Symbolic Model Checking for Simply Timed Systems

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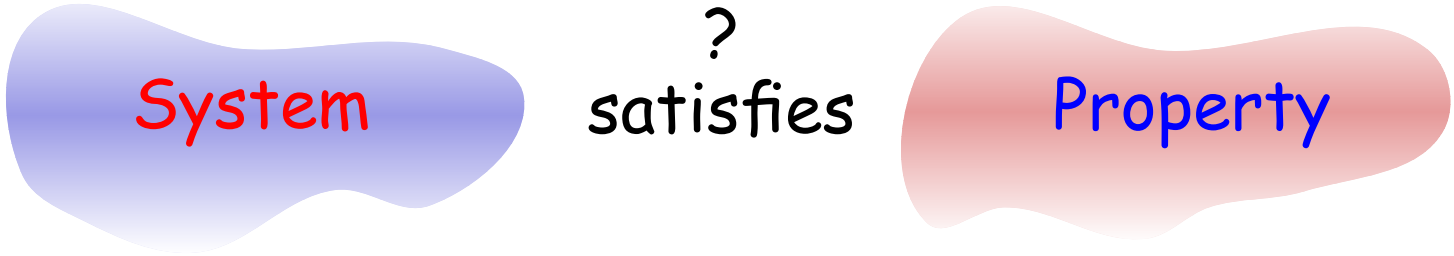
Model Checking



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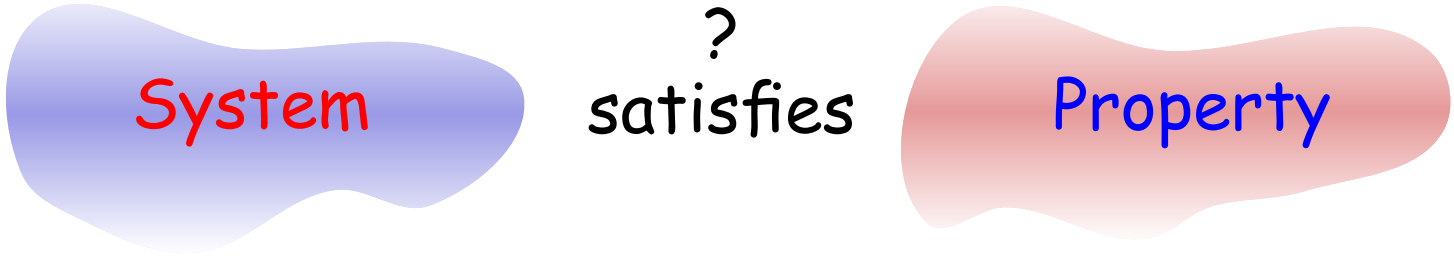

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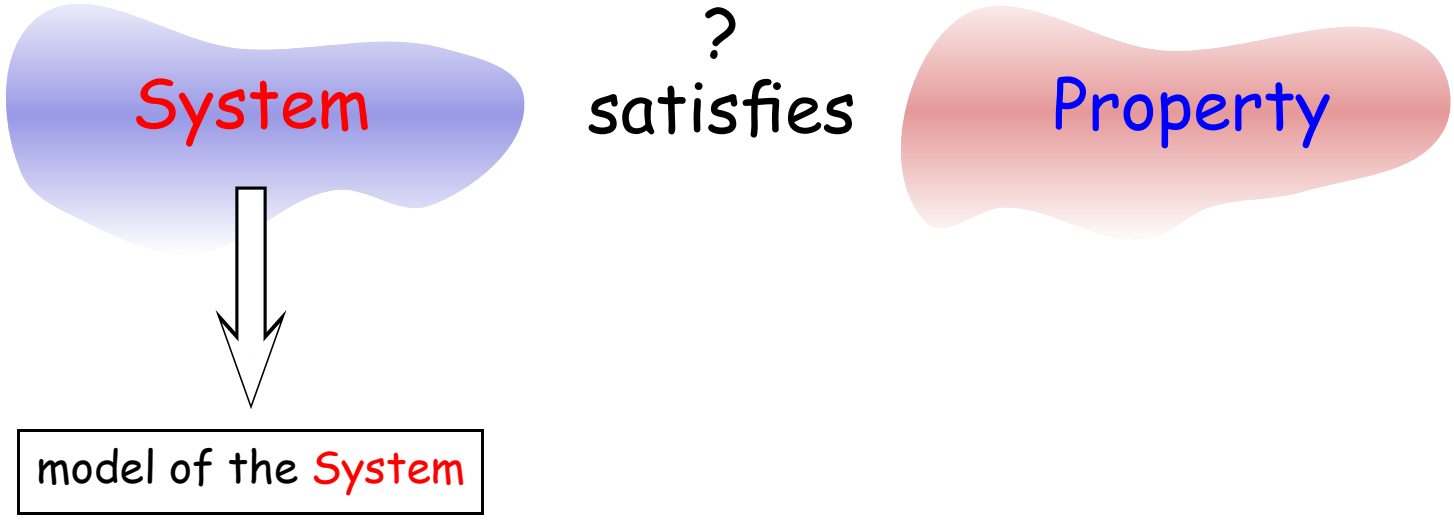

