# Model Checking a Path

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Verify that a specification holds along all paths of a model

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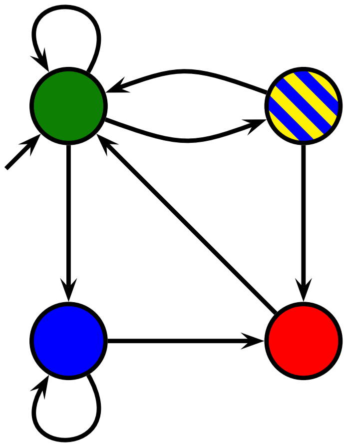

$$LTL \ni \varphi ::= \neg\varphi \mid \varphi \vee \varphi \mid X\,\varphi \mid \varphi\, U\, \varphi \mid \quad \mid \ldots$$

Abbreviations:

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$$G\varphi = \neg F \neg \varphi$$

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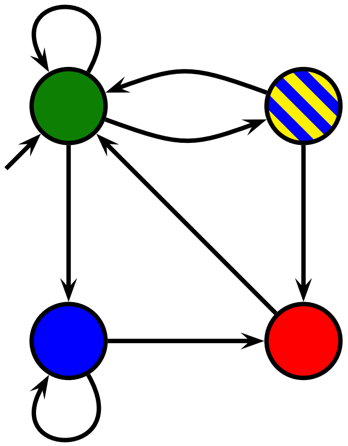

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Example:

$G(● \vee (● \, U \, ●))$

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Several interesting occurences:

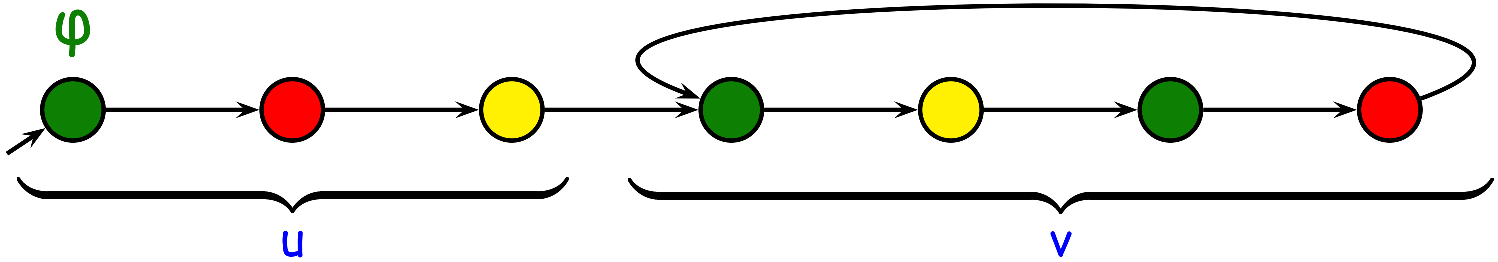
- Testing;
- Model checking probabilistic systems;
- On-the-fly and runtime model checking;
- Analysis of counter-examples;
- Analysis of log files;
- ...

# The PMC problem

**PMC:**

Input: two finite words $u$ and $v$, and a formula $\varphi$.

Output: yes iff $u \cdot v^\omega \models \varphi$.



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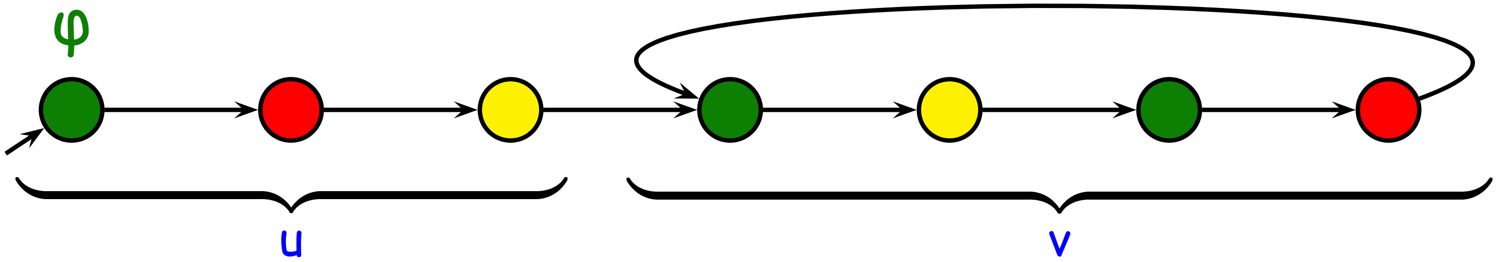


Note: v can be empty, i.e. we deal with finite paths as well as with infinite paths.

# Our results

- PMC is easy: much easier than standard MC
    - can efficiently handle more expressive logics,
    - can efficiently handle compressed paths.
- Ultimately periodic paths are not more difficult than finite paths,
- Polynomial time algorithms by using dynamic programming techniques.

# Outline of the talk

- LTL Path Model Checking
- PMC for richer logics:
  - LTL + Past, LTL + Past + Now
  - LTL + Chop
  - first order logic
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PTIME

LOG(CFL)

NLOGSPACE

LOGSPACE

$NC^1$ = ALOGTIME

$AC^0$

Large (?) gap between upper and lower bound.

