Cryptographic Protocols
Formal and Computational Proofs
Introduction

David Baelde

LSV, ENS Paris-Saclay

2019

(based on slides by B. Blanchet & S. Delaune)

Cryptographic protocols

Paiement Internet

Cryptographic protocols

Communicating processes that use cryptographic primitives to meet security properties in a hostile environment.

Security properties (1)

- Secrecy: May an intruder learn some secret message?
- Authentication: Is the agent Alice really talking to Bob?
- Non-repudiation: Alice sends a message to Bob. Alice cannot later deny having sent this message. Bob cannot deny having received the message.
- Forward security: Past communications remain secure if long-term keys are compromised.
- Post-compromise security: Future communications are secure soon after long-term keys are compromised.

Security properties: E-voting (2)

Eligibility: only legitimate voters can vote

Fairness: no early results can be obtained which could influence the remaining voters

Individual verifiability:
a voter can verify that her vote was really counted

Universal verifiability:
anyone can verify that the outcome really is the sum of all the votes

Belgique - Election 2004 - http://www.poureva.be/ - (C) Kanar

Security properties: E-voting (3)

Privacy: the fact that somebody voted in a particular way is not revealed to anyone

Receipt-freeness: a voter cannot prove that she voted in a certain way (this is important to protect voters from coercion)

Coercion-resistance: same as receipt-freeness, but the coercer interacts with the voter during the protocol, e.g. by preparing messages

Cryptographic primitives

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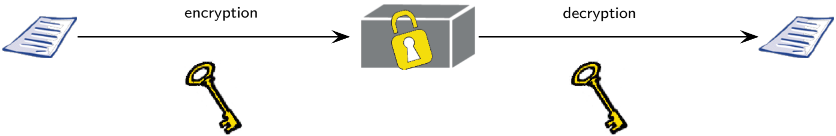
Symmetric encryption



$\longrightarrow$ Examples: Caesar encryption, DES, AES, ...

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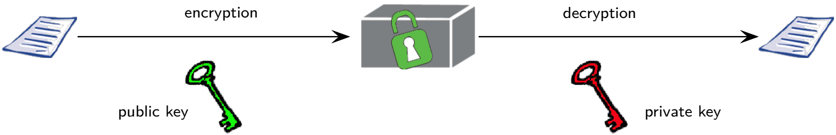
Asymmetric encryption



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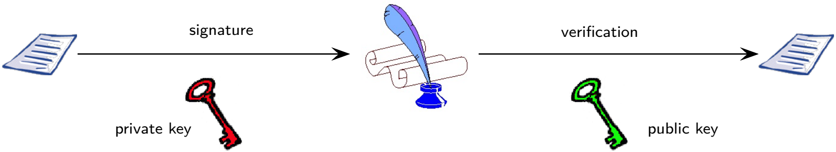
Signature



Credit Card Payment Protocol

Example: credit card payment

- The client Cl puts his credit card C in the terminal T.
- The merchant enters the amount M of the sale.

- The terminal authenticates the credit card.
- The client enters his PIN.
  If $M \geq 100$€, then in 20% of cases,
  - The terminal contacts the bank B.
  - The bank gives its authorisation.

More details

The Bank B , the Client Cl, the Credit Card C and the Terminal T

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- a private signature key – $\mathrm{priv}(B)$
- a public key to verify a signature – $\mathrm{pub}(B)$
- a secret key shared with the credit card – K_{CB}

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Credit Card

- some *Data*: name of the cardholder, expiry date ...
- a signature of the *Data* – $\{hash(Data)\}_{\mathrm{priv}(B)}$
- a secret key shared with the bank – K_{CB}

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Terminal

- the public key of the bank – $\mathrm{pub}(B)$

Payment protocol

The terminal T checks the authenticity of the credit card C:

$$1. \quad C \rightarrow T : \quad Data, \{hash(Data)\}_{\mathsf{priv}(B)}$$

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The terminal T asks the code:

$$\begin{array}{lccll} 2. & T & \rightarrow & Cl : & PIN? \\ 3. & Cl & \rightarrow & C : & 1234 \\ 4. & C & \rightarrow & T : & ok \end{array}$$