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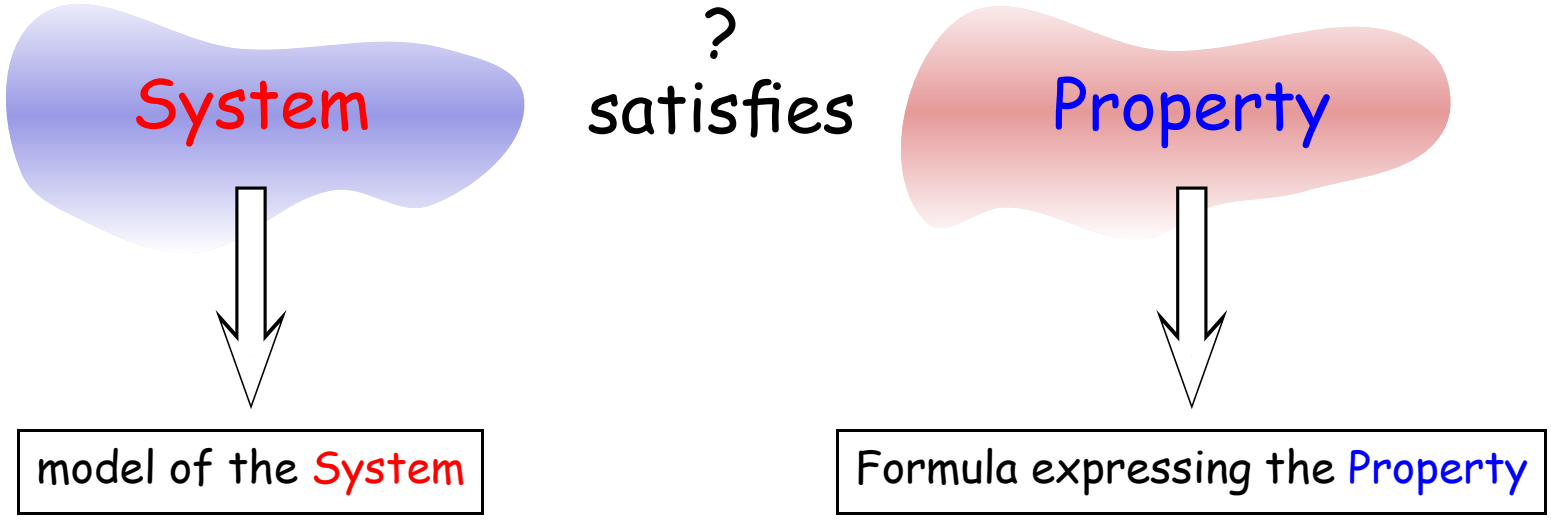

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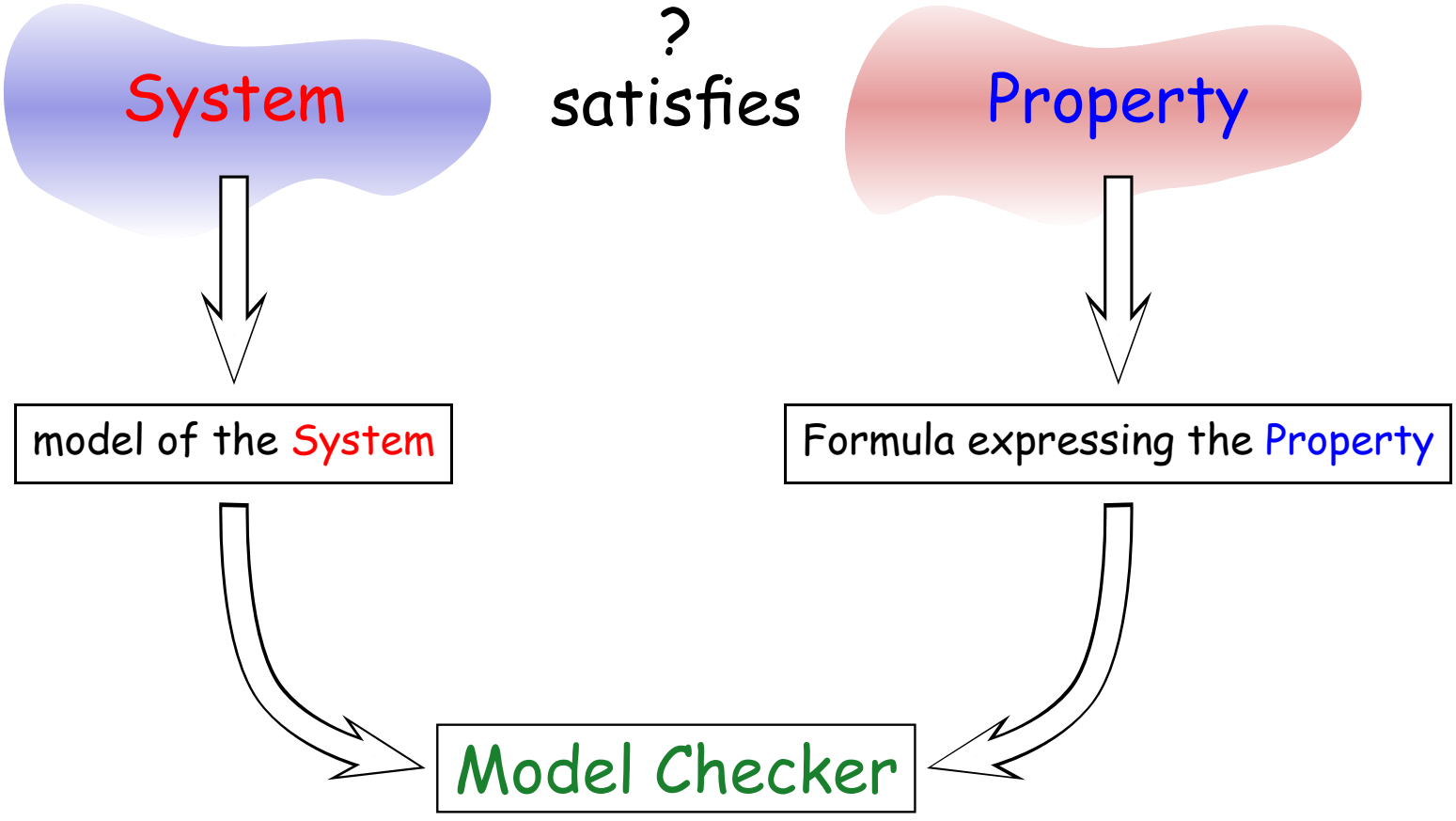