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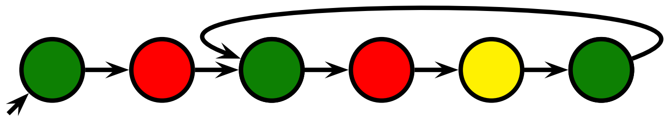
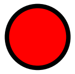


G F 

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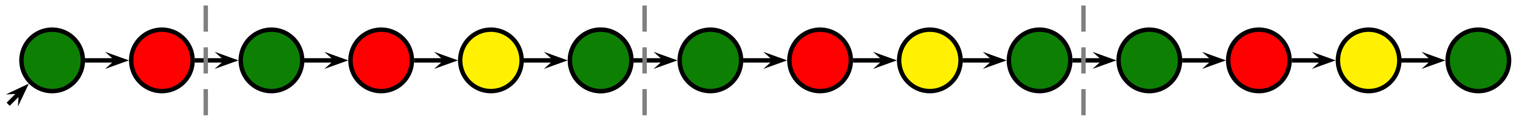
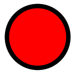


G F 

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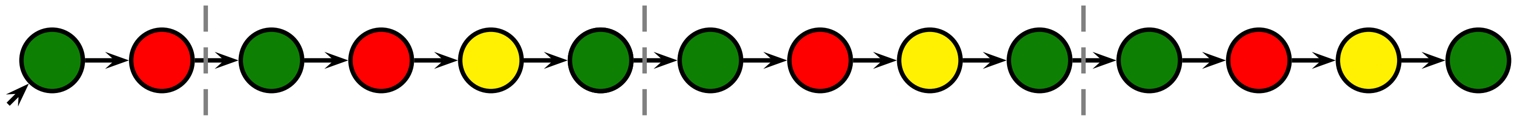

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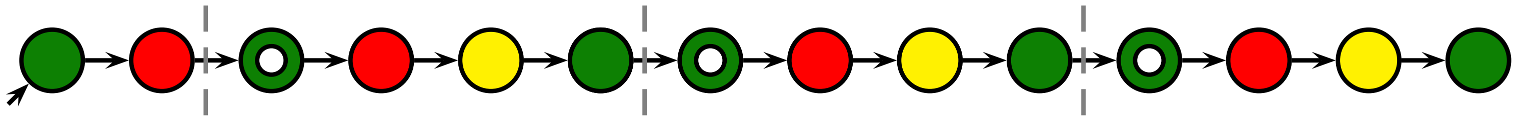

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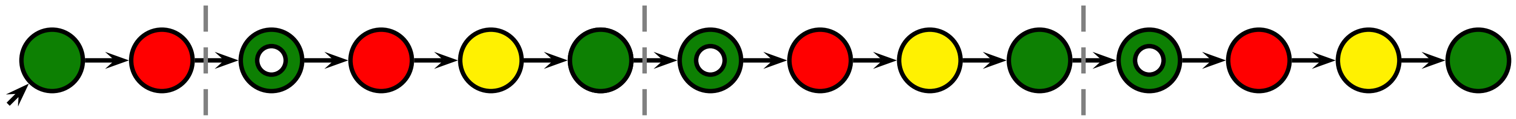

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For example:

$$\overline{G\,F\,\bullet} = G\,(F\,(\bullet \wedge \gamma_1) \vee \neg\gamma_2)$$

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# Complexity of PMC

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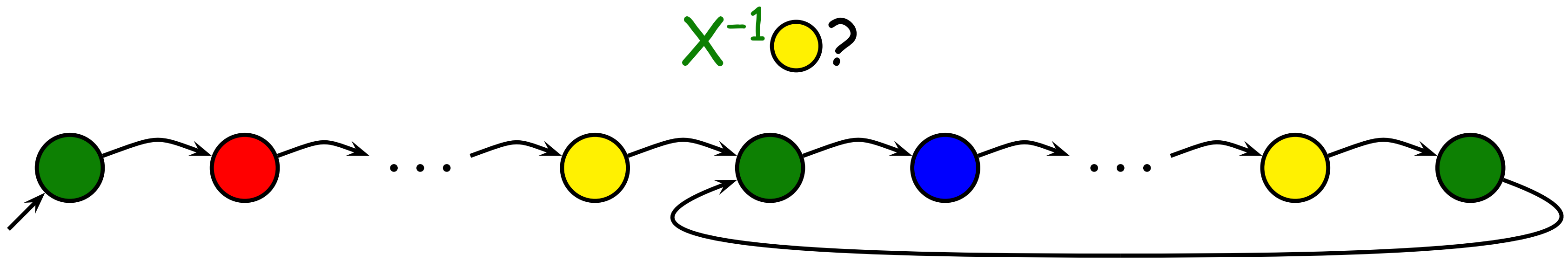


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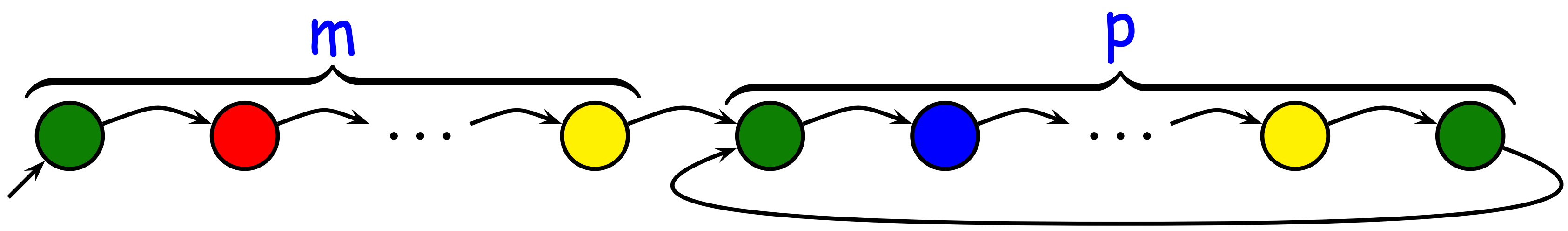


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Lemma: For any loop $(u, v)$ with $|u| = m$ and $|v| = p$, for any LTL+Past formula $\varphi$ with at most $h$ nested past-time modalities, and for any $k \geq m + p \cdot h$, we have

