4, \overline{b}, \perp, q_5)
\end{array}$$



$$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$$

q_3

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

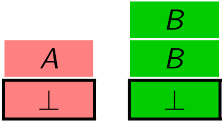
Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\mathsf{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\mathsf{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_3



Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

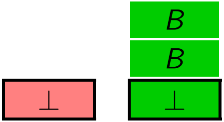
Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$

$$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$$

q_3



Example of a 2-VPA

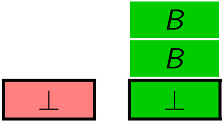
$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

δ_{push} :	(q_0, a, A, q_2)	δ_{pop} :	$(q_3, \overline{a}, A, q_3)$
	(q_1, a, A, q_2)		$(q_3, \overline{a}, \perp, q_4)$
	(q_2, b, B, q_1)		$(q_4, \overline{b}, B, q_4)$
	(q_2, b, B, q_3)		$(q_4, \overline{b}, \perp, q_5)$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_4



Example of a 2-VPA

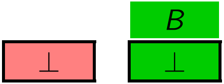
$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

δ_{push} :	(q_0, a, A, q_2)	δ_{pop} :	$(q_3, \overline{a}, A, q_3)$
	(q_1, a, A, q_2)		$(q_3, \overline{a}, \perp, q_4)$
	(q_2, b, B, q_1)		$(q_4, \overline{b}, B, q_4)$
	(q_2, b, B, q_3)		$(q_4, \overline{b}, \perp, q_5)$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_4



Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\begin{array}{llll}
\delta_{\text{push}}: & (q_0, a, A, q_2) & \delta_{\text{pop}}: & (q_3, \overline{a}, A, q_3) \\
 & (q_1, a, A, q_2) & & (q_3, \overline{a}, \perp, q_4) \\
 & (q_2, b, B, q_1) & & (q_4, \overline{b}, B, q_4) \\
 & (q_2, b, B, q_3) & & (q_4, \overline{b}, \perp, q_5)
\end{array}$$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_4

$\boxed{\perp}$ $\boxed{\perp}$

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\begin{array}{llcll}
\delta_{\text{push}}: & (q_0, a, A, q_2) & \qquad & \delta_{\text{pop}}: & (q_3, \overline{a}, A, q_3) \\
& (q_1, a, A, q_2) & & & (q_3, \overline{a}, \perp, q_4) \\
& (q_2, b, B, q_1) & & & (q_4, \overline{b}, B, q_4) \\
& (q_2, b, B, q_3) & & & (q_4, \overline{b}, \perp, q_5)
\end{array}$$

$a \quad b \quad a \quad b \quad \overline{a} \quad \overline{a} \quad \overline{a} \quad \overline{b} \quad \overline{b} \quad \overline{b}$

q_5

$\boxed{\perp}$ $\boxed{\perp}$

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\begin{array}{llcll}
\delta_{\text{push}}: & (q_0, a, A, q_2) & \quad & \delta_{\text{pop}}: & (q_3, \overline{a}, A, q_3) \\
 & (q_1, a, A, q_2) & & & (q_3, \overline{a}, \perp, q_4) \\
 & (q_2, b, B, q_1) & & & (q_4, \overline{b}, B, q_4) \\
 & (q_2, b, B, q_3) & & & (q_4, \overline{b}, \perp, q_5)
\end{array}$$

$$L(\mathcal{A}) = \{(ab)^n \, \overline{a}^{\,n+1} \, \overline{b}^{\,n+1} \mid n \geq 1\}$$

$\boxed{\perp}$ $\boxed{\perp}$

Example of a 2-VPA

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of 2-VPA $\mathcal{A}$ with initial state q_0 and final state q_5:

$$\delta_{\text{push}}: \quad (q_0, a, A, q_2) \quad (q_1, a, A, q_2) \quad (q_2, b, B, q_1) \quad (q_2, b, B, q_3)$$

$$\delta_{\text{pop}}: \quad (q_3, \overline{a}, A, q_3) \quad (q_3, \overline{a}, \perp, q_4) \quad (q_4, \overline{b}, B, q_4) \quad (q_4, \overline{b}, \perp, q_5)$$

$$L(\mathcal{A}) = \{(ab)^n \, \overline{a}^{\,n+1} \, \overline{b}^{\,n+1} \mid n \geq 1\}$$