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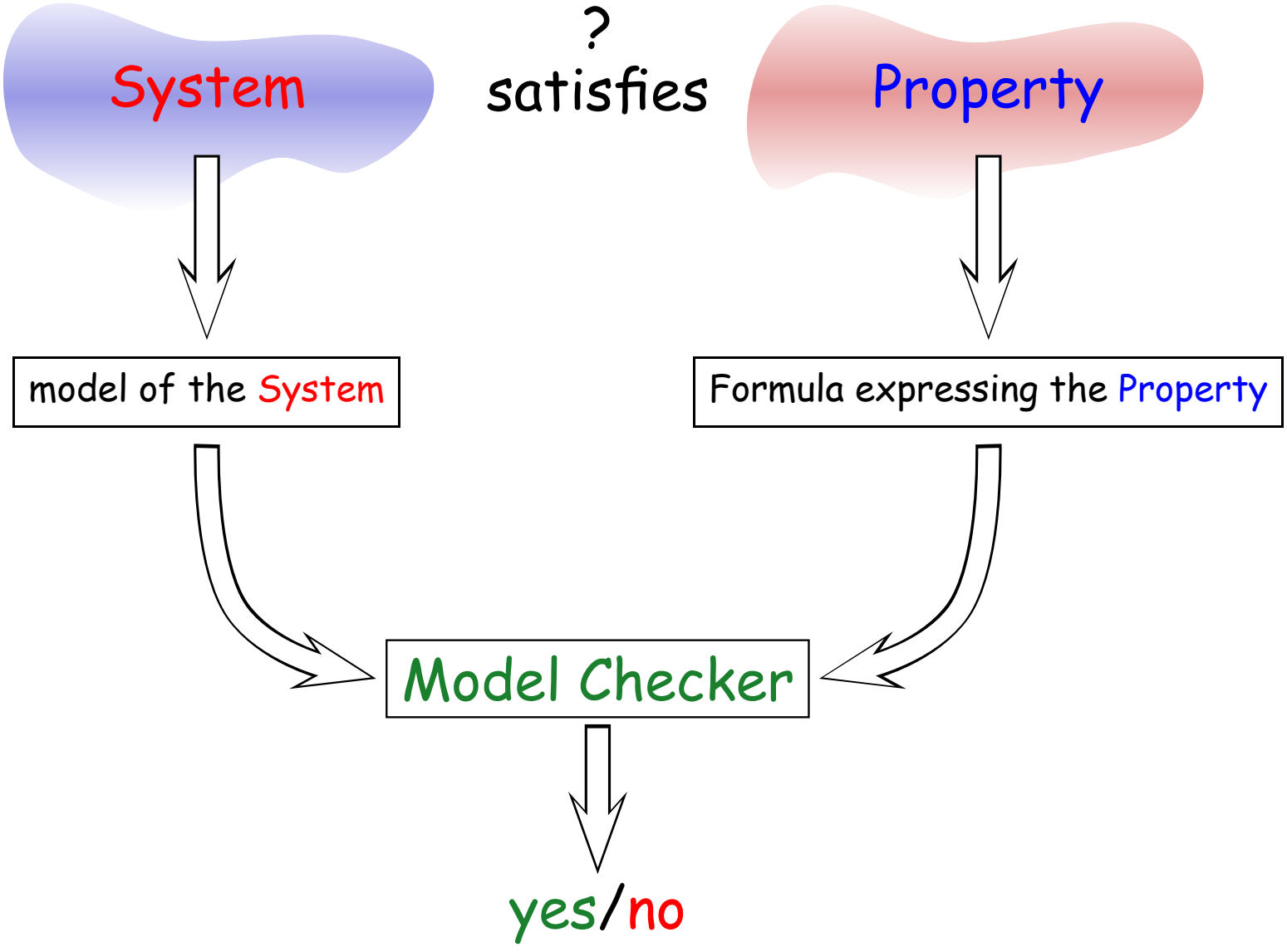

Timed Model Checking

We must sometimes take time into account:

$$AG\ (\text{problem} \rightarrow EF_{\leq 15}\ \text{alarm})$$

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Timed Model Checking

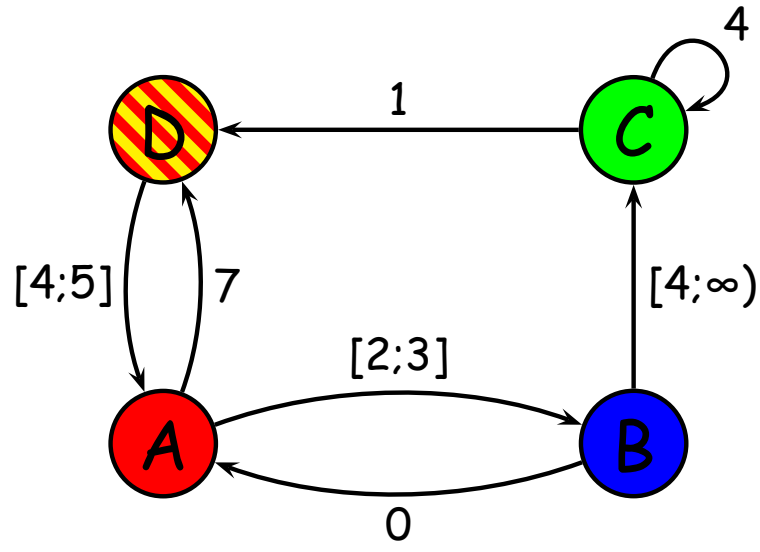
We must sometimes take time into account:

$$AG\,(\text{problem} \rightarrow EF_{\leq 15}\,\text{alarm})$$

- Timed automata: Expressive but expensive
- Kripke structures: One time unit per transition.
  $\rightarrow$ more efficient algorithms, but larger models
- Timed Kripke structures: An interesting alternative...