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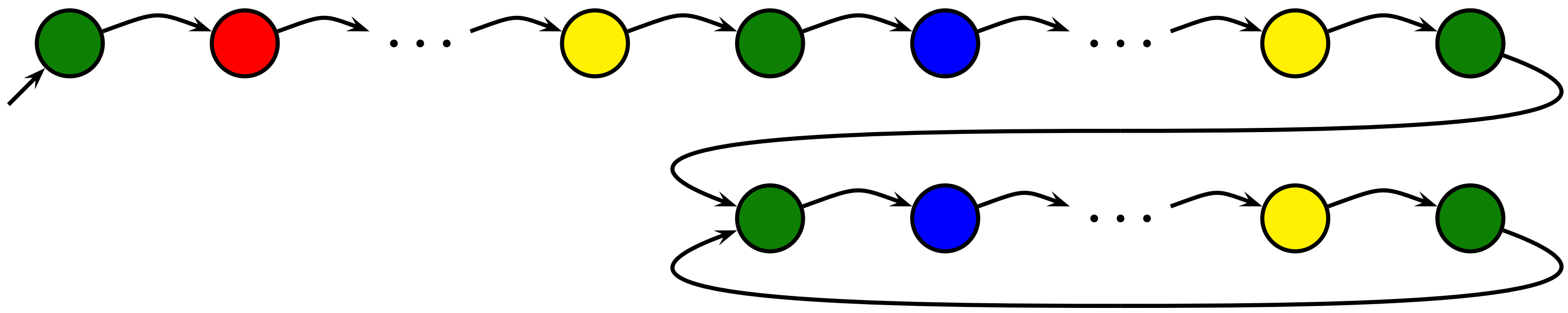

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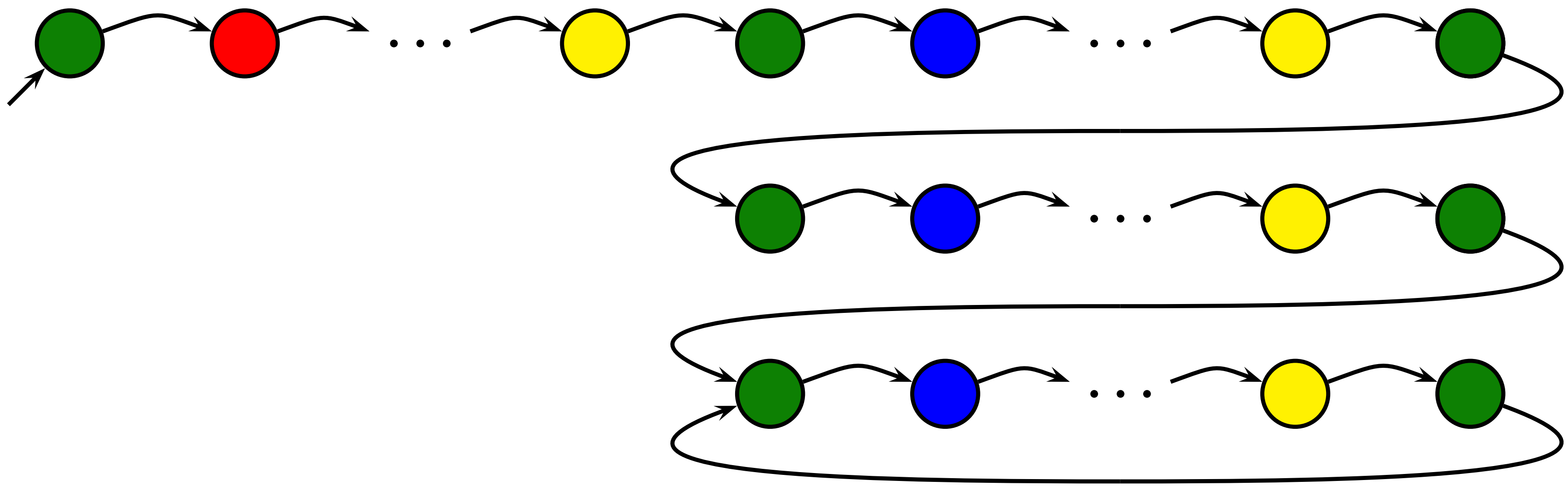

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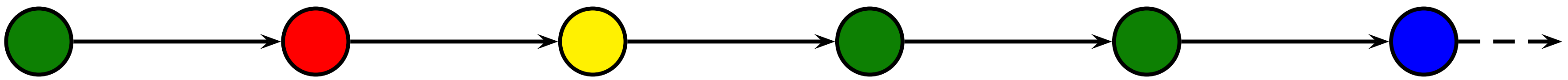

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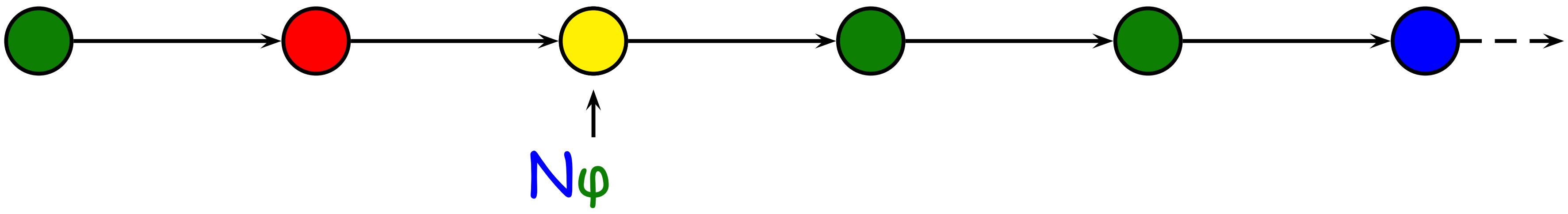