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The terminal T requests authorisation from the bank B:

$$\begin{array}{lcccl} 5. & T & \rightarrow & B: & auth? \\ 6. & B & \rightarrow & T: & 4528965874123 \\ 7. & T & \rightarrow & C: & 4528965874123 \\ 8. & C & \rightarrow & T: & \{4528965874123\}_{K_{CB}} \\ 9. & T & \rightarrow & B: & \{4528965874123\}_{K_{CB}} \\ 10. & B & \rightarrow & T: & ok \end{array}$$

Attack against credit cards

Initially, security was guaranteed by:

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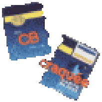
However, there are attacks!

- cryptographic attack: 320-bit keys are no longer secure,
- logical attack: no link between PIN and authentication,
- hardware attack: replication of cards.



→ "YesCard" made by Serge Humpich in 1997

The "YesCard": how does it work?

Logical attack

$$
\begin{array}{llll}
1. C & \rightarrow T & : \mathsf{Data}, \{\mathsf{hash}(\mathsf{Data})\}_{\mathsf{priv}(B)} \\
2. T & \rightarrow CI & : PIN? \\
3. CI & \rightarrow C & : 1234 \\
4. C & \rightarrow T & : ok
\end{array}
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Logical attack

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1. C & \rightarrow T & : \text{Data}, \{\text{hash}(\text{Data})\}_{\text{priv}(B)} \\
2. T & \rightarrow CI & : PIN? \\
3. CI & \rightarrow C' & : 2345 \\
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$$

The "YesCard": how does it work?

Logical attack

$$
\begin{aligned}
1.\,C &\quad \rightarrow T &&: \text{Data}, \{\text{hash}(\text{Data})\}_{\text{priv}(B)}\\
2.\,T &\quad \rightarrow CI &&: PIN?\\
3.\,CI &\quad \rightarrow C' &&: 2345\\
4.\,C' &\quad \rightarrow T &&: ok
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$$

$\rightarrow$ copy card data and certificate on card that accepts all PINs

The "YesCard": how does it work?

Logical attack

$$\begin{array}{llll} 1.C & \rightarrow T & : \text{Data}, \{\text{hash}(\text{Data})\}_{\text{priv}(B)} \\ 2.T & \rightarrow CI & : PIN? \\ 3.CI & \rightarrow C' & : 2345 \\ 4.C' & \rightarrow T & : ok \end{array}$$

$\rightarrow$ copy card data and certificate on card that accepts all PINs

$$\begin{array}{llll} 1.C' & \rightarrow T & : \text{XXX}, \{\text{hash}(\text{XXX})\}_{\text{priv}(B)} \\ 2.T & \rightarrow CI & : PIN? \\ 3.CI & \rightarrow C' & : 0000 \\ 4.C' & \rightarrow T & : ok \end{array}$$

Credit cards: going further

Preventing the YesCard attack

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Flaw fixed with the more recent Dynamic Data Authentication mode.
Each card has signature key pair (whose public key is authentified) used to sign Data together with a random value (challenge).

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New "attacks": contact-less cards

The same protocol is used through wireless communications:

$\rightsquigarrow$ Card data easily harvested!

Included card holder name and record of purchase in France until 2013.

Conclusion: security not the main goal of credit card consortium.

Formal Methods

The problem

In summary:

- Protocol design is error prone.
  Hard to clearly define threats and security properties.
- Flaws undetected by testing appear in presence of adversary.
- Errors can have serious consequences.

$\rightsquigarrow$ Formal methods and formal proofs are needed!

Active and successful research for several decades.

Models of protocols: communication assumptions

Attacker capabilities

- The attacker can intercept all messages sent on the network.
- He can compute messages using crypto primitives.
- He can send messages on the network.

A worst case scenario where

the attacker is (perhaps maliciously) executing the protocol.

Models of protocols: the computational model

The computational model has been developed in the early 80'
by Goldwasser, Micali, Rivest, Yao, and others.

- Messages are bitstrings.
- Cryptographic primitives are computations on bitstrings.
- The attacker is any probabilistic (polynomial-time) Turing machine.
- Cryptographic assumptions on primitives: examples for encryption, hash functions, signatures.