Timed Kripke Structures

Timed Kripke Structure = Kripke Structure with integer durations on transitions:



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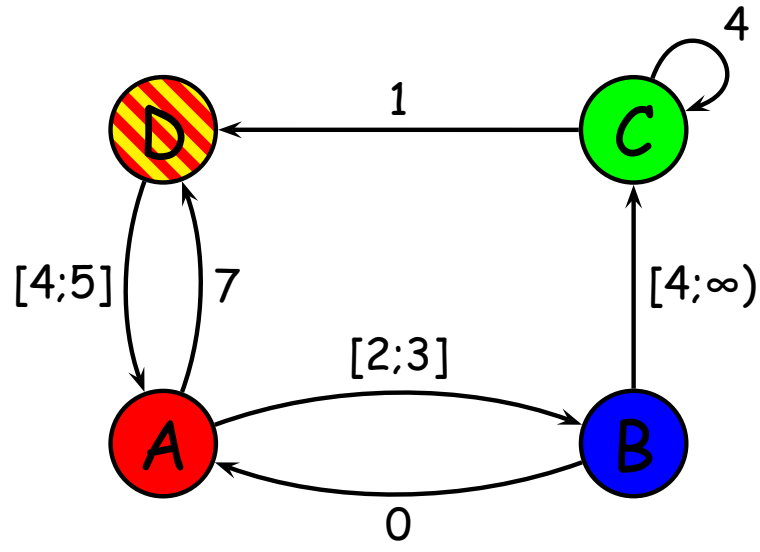

Semantics: instantaneous steps, no intermediate state.

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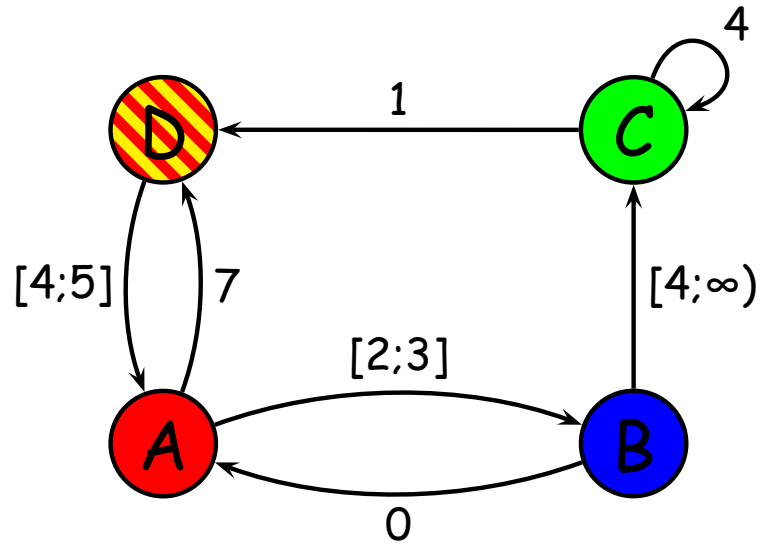

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$$A \models E\,(\text{true}\ U_{=11}\ \text{red})$$

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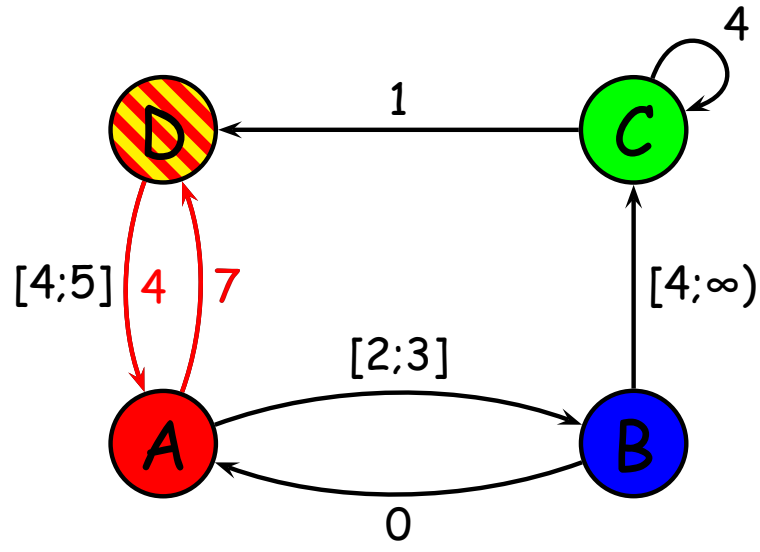

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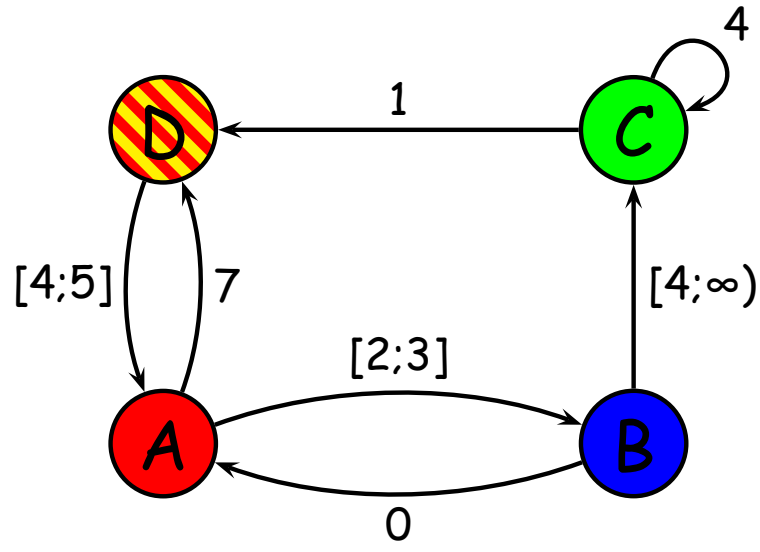