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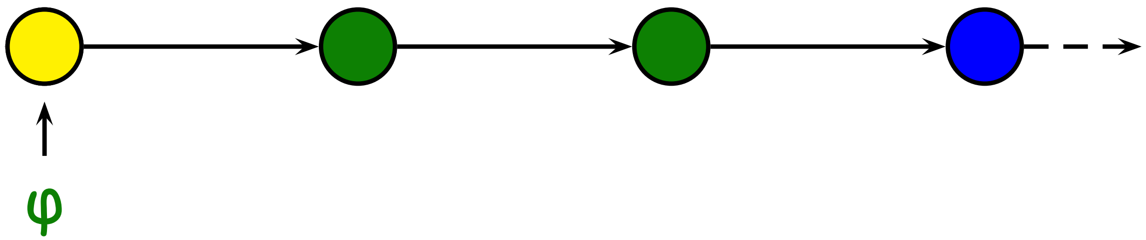

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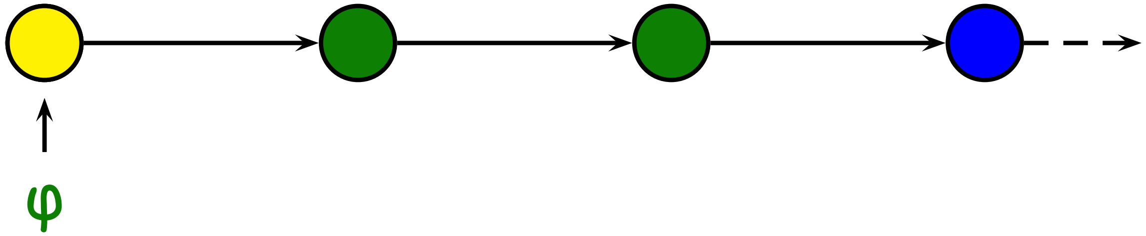


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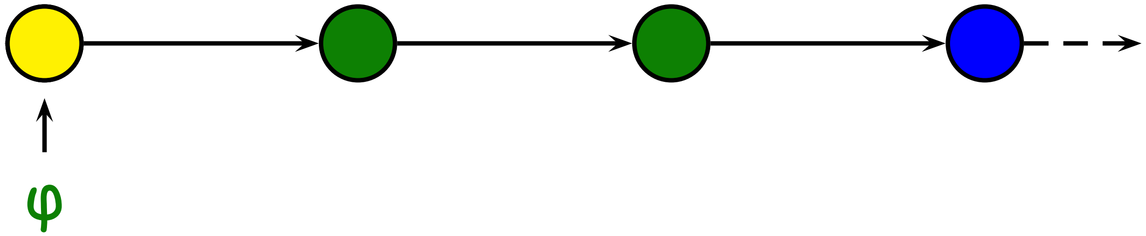


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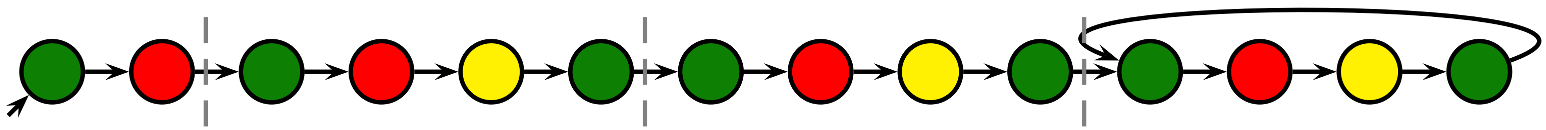

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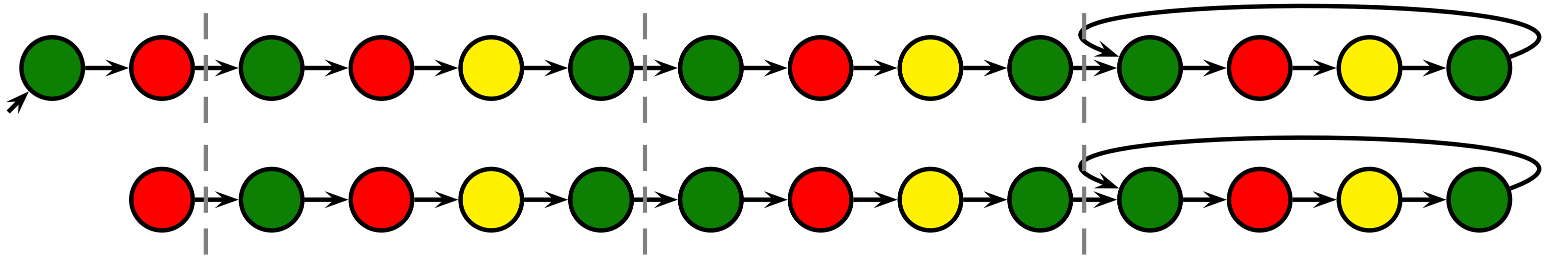