2-phase words $\Longrightarrow$ 2-phase 2-VPA

$\boxed{\perp}$ $\boxed{\perp}$

k-phase 2-VPA vs. 2-VPA

k-phase 2-VPA are "robust" [La Torre-Madhusudan-Parlato 2007]

- Decidable emptiness, inclusion, and equivalence problem
- Closed under all boolean operations
- Not determinizable
- MSO characterization

$\Longrightarrow$ upcoming talk

2-VPA

- Undecidable emptiness problem
- Closed under union and intersection
- Not closed under complementation
- EMSO characterization

Key technique: Nested words [Alur-Madhusudan 2006]

Word over $\Sigma = \{a, \overline{a}, b, \overline{b}\}$:

$$a \longrightarrow b \longrightarrow a \longrightarrow b \longrightarrow \overline{a} \longrightarrow \overline{a} \longrightarrow \overline{a} \longrightarrow \overline{b} \longrightarrow \overline{b} \longrightarrow \overline{b}$$

Key technique: Nested words [Alur-Madhusudan 2006]

Nested word over $\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$:

$$a \longrightarrow b \longrightarrow a \longrightarrow b \longrightarrow \overline{a} \longrightarrow \overline{a} \longrightarrow \overline{a} \longrightarrow \overline{b} \longrightarrow \overline{b} \longrightarrow \overline{b}$$

Key technique: Nested words [Alur-Madhusudan 2006]

Nested word over $\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$:

$$
\begin{tikzcd}
a \arrow[r] & b \arrow[r] & a \arrow[r] \arrow[rr, bend left] & b \arrow[r] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{b} \arrow[r] & \overline{b} \arrow[r] & \overline{b}
\end{tikzcd}
$$

Key technique: Nested words [Alur-Madhusudan 2006]

Nested word over $\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$:

$$\begin{tikzcd}
a \arrow[r] \arrow[rrrrr, bend left] & b \arrow[r] & a \arrow[r] \arrow[rr, bend left] & b \arrow[r] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{b} \arrow[r] & \overline{b} \arrow[r] & \overline{b}
\end{tikzcd}$$

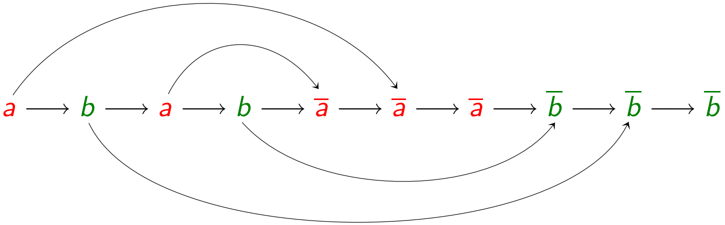
Key technique: Nested words [Alur-Madhusudan 2006]

Nested word over $\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$:

$$\begin{tikzcd}
a \arrow[r] \arrow[rrrrr, bend left] & b \arrow[r] & a \arrow[r] \arrow[rr, bend left] & b \arrow[r] \arrow[rrrr, bend right] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{a} \arrow[r] & \overline{b} \arrow[r] & \overline{b} \arrow[r] & \overline{b}
\end{tikzcd}$$

Key technique: Nested words [Alur-Madhusudan 2006]

Nested word over $\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$:



2-Stack Nested Word Automata (2-NWA)