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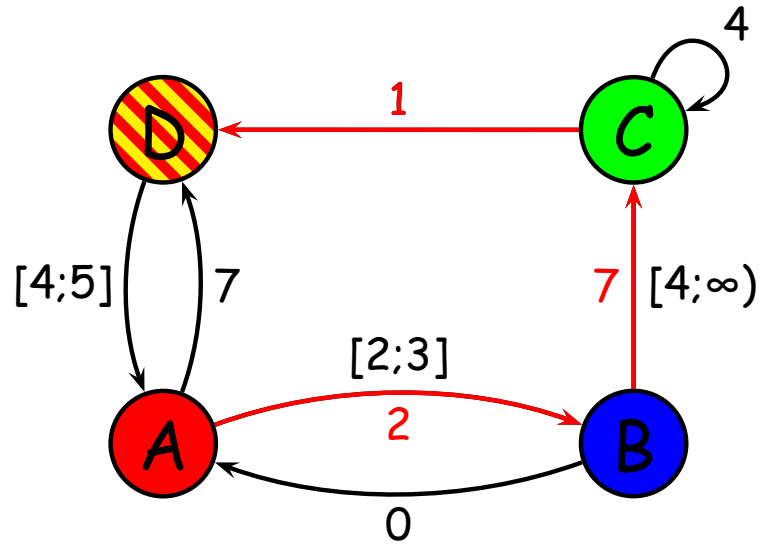

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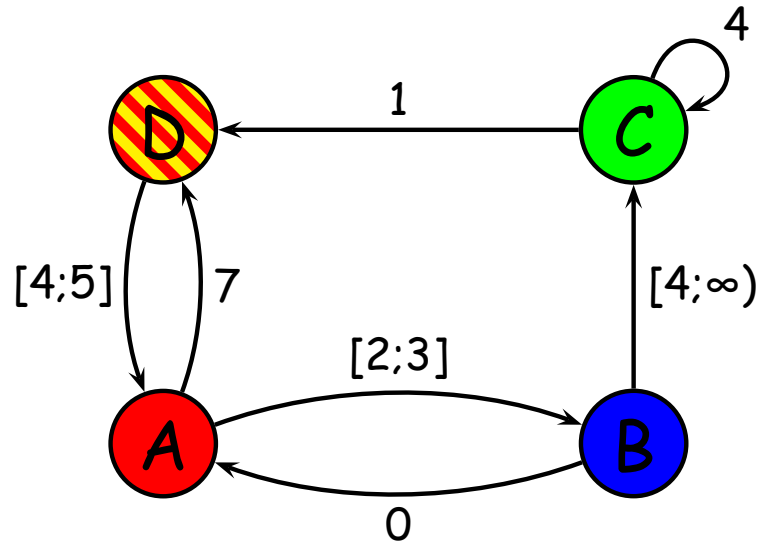

Semantics: instantaneous steps, no intermediate state.

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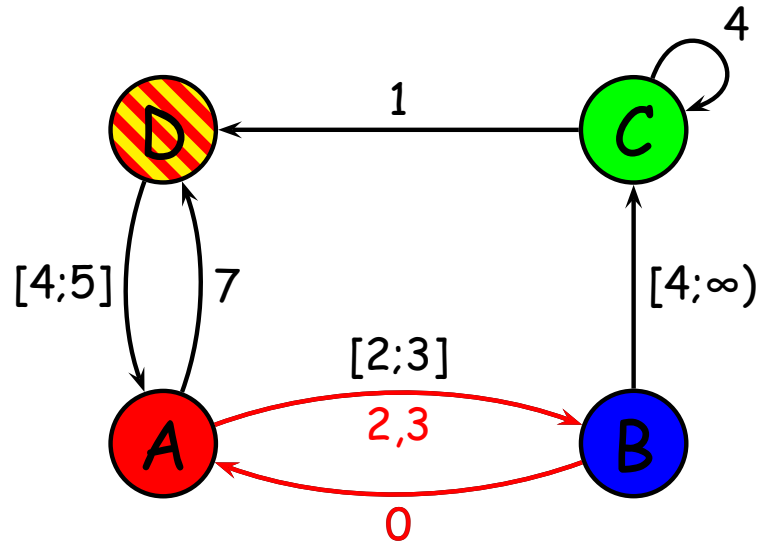

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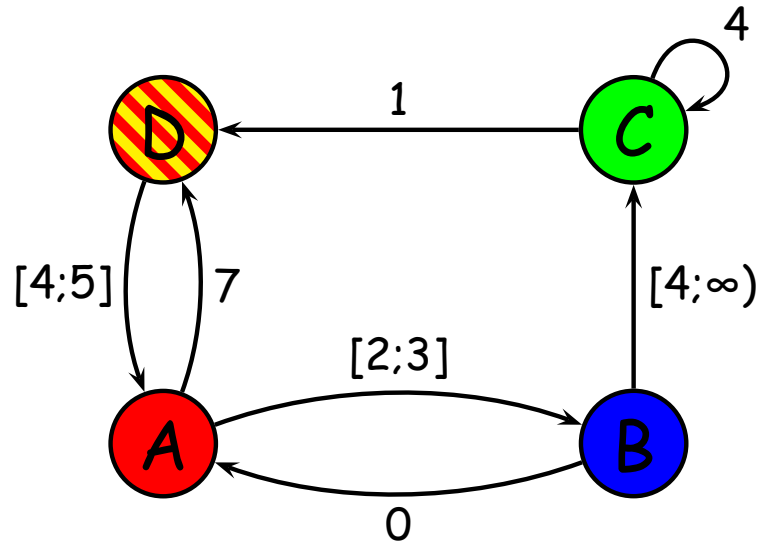

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$$A \not\models EF_{=5}\ (red \wedge blue)$$