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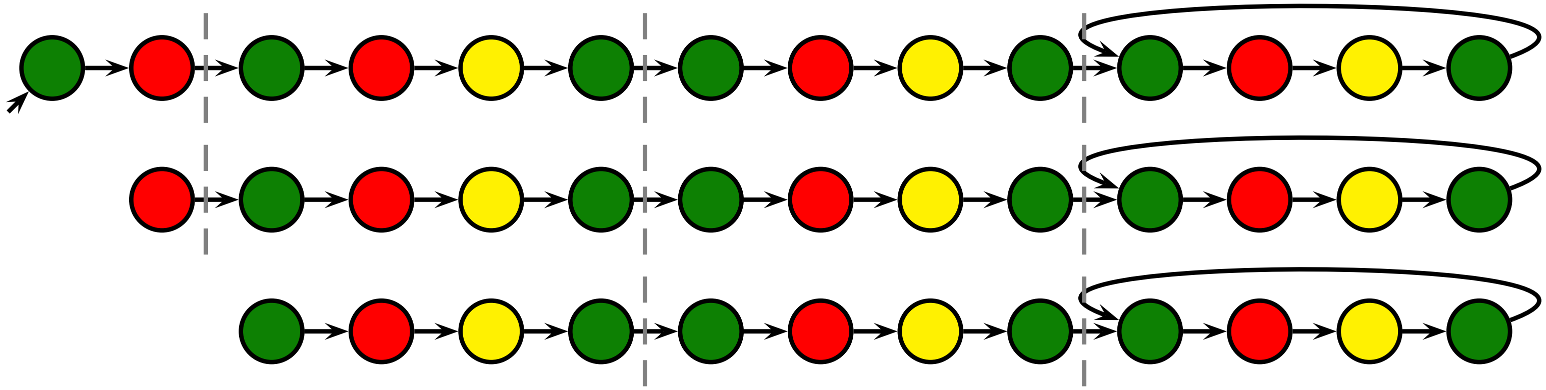

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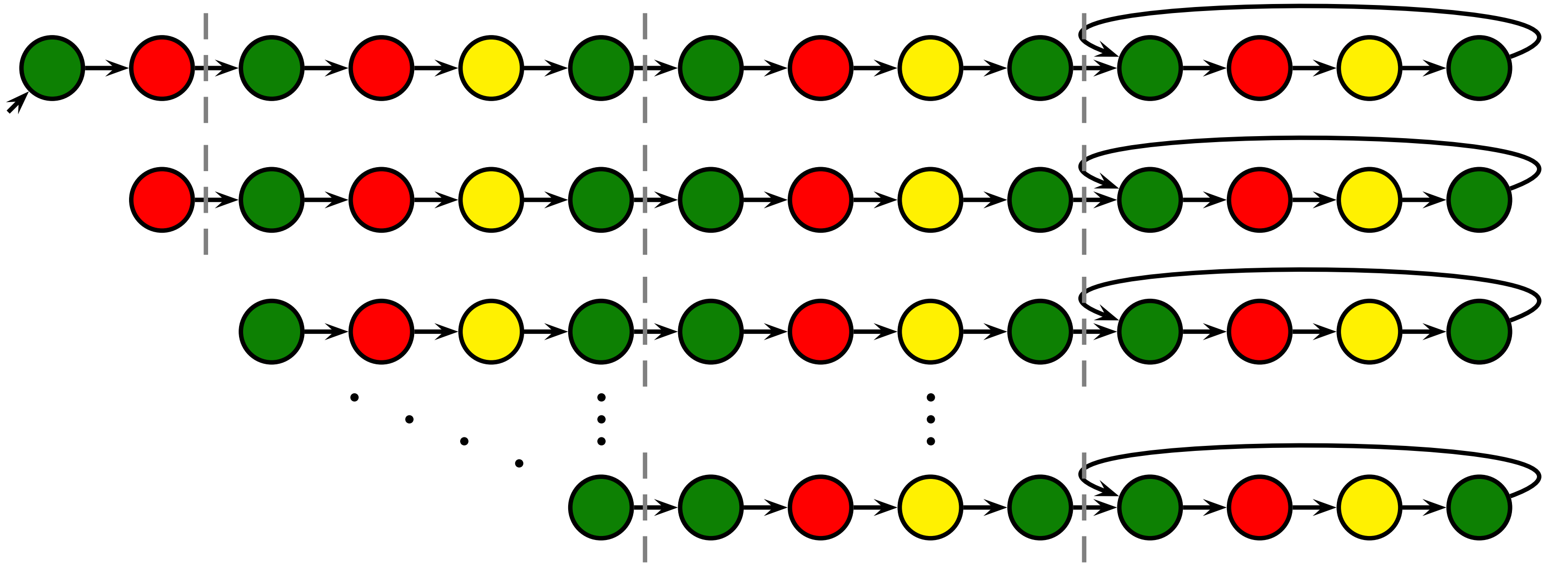

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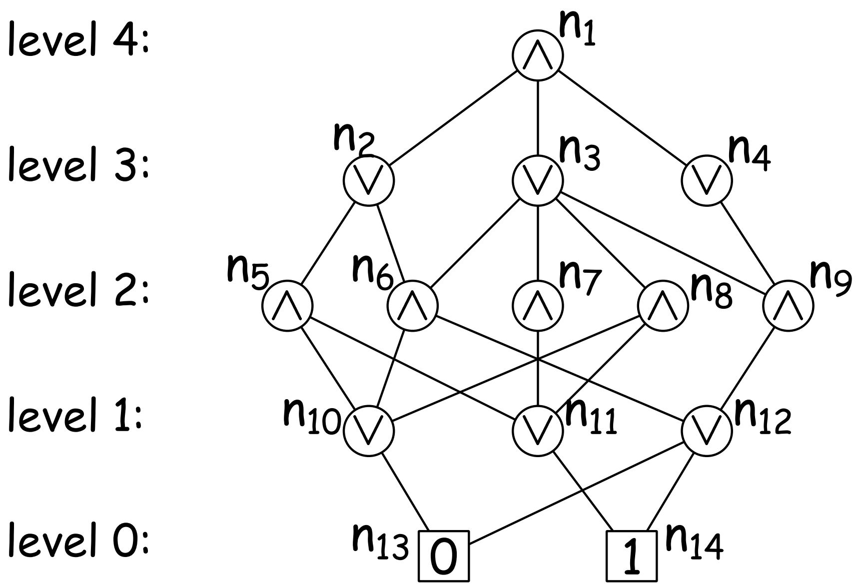
Reduction of Circuit-Value:



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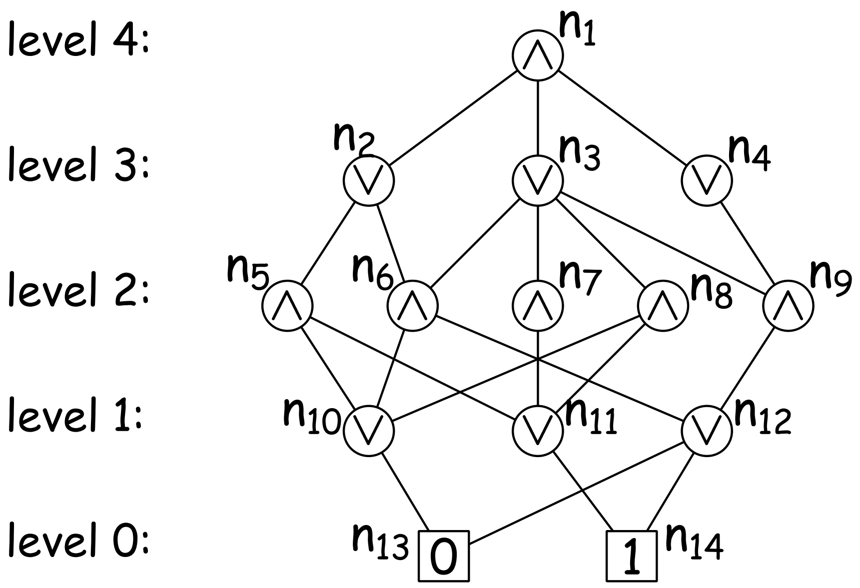


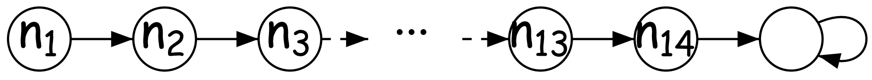

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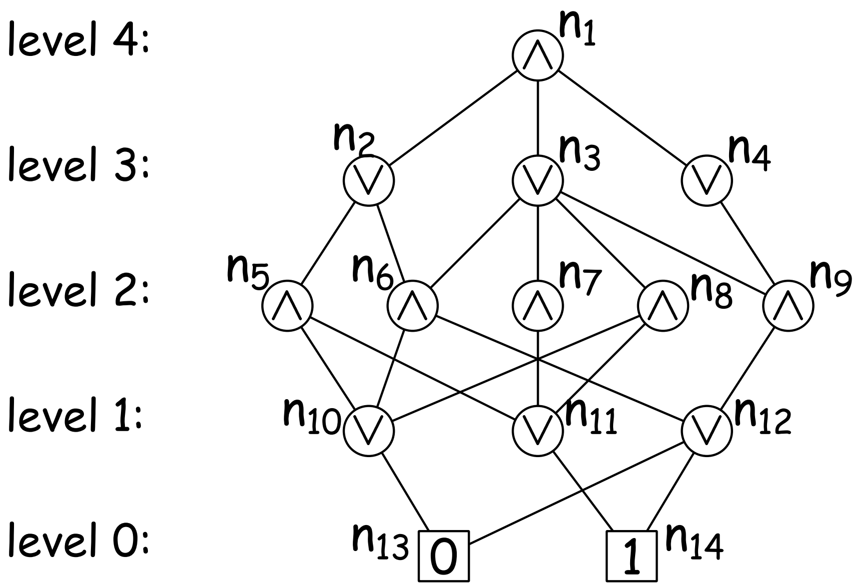


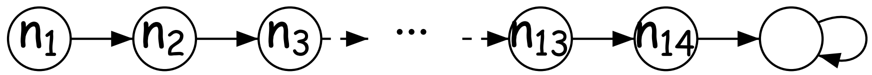


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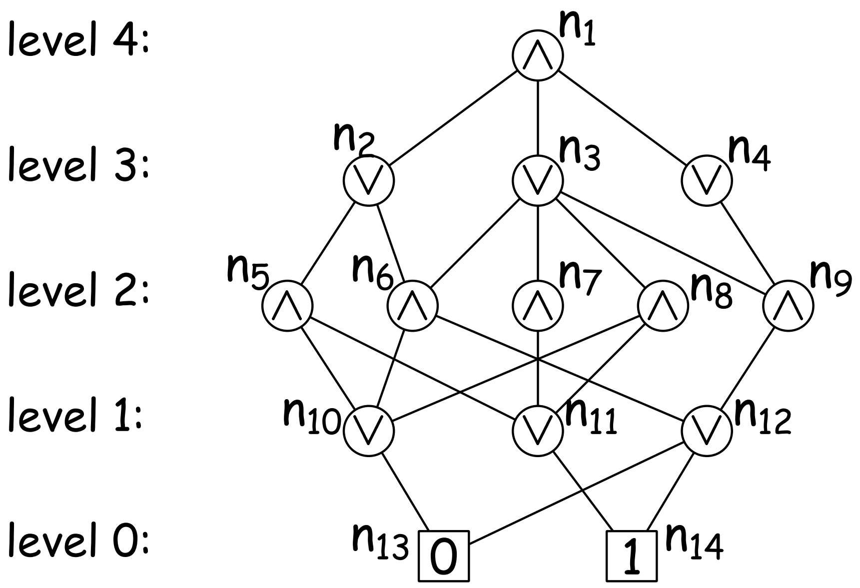