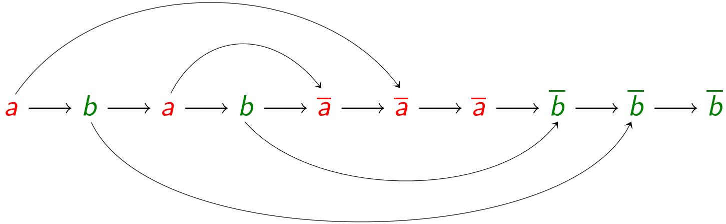
$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\delta_1 : \quad (q_0, a, q_2) \qquad\qquad \delta_2 : \quad (q_2, q_3, \overline{a}, q_3)$$

$$(q_2, b, q_1) \qquad\qquad (q_3, q_4, \overline{b}, q_4)$$

$$\vdots \qquad\qquad (q_1, q_4, \overline{b}, q_4)$$

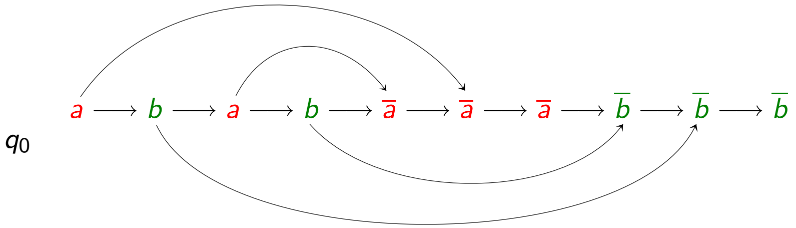


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll}
\delta_1: & (q_0, a, q_2) & \qquad & \delta_2: & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$

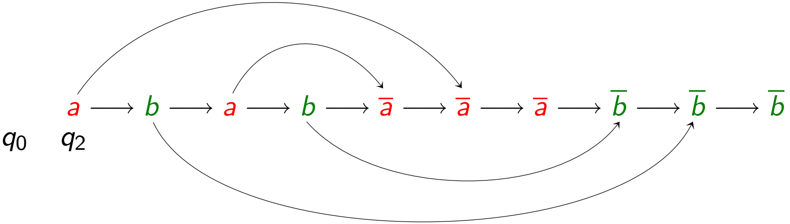


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll}
\delta_1 : & (q_0, a, q_2) & \qquad & \delta_2 : & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$



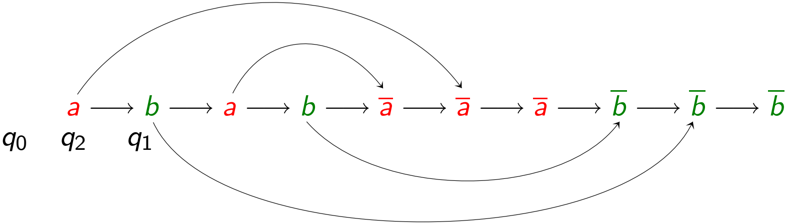
2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

δ_1 : (q_0, a, q_2), (q_2, b, q_1), $\vdots$

δ_2 : $(q_2, q_3, \overline{a}, q_3)$, $(q_3, q_4, \overline{b}, q_4)$, $(q_1, q_4, \overline{b}, q_4)$

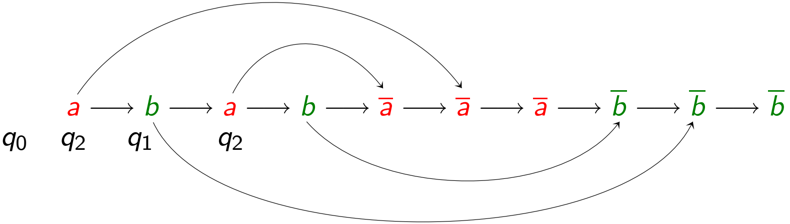


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll}
\delta_1 : & (q_0, a, q_2) & \qquad & \delta_2 : & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$

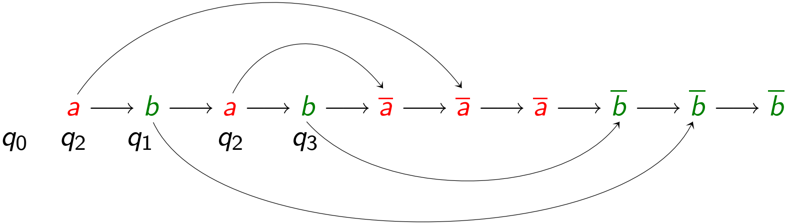


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llll}
\delta_1: & (q_0, a, q_2) & \delta_2: & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$