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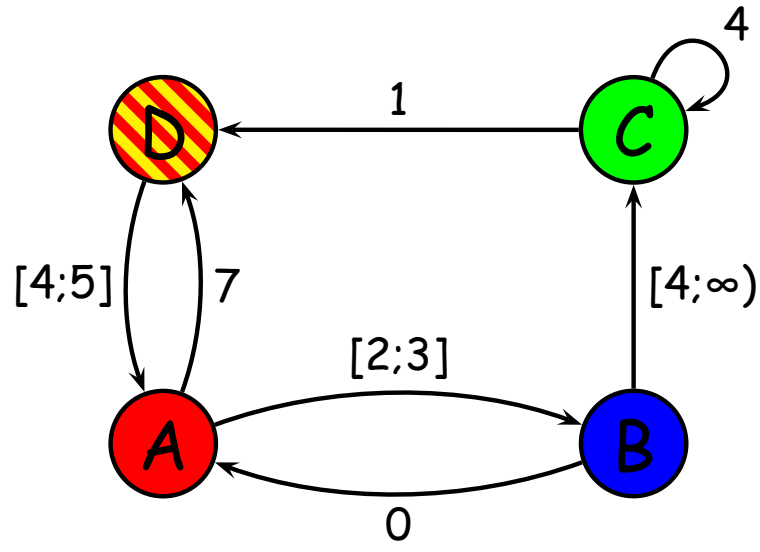

Semantics: instantaneous steps, no intermediate state.

$$A \not\models AG\,(EF_{9..10}\ \text{green})$$

Complexity of TCTL over TKS

	KS, TKS 0/1 [EMSS90], [CC01]	TKS [LMS02]
CTL	P-complete	P-complete
TCTL $_{\leq,\geq}$	P-complete	P-complete
TCTL	P-complete	Δ_2^P-complete

$$P \subseteq NP \subseteq \Delta_2^P \stackrel{def}{=} P^{NP} \subseteq PSPACE$$

Outline

- Symbolic algorithms for TCTL model checking
  - for $EU_{=0}$ and $AU_{=0}$
  - for $EU_{[a..b]}$ and $AU_{[a..b]}$
  - special algorithms for $TCTL_{\leq,\geq}$
  - computing minimum and maximum delays
- Implementation on top of NuSMV

Algorithm for $EU_{=0}$ and $AU_{=0}$

For $E[\varphi\ U_{=0}\ \psi]$:

either ψ holds, or φ holds and some successor satisfies $E[\varphi\ U\ \psi]$ in the sub-TKS containing only transitions whose duration is zero.

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For $EG_{=0}\ \varphi$:

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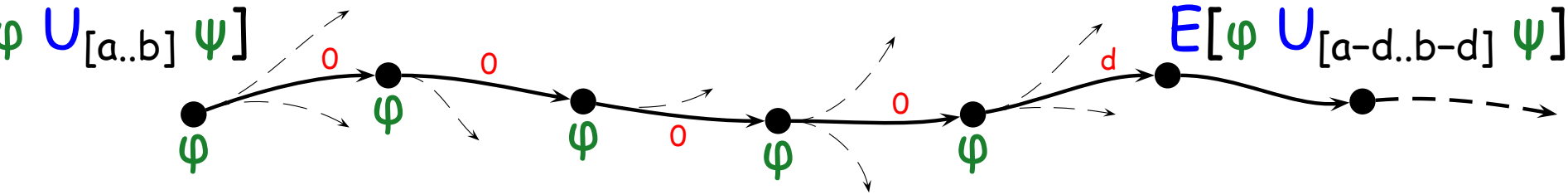
For $EG_{=0}\ \varphi$:

either there is a path where φ always holds, or there is a path verifying φ until a non-zero transition is eventually reached.

For $A[\varphi\ U_{=0}\ \psi]$:

both $A[\varphi\ U\ \psi]$ and $AF_{=0}\ \psi$ must hold.

Algorithm for $EU_{[a..b]}$



At each step i between 0 and b, we iteratively compute max_duration + 1 sets of states s.t.

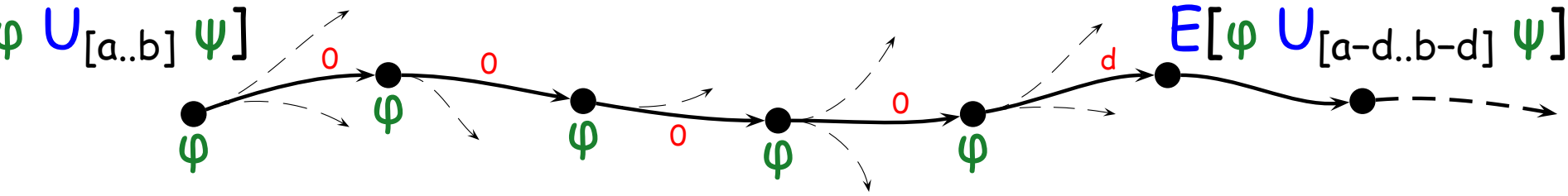
$Set_i[j] = E[(\varphi\ U_{[i+j-1+a-b..i+j-1]}\ \psi) \wedge$ first non-neg. duration $\geq j]$

Thus

$$Set_i[1] = E[(\varphi\ U_{[i+a-b..i]}\ \psi)$$

and

Algorithm for $EU_{[a..b]}$



At each step i between 0 and b, we iteratively compute $max_duration + 1$ sets of states s.t.

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First step:

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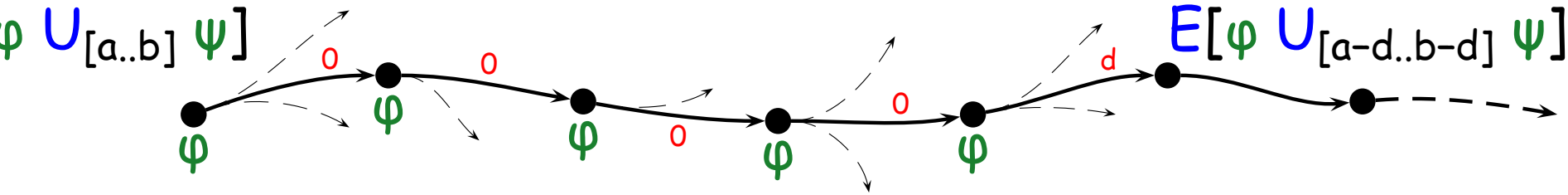
Algorithm for $EU_{[a..b]}$