Realistic but (until recently) does not lend itself to fully formal proofs.

Models of protocols: the formal model

The formal model, aka. symbolic or "Dolev-Yao model"
is due to Needham and Schroeder [1978] and Dolev and Yao [1983].

- Cryptographic primitives are blackboxes.
- Messages are terms on these primitives.
  $\hookrightarrow$ $\{m\}_k$ encryption of the message m with key k,
  $\hookrightarrow$ $\langle m_1, m_2 \rangle$ pairing of messages m_1 and m_2, ...
- The attacker is restricted to compute only using these primitives, according to some equations.
  $\hookrightarrow$ $\mathrm{dec}(\{m\}_k, k) = m$
  $\hookrightarrow$ $\mathrm{fst}(\langle x, y \rangle) = x$
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Lends itself well to automatic proofs: AVISPA, ProVerif, ...
Delicate hypothesis: the only equalities are those given by the equations.

Relationship between the two models

An attack in the formal model is an attack in the computational model.
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Computational soundness (under very specific assumptions)

Proof in the formal model $\Rightarrow$ proof in the computational model

Approach pioneered by Abadi&Rogaway [2000]; many works since then.

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The computational model is still a model:

beware side channel attacks and implementation flaws!

Verifying protocols in the formal model

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- Bounded number of sessions but unbounded messages
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- Unbounded messages and number of sessions $\Rightarrow$ undecidable
  - Interactive theorem proving: Isabelle [Paulson, JCS'98]
  - Approximations:
    - abstract interpretation [Monniaux'03], TA4SP integrated in AVISPA
    - typing [Abadi'99], [Gordon & Jeffrey '02]
  - Semi-decision procedures (and approximations): Proverif

Proofs in the computational model

Proof by sequence of game reductions [Shoup, Bellare & Rogaway].
Games correspond to security property of protocol or primitive,
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Example: IND-CPA

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- Allow attacker to (polynomially) perform encryptions.
- Attacker chooses m_0, m_1.
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Automation

CryptoVerif, Certicrypt, F$\star$...

State of the art

A growing list of successes:

1996 Needham-Schroeder public-key protocol attack, 17 years later

2008 Google's non-compliant SSO implem. attack (AVISPA)

2010 Freak: trick SSL implems to choose export crypto (MiTLS)

2016 Proofs of TLS (MiTLS, Tamarin)

2016 Proof of BAC e-passport protocol (UKano/Proverif)

Some industrial acceptation:

- Involvement of academic community in the design of TLS 1.3
- Some collaborations around 5G protocol design.
- Mozilla has incorporated certified crypto implementations into its library.

Conclusion

Difficulties: a big picture

- Communicating agents: concurrent, bounded or not
- Crypto primitives: symbolic vs. computational, algebraic properties
- Hostile environment: passive vs. active, deduction, equivalence

Upcoming lectures

- Verification in the symbolic model (D. Baelde, 8 sessions)
- Mechanizing computational security proofs (B. Blanchet, 4 sessions)
- Verification of protocol implementations (K. Bhargavan, 4 sessions)

Conclusion

Upcoming lectures

- Verification in the symbolic model (D. Baelde, 8 sessions)
  - Computational and symbolic models (1/8)
  - Verification in the symbolic model (5/8)
    - The deduction problem
    - Symbolic model-checking
    - Indistinguishability
    - Proverif: the unbounded case, correspondences, diff-equiv.
  - Computationally complete symbolic attacker (1/8)
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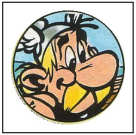
Exercises

Needham-Schroeder's Protocol (1978)



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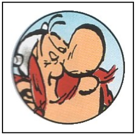

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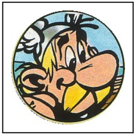

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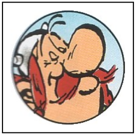

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Questions

- Is N_b secret between A and B ?
- When B receives $\{N_b\}_{\mathrm{pub}(B)}$, does it really come from A ?