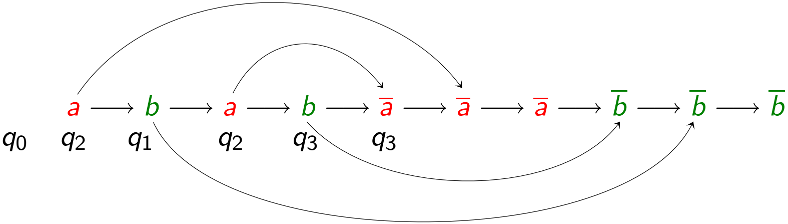


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll} \delta_1 : & (q_0, a, q_2) & \quad & \delta_2 : & (q_2, q_3, \overline{a}, q_3) \\ & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\ & \vdots & & & (q_1, q_4, \overline{b}, q_4) \end{array}$$

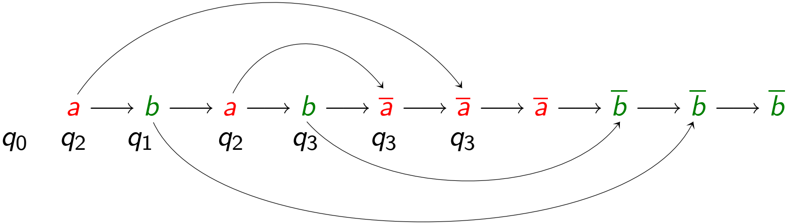


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\delta_1 : \quad (q_0, a, q_2) \qquad\qquad \delta_2 : \quad (q_2, q_3, \overline{a}, q_3)$$
$$(q_2, b, q_1) \qquad\qquad (q_3, q_4, \overline{b}, q_4)$$
$$\vdots \qquad\qquad (q_1, q_4, \overline{b}, q_4)$$



2-Stack Nested Word Automata (2-NWA)

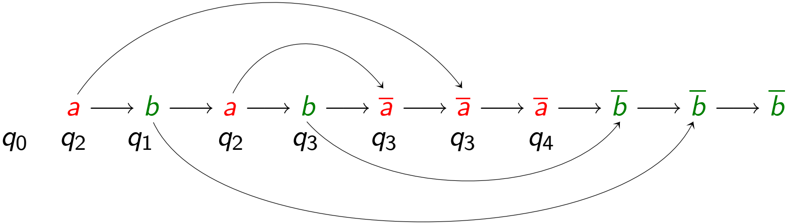
$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\delta_1 : \quad (q_0, a, q_2) \qquad\qquad \delta_2 : \quad (q_2, q_3, \overline{a}, q_3)$$

$$(q_2, b, q_1) \qquad\qquad (q_3, q_4, \overline{b}, q_4)$$

$$\vdots \qquad\qquad (q_1, q_4, \overline{b}, q_4)$$

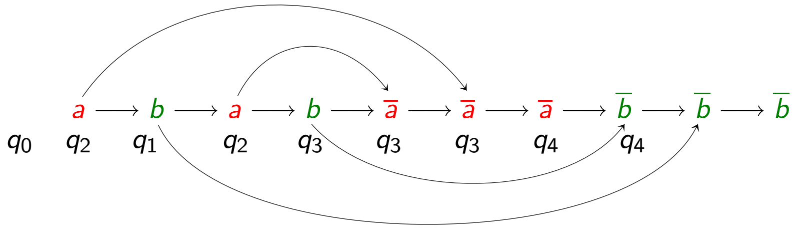


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll}
\delta_1: & (q_0, a, q_2) & \qquad & \delta_2: & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$

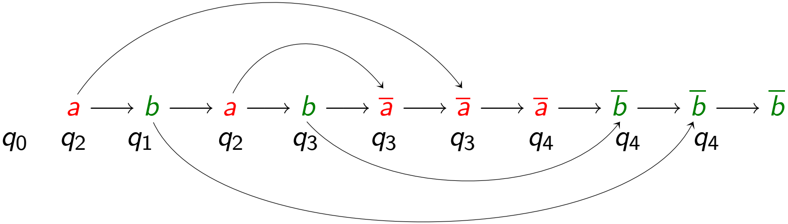


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

$$\begin{array}{llcll}
\delta_1: & (q_0, a, q_2) & \quad & \delta_2: & (q_2, q_3, \overline{a}, q_3) \\
 & (q_2, b, q_1) & & & (q_3, q_4, \overline{b}, q_4) \\
 & \vdots & & & (q_1, q_4, \overline{b}, q_4)
\end{array}$$