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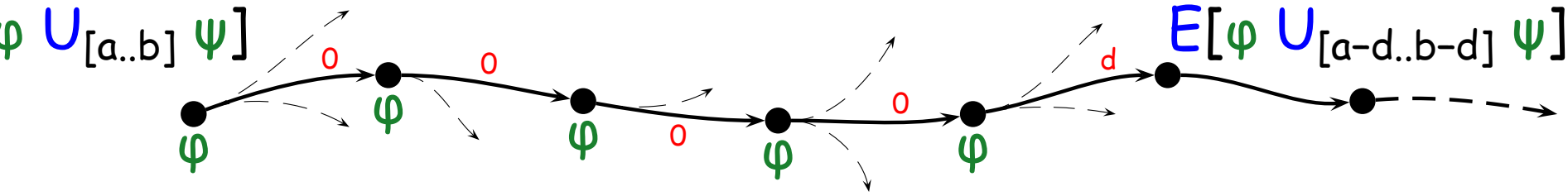
$= E[(\varphi\ U_{=0}\ \psi) \wedge$ first non-neg. duration $\geq j]$

Algorithm for $EU_{[a..b]}$



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Step i:

$$Set_i[j] = Set_{i-1}[j+1] \cup E[(\varphi\ U_{=0}(\varphi \wedge \xrightarrow{j} \wedge Set_{i-1}[1]))]$$

Algorithm for $EU_{[a..b]}$

```
if (b < a) { return(∅); }

/* Initialization */
for (j = 1; j ≤ max_duration; j++) {
    if (j ≤ b − a + 1) { Set[j] ← E[ φ U=0 (ψ ∧ EX EF=0 duration ≥ j) ] }
    if (j > b − a + 1) { Set[j] ← ⊥ }
}
Res ← E[ φ U=0 ψ ];

/* Loop */
for (i = 1; i ≤ b; i++) {
   for (j = 1; j ≤ max_duration; j++) {
      Set[j] ← Set[j + 1] ∪ E[ φ U=0 (φ ∧ EX (Res ∧ duration = j)) ]
   }
   Res ← Set[1];
}
if (i ≤ b − a) { Res ← Res ∪ ψ; }

/* End */
return(Res);
```

Algorithm for $EU_{\leq a}$

We could use $EU_{[0..a]}$, but it would be expensive: theoretically, $TCTL_{\leq,\geq}$ is easier than TCTL.

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We design specific algorithms for $TCTL_{\leq,\geq}$, using only the "minimal duration" transition relation:

$$q \xrightarrow{d} q' \quad \text{iff} \quad \begin{cases} q \xrightarrow{d} q' \\ \forall d'.\ (d' \geq d \ \vee \ q \not\xrightarrow{d'} q') \end{cases}$$

This relation can be computed efficiently with BDDs.

Algorithm for $EU_{\leq a}$

```
/* Initialization */
for (j = 1; j ≤ max_duration; j++) {
    Incr[j] ← ⊥;
}
Res ← E[ φ U=0 ψ ];
/* Loop */
for (i = 1; i ≤ a; i++) {
   for (j = 1; j ≤ max_duration; j++) {
      Incr[j] ← Incr[j + 1] ∪ E[ φ U=0 (φ ∧ EX→ (Res ∧ duration = j)) ]
   }
   Res ← Res ∪ Incr[1];
}
/* End */
return(Res);
```

$$Set_i[j] = E[(\varphi\ U_{[j..i+j-1]}\ \psi) \wedge \text{first duration} \geq j]$$

$$Res_i = E[\varphi\ U_{\leq i}\ \psi]$$

Algorithm for $EU_{\geq a}$

```
/* Initialization */
Res ← E[ φ U ψ ];
for (j = 1; j ≤ max_duration; j++) {
    Incr[j] ← E[ φ U=0 (φ ∧ EX⁻ (Res ∧ duration ≥ j)) ];
}
/* Loop */
for (i = 1; i ≤ a; i++) {
   for (j = 1; j ≤ max_duration; j++) {
      Incr[j] ← Incr[j + 1] ∪ E[ φ U=0 (φ ∧ EX⁻ (Res ∧ duration = j)) ]
   }
   Res ← Incr[1];
}
/* End */
return(Res);
```

$$Incr[j] = E[(\varphi\ U_{\geq i+j-1}\ \psi) \wedge \text{first duration} \geq j]$$

$$Res = E[\varphi\ U_{\geq i}\ \psi]$$

Minimum and maximum delays

In order to compute the minimum delay between a set S and a set T, we recursively compute $EF_{\leq a}T$ until it intersects with S or it reaches a fixpoint.

We compute maximum delays by computing $EG_{\leq a}\neg T$, until it does not intersect with S anymore, or until it reaches a fixpoint.

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What is NuSMV?

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SMV: "Symbolic Model Verifier".
Developped at CMU (Pittsburgh, USA)
by Ken McMillan et al.

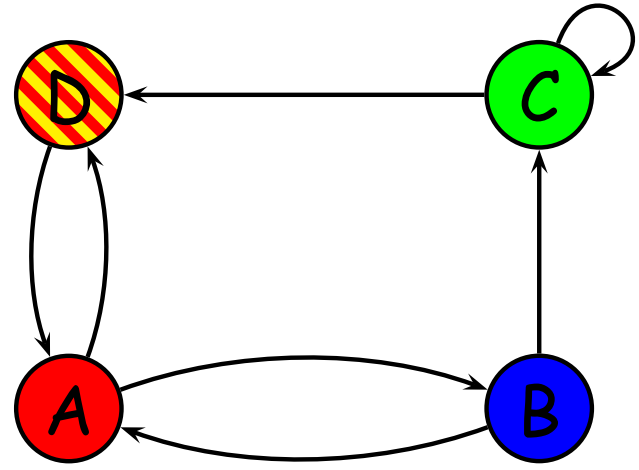
NuSMV: Open-source (LGPL) implementation of SMV
Developped at ITC-IRST (Trento, Italy)
by Alessandro Cimatti et al.