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Attack

An attack was found 17 years after its publication! [Lowe 96]

Example: Man in the middle attack

Agent A

Intruder I

Agent B

Attack

- involving 2 sessions in parallel,
- an honest agent has to initiate a session with I.

$$\begin{array}{lcl} A \to B & : & \{A, N_a\}_{\mathrm{pub}(B)} \\ B \to A & : & \{N_a, N_b\}_{\mathrm{pub}(A)} \\ A \to B & : & \{N_b\}_{\mathrm{pub}(B)} \end{array}$$

Example: Man in the middle attack

Agent A

$$\xrightarrow{\{A, N_a\}_{\mathsf{pub}(I)}}$$

Intruder I

$$\xrightarrow{\{A, N_a\}_{\mathsf{pub}(B)}}$$

Agent B

$$
\begin{array}{lcl}
\mathsf{A} \rightarrow \mathsf{B} & : & \{A, N_a\}_{\mathsf{pub}(B)} \\
\mathsf{B} \rightarrow \mathsf{A} & : & \{N_a, N_b\}_{\mathsf{pub}(A)} \\
\mathsf{A} \rightarrow \mathsf{B} & : & \{N_b\}_{\mathsf{pub}(B)}
\end{array}
$$

Example: Man in the middle attack

$\{A, N_a\}_{\mathrm{pub}(I)} \longrightarrow$

$\longleftarrow \{N_a, N_b\}_{\mathrm{pub}(A)}$

$\{A, N_a\}_{\mathrm{pub}(B)} \longrightarrow$

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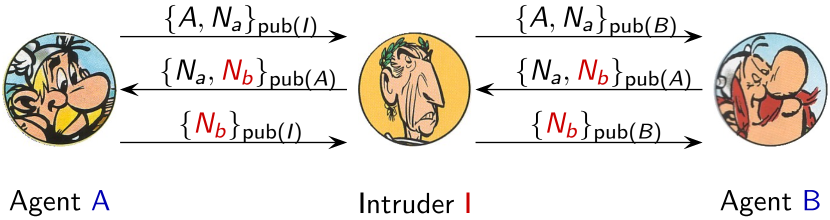

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$\{A, N_a\}_{\mathrm{pub}(B)} \longrightarrow$

$\longleftarrow \{N_a, N_b\}_{\mathrm{pub}(A)}$

$\{N_b\}_{\mathrm{pub}(B)} \longrightarrow$

Agent A — Intruder I — Agent B

Attack

- The intruder knows N_b.
- When B finishes his session (apparently with A), A has never talked with B.

$$
\begin{array}{lcl}
A \rightarrow B & : & \{A, N_a\}_{\mathrm{pub}(B)} \\
B \rightarrow A & : & \{N_a, N_b\}_{\mathrm{pub}(A)} \\
A \rightarrow B & : & \{N_b\}_{\mathrm{pub}(B)}
\end{array}
$$

Exercise: Needham-Schroder

$$
\begin{array}{lcl}
\mathrm{A} \rightarrow \mathrm{B} & : & \{A, N_a\}_{\mathrm{pub}(B)} \\
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\mathrm{A} \rightarrow \mathrm{B} & : & \{N_b\}_{\mathrm{pub}(B)}
\end{array}
$$

Question

Propose a fix to ensure mutual authentication.

Exercise: LAK

RFID tags must be authenticated by a reader.
Each tag owns a secret key k, shared with the reader.

$$
\begin{array}{lcl}
R & \rightarrow & T & : & n_R \\
T & \rightarrow & R & : & n_T, h(n_R \oplus n_T \oplus k) \\
R & \rightarrow & T & : & h(h(n_R \oplus n_T \oplus k) \oplus k \oplus n_R)
\end{array}
$$

Questions

- Show that the reader does not properly authenticate the tag. It is enough to consider a scenario with one tag and one reader, with the same shared secret k.
- Propose a fix.

Exercise: Private Authentication

A and B each have a secret key sk_X and a public key pk_X.

$$\begin{array}{ccccc} A & \rightarrow & B & : & \{n_A, pk_A\}_{pk_B} \\ B & \rightarrow & A & : & \{n_A, n_B, pk_B\}_{pk_A} \end{array}$$

Question

Can an attacker distinguish between:

- a role B willing to talk to A, and
- a role B willing to talk to A' ?