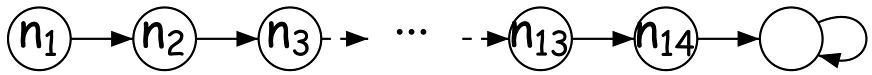


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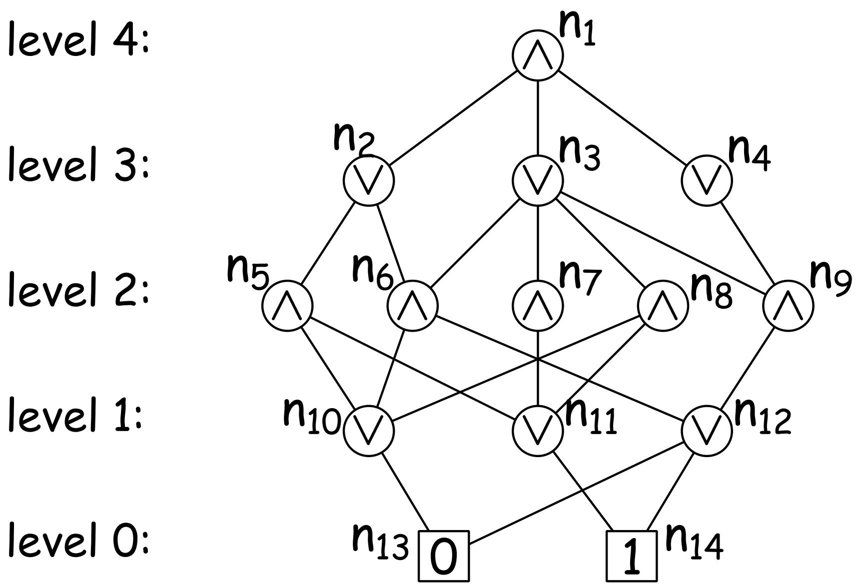


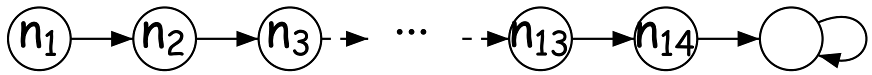


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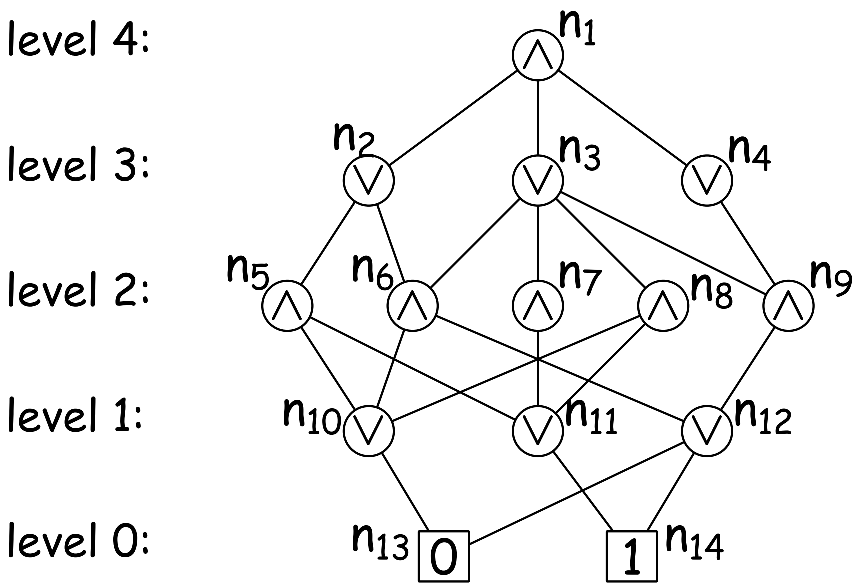


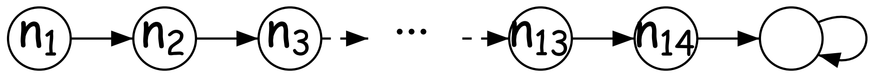


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where $\varphi_{next} = \bigvee_{(n,n')\in E} (n' \wedge F^{-1}G^{-1}n)$

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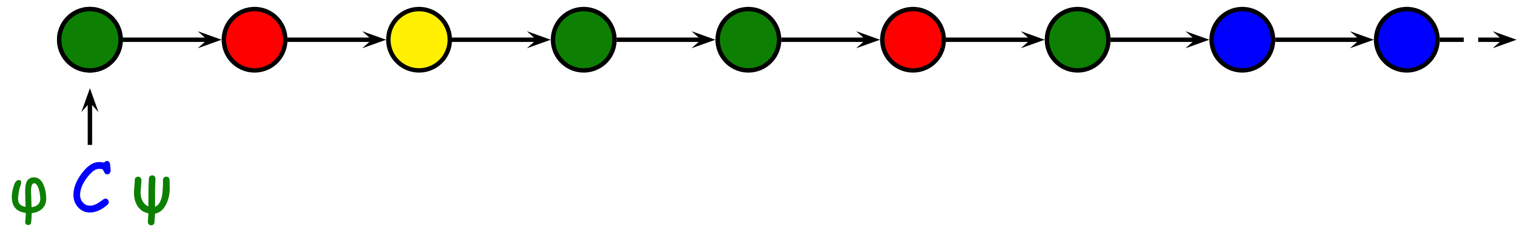