What is NuSMV?

(Nu)SMV = symbolic CTL model-checker.

models defined "symbolically", and encoded using BDD:

```
MODULE automate
  VAR
    red: boolean;
    blue: boolean;
    green: boolean;
    yellow: boolean;
  INIT
    red & !blue & !green & !yellow
  TRANS
    case
       red & !blue & !green & !yellow:
          next(!red &  blue & !green & !yellow) |
          next( red & !blue & !green &  yellow)
      !red &  blue & !green & !yellow:
          next( red & !blue & !green & !yellow) |
          next(!red & !blue &  green & !yellow)
      !red & !blue &  green & !yellow:
          next( red & !blue & !green &  yellow) |
          next(!red & !blue &  green & !yellow)
       red & !blue & !green &  yellow:
          next( red & !blue & !green & !yellow)
```



What is NuSMV?

(Nu)SMV = symbolic CTL model-checker.

$CTL \ni \varphi ::= p \mid \varphi \vee \varphi \mid \neg\varphi \mid EX\,\varphi \mid AX\,\varphi \mid E[\varphi U \varphi] \mid A[\varphi U \varphi]$

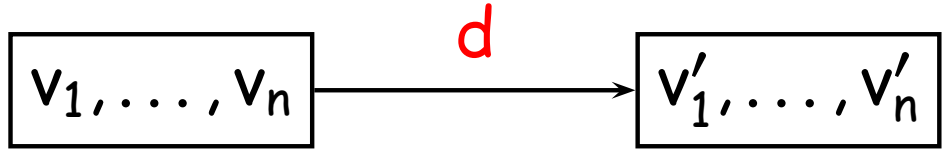
NuSMV also handles TCTL, assuming that one transition takes one time unit.

$TCTL \ni \psi ::= \psi \in CTL \mid E[\psi U_{[c,d]} \psi] \mid A[\psi U_{[c,d]} \psi]$

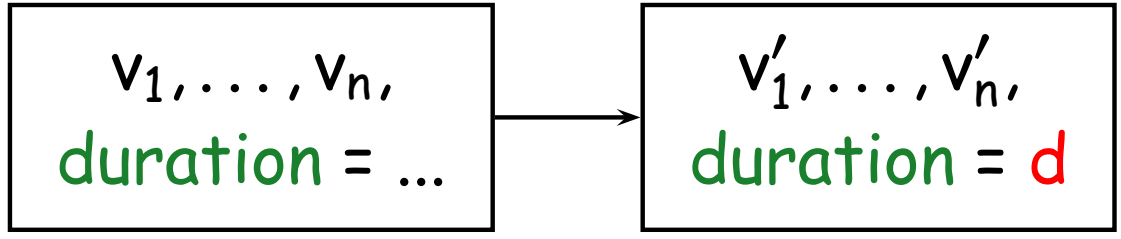
Coding TKS in NuSMV

We add one special integer variable, named duration.

We encode



as



Bridge crossing at night

- 4 persons are to cross the bridge.
- At most two at a time on the bridge.
- They have one lamp, needed when crossing.
- It takes 5, 10, 20 and 25 seconds for each of them to cross.

What is the optimal combination?

Bridge crossing at night

```
MODULE hiker(time,lamp,duration)

VAR
  state: boolean;                -- true = left of the bridge, false = right
  changed: boolean;              -- true if the person just crossed the bridge

INIT
  !state & !changed

TRANS
  ((!state & !lamp) -> ((next(state) & next(duration)>=time & next(changed)) |
                        (next(!state) & next(!changed))))
&
  ((state & lamp) -> ((next(!state) & next(duration)>=time & next(changed)) |
                       (next(state) & next(!changed))))
&
  ((!state & lamp) -> (next(!state) & next(!changed)))
&
  ((state & !lamp) -> (next(state) & next(!changed)))
```

Bridge crossing at night

```
MODULE main

VAR
  v1: hiker(5,lamp,duration);
  v2: hiker(10,lamp,duration);
  v3: hiker(10,lamp,duration);
  v4: hiker(25,lamp,duration);
  lamp: boolean;                -- true = left of the bridge, false = right
  duration: 0..25;

DEFINE
  bridge :=  v1.changed +  v2.changed +  v3.changed +  v4.changed ;
  initial:= (!v1.state) & (!v2.state) & (!v3.state) & (!v4.state);
  final  := ( v1.state) & ( v2.state) & ( v3.state) & ( v4.state);

INIT
  lamp & duration=0

TRANS
  (next(bridge=0)) <-> (next(lamp)=lamp)

COMPUTE MIN[ initial, final ]

SPEC !EF<=60 final
```

Bridge crossing at night

This example can easily be translated into NuSMV, but...

Bridge crossing at night

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	NuSMV		TSMV	
	time	memory	time	memory
bridge	0.13 sec.	9 112 KB	0.02 sec.	1 280 KB
bridge × 10	4.14 sec.	18 860 KB	0.04 sec.	1 280 KB
bridge × 20	22.44 sec.	19 508 KB	0.07 sec.	1 280 KB
bridge × 50	176.55 sec.	28 640 KB	0.22 sec.	9 920 KB
bridge × 100	844.25 sec.	50 964 KB	0.45 sec.	11 580 KB
bridge × 200	4889.93 sec.	98 212 KB	1.03 sec.	14 956 KB

TSMV is (more or less) scale-insensitive.

Encouraging results

- relatively easy implementation into NuSMV,
- efficient model checking of timed Kripke structures,
- scale-insensitivity, memory and time consumption equivalent to those of NuSMV on classical Kripke Structure (with duration 1).

Future work

- more examples (NuSMV classical examples, UppAal examples, scheduling examples, ...)
- clean up the code, find better algorithms,
- counter-example generation.