Coq Tutorial : Basic Tactics

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Figure 1 describes a few tactics as inference rules: a tactic replaces a goal of the form shown in conclusion of the rule with a number of new subgoals described by the premisses of the rule.

Goals are written as the sequents of first-order natural deduction, but are richer in that they also contain declarations for universally quantified variables, e.g. $\mathrm{x} : \mathsf{nat}, \mathrm{H} : x \neq 0 \vdash 0 < x$.

Negation and equivalence. Negation $\neg P$ is defined as $P \to \bot$ and, accordingly, you can use the tactics available for implication directly on $\neg P$. The same goes for $P \leftrightarrow Q$ which is defined as $(P \to Q) \wedge (Q \to P)$.

Complex tactic invocations. Several tactics use an hypothesis name H to refer to an item in the current goal's context. In such cases H can also be the name of a previously proved result (e.g. **apply** strong_induction) or the constructor of an inductively defined predicate (e.g. **apply** le_n).

If H corresponds to a formula it is also possible to specify how universally quantified variables and hypotheses must be instantiated: for example, with mylemma : $\forall x.\ \neg P$ and H : P one can directly do **elim** (mylemma t H).

Equality. There are several ways to use an hypothesis H : $u = v$.

- **inversion** H should only be used when the equal terms feature a constructor (e.g. $0 = \mathsf{S}x$, $\mathsf{S}x = \mathsf{S}y$).
- **rewrite** H replaces all occurrences of u by v in the goal.
- **rewrite** H **in** H′ performs the replacement in hypothesis H′.
- **rewrite**<− replaces v by u rather than u by v.

Other tactics. A few tactics cannot easily be described as a single inference rule.

- **unfold** f unfolds the definition of f in the current goal. One can also use the variant **unfold** ... **in** H. It is sometimes useful to understand a goal but never necessary (*except* before rewriting) since Coq implicitly performs the required unfolding before other tactic applications.
- **destruct** x can be used to perform a case analysis on x if it belongs to an inductive type, e.g. nat.
- **inversion** H performs a case analysis on H : P when P is an instance of an inductively defined predicate (e.g. equality eq, le, multiple).
- **simpl** performs all possible computations to simplify the current goal.
- **clear** H x H′ ... drops unused items. An item can only be dropped if (after having dropped the previous items) it is unused in the context.

exact H.

$$\frac{}{\ldots,\ \mathrm{H} : P \vdash P}$$

split.

$$\frac{\ldots \vdash P \quad \ldots \vdash Q}{\ldots \vdash P \wedge Q}$$

destruct H.

$$\frac{\ldots,\ \mathrm{H} : P,\ \mathrm{H}' : Q \vdash ...}{\ldots,\ \mathrm{H} : P \wedge Q \vdash ...}$$

left.

$$\frac{\ldots \vdash P}{\ldots \vdash P \vee Q}$$

right.

$$\frac{\ldots \vdash Q}{\ldots \vdash P \vee Q}$$

destruct H.

$$\frac{\ldots,\ \mathrm{H} : P \vdash ... \quad \ldots,\ \mathrm{H} : Q \vdash ...}{\ldots,\ \mathrm{H} : P \vee Q \vdash ...}$$

intro ⟨H⟩.

$$\frac{\ldots,\ \mathrm{H} : P \vdash Q}{\ldots \vdash P \to Q}$$

intro ⟨x⟩.

$$\frac{\ldots,\ \mathrm{x} : t \vdash P\{y \mapsto x\}}{\ldots \vdash \forall y : t.P}$$

exists u.

$$\frac{\ldots \vdash P\{x \mapsto u\}}{\ldots \vdash \exists x : t.P}$$

apply H.

$$\frac{\ldots \vdash P_1\{x_i \mapsto t_i\} \quad \ldots \quad \ldots \vdash P_n\{x_i \mapsto t_i\}}{\ldots,\ \mathrm{H} : \forall \vec{x_i}.\ \vec{P_j} \to Q \vdash Q\{x_i \mapsto t_i\}}$$

destruct H.

$$\frac{\ldots,\ \mathrm{x} : t,\ \mathrm{H} : P \vdash ...}{\ldots,\ \mathrm{H} : \exists x : t.P \vdash ...}$$

reflexivity.

$$\frac{}{\ldots \vdash u = u}$$

rewrite H.

$$\frac{\ldots,\ \mathrm{H} : u = v \vdash P\{x \mapsto v\}}{\ldots,\ \mathrm{H} : u = v \vdash P\{x \mapsto u\}}$$

inversion H.

$$\frac{\text{see § on equality}}{\ldots,\ \mathrm{H} : u = v \vdash ...}$$

elim H.

$$\frac{\ldots, \vdash P}{\ldots,\ \mathrm{H} : \neg P \vdash ...}$$

assert P.

$$\frac{\ldots \vdash P \quad \ldots,\ \mathrm{H} : P \vdash ...}{\ldots \vdash ...}$$

Figure 1: Description of basic tactics as inference rules.