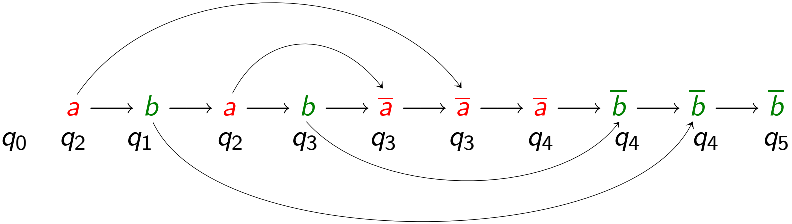


2-Stack Nested Word Automata (2-NWA)

$$\Sigma = (\{a\}, \{\overline{a}\}, \{b\}.\{\overline{b}\}, \emptyset)$$

Transitions of a 2-NWA $\mathcal{A}$:

δ_1 :	(q_0, a, q_2)	δ_2 :	$(q_2, q_3, \overline{a}, q_3)$
	(q_2, b, q_1)		$(q_3, q_4, \overline{b}, q_4)$
	$\vdots$		$(q_1, q_4, \overline{b}, q_4)$

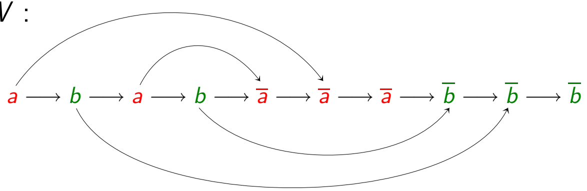


Logical characterization of 2-NWA/2-VPA

EMSO Logic

$$\varphi ::= a(x) \mid \mathrm{succ}(x,y) \mid \mathrm{match}(x,y) \mid x = y \mid x \in X$$
$$\neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \exists x \varphi \mid \exists X \varphi \ \text{(restricted use)}$$

W :



Example

- $W \models \forall x.(a(x) \rightarrow \exists y.\mathrm{match}(x,y))$
- $W \not\models \exists x.\exists y.(\mathrm{succ}(x,y) \wedge \mathrm{match}(x,y))$

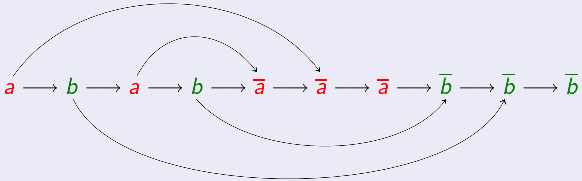
Logical characterization of 2-NWA/2-VPA

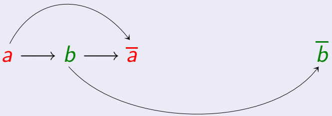
Theorem

2-NWA are expressively equivalent to EMSO logic.

Proof.

Construct 2-NWA that detects spheres, then apply Hanf's Theorem.





1-sphere around
4th position

□

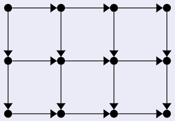
2-NWA and complementability

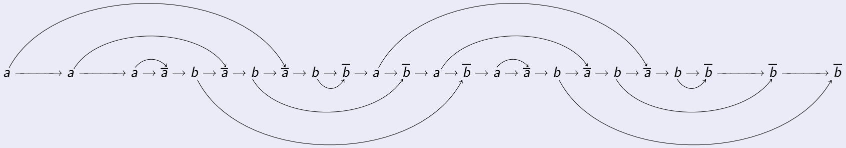
Theorem

2-NWA/2-VPA cannot be complemented in general.

Proof.

FO reduction from grids to nested words, then apply [Matz-Thomas 1997].





□

Future work

- Logical characterization of general multi-stack VPA?
- How about more general visibly pushdown alphabets?
  [Carotenuto-Murano-Peron 2007]
- How about $\leq$ in EMSO logic?