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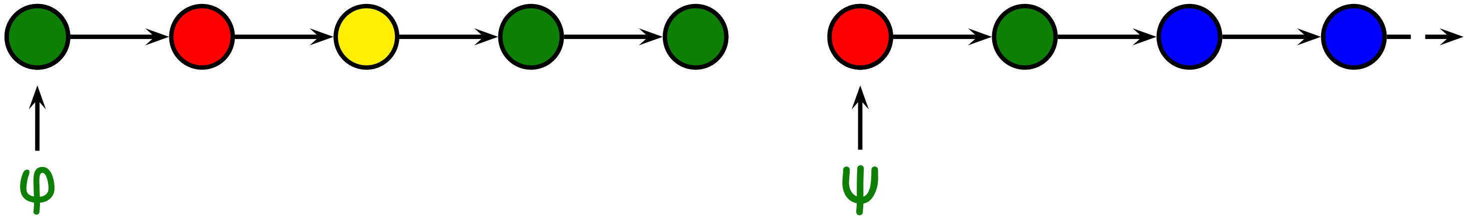

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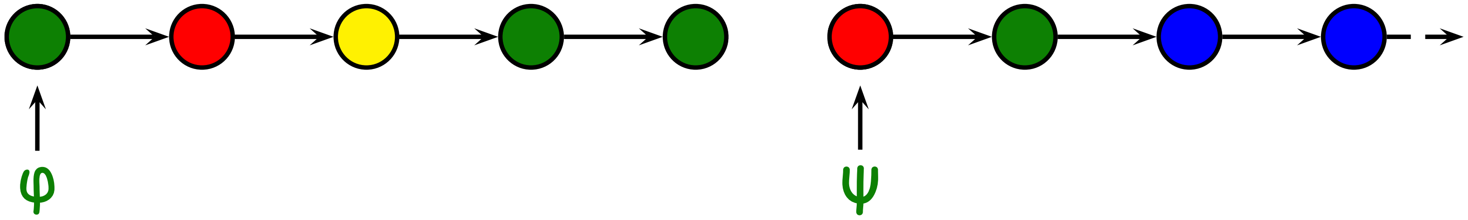


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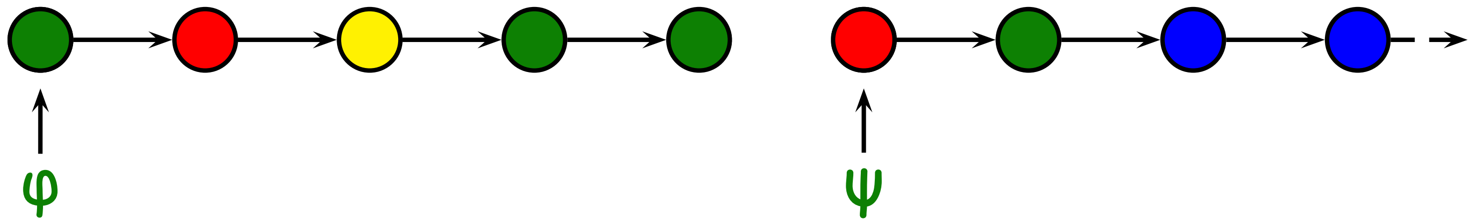


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# PMC for First Order Logic

Model checking FOMLO properties is nonelementary ([Sto74])

Theorem:
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Theorem:
Path Model Checking FOMLO is PSPACE-complete.

- PSPACE-hardness: reduction from QBF.
- PSPACE-easiness:
  - a finite (exponential) prefix of the path is sufficient (but, as for LTL, we have to modify the formula)
  - store values of variables in binary.

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# Compressed Paths

Useful in analysis of log files, DNA strings, ...

Exponential compression:

$$[(a \cdot b^{22})^5 \cdot c \cdot a^{32}]^{96} \cdot a \cdot [(b^5 \cdot c)^{34} \cdot a]^{19}$$

Possible compression methods:

- Lempel-Ziv (LZ):
  - reuse subwords of what has already been compressed;
  - implemented in **gzip**,
- Straight-Line Programs (SLP): special context-free grammars;
- ...

# Straight Line Programs

example of a SLP:

$$P = \begin{cases} N_1 \rightarrow N_2 \cdot N_3 \\ N_2 \rightarrow N_3 \cdot N_4 \\ N_3 \rightarrow N_4 \cdot N_5 \\ N_4 \rightarrow N_5 \cdot N_6 \end{cases} \quad \begin{array}{l} N_5 \rightarrow a \cdot N_6 \\ N_6 \rightarrow N_7 \cdot b \\ N_7 \rightarrow c \cdot b \end{array}$$

This SLP encodes the following word:

$$w(P) = acbbcbbacbbacbbcbbacbbcbbacbb$$

# SLP and Finite State Automata

Theorem:
Given a SLP P and a finite state automaton $\mathcal{A}$, deciding whether w(P) is accepted by $\mathcal{A}$ is PTIME-complete.

- easiness [PR99]: fill a boolean table T[r,s,i] s.t. T[r,s,i] is true iff there is a path from state r to state s labeled with $w(N_i)$.

$$N_i \to N_j \cdot N_k \;\Rightarrow\; T[r,s,i] = \bigvee_u (T[r,u,j] \wedge T[u,s,k])$$

- hardness: encode NAND-CIRCUIT VALUE, which is PTIME-complete.

# Compressed Path Model Checking

| | PMC for LTL | Accept. by FSA |
|---|---|---|
| uncompressed | PTIME-easy | NL-complete |
| compressed | PSPACE-complete | PTIME-complete |

PSPACE-hardness is “due to” the formula:

compressed PMC can be achieved in time $O(|P| \times 2^{O(|\varphi|)})$.

# Conclusion

- "new" problem: Path Model Checking;
- efficient algorithms for rich logics:
  - PTIME-complete for LTL+Past+Now and LTL+Chop;
  - PSPACE-complete for First Order Logic,
- Efficient LTL PMC on compressed paths.