# On the Expressiveness of TPTL and MTL

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February 16, 2006

# Model checking

Does a system satisfy some property?

Modelling

Specification



*E.g.* with temporal logics [Pnu77]:

Always(`problem` $\Rightarrow$ Eventually `alarm`)

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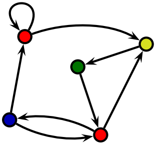

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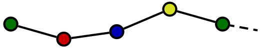

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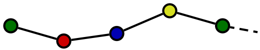

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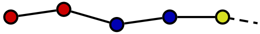

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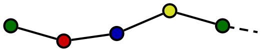

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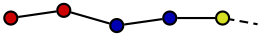

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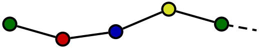

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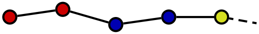

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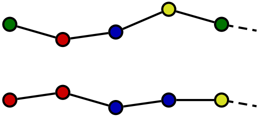

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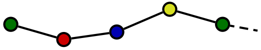

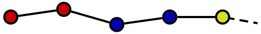

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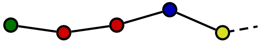

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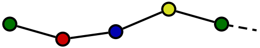

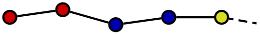

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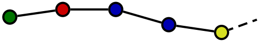

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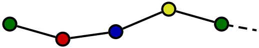

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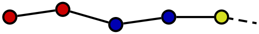

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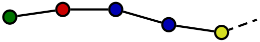

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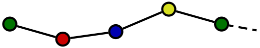

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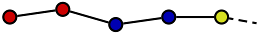

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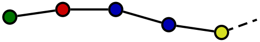

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**LTL+Past**

**Theorem ([Kam68, GPSS80])**

**LTL** and **LTL+Past** are equally expressive.

# Adding real-time constraints

- Timed automata: Finite automata + real-valued clocks [AD90]
- Timed **LTL**: 2 standard extensions:
  - **MTL** [Koy90]: adding subscripts to modalities:

$$\mathbf{G}\,(\texttt{problem} \Rightarrow \texttt{problem}\ \mathbf{U}_{\leq 5}\ \texttt{alarm})$$

  - **TPTL** [AH89]: adding clocks in formulas:

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# Outline of the talk

1. Introduction
2. Definitions
   - Two frameworks: pointwise and continuous
   - Semantics of **TPTL** and **MTL**
3. Expressiveness of **TPTL** and **MTL**
   - Alur and Henzinger's conjecture
   - The formula can be expressed...
   - ...but the conjecture holds!
4. What if we only allow the **F** modality?
   - $\mathbf{TPTL_F}$ and $\mathbf{MTL_F}$ are equally expressive
   - An interesting corollary

# Outline of the talk

# Two possible semantics for a timed path

- pointwise semantics: discrete view of a continuous system:

  timed word: $((w_i)_{i\in\mathbb{N}}, (t_i)_{i\in\mathbb{N}})$

  word in $(2^{\mathsf{AP}})^{\mathbb{N}}$

  sequence of dates in $(\mathbb{Q}^+)^{\mathbb{N}}$

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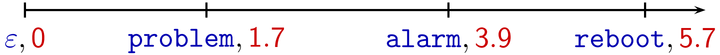


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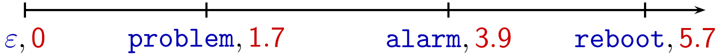


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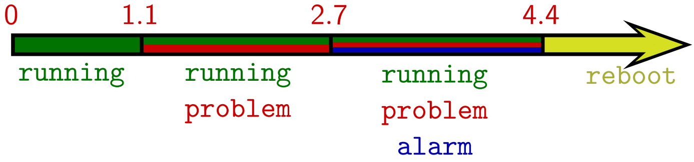

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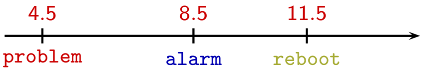


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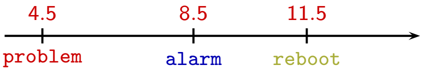
✔ 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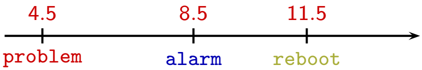


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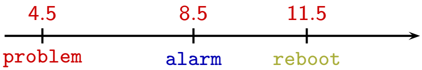
✔ 


false **U** alarm

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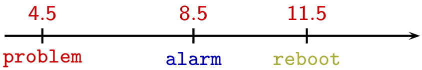
✔ 


$\texttt{false} \; \mathbf{U} \; \texttt{alarm}$
$\equiv \mathbf{X} \; \texttt{alarm}$

# **TPTL** and **MTL** in the pointwise semantics

- Syntax of **MTL**:

$$\mathbf{MTL} \ni \phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid \phi\ \mathbf{U}_I\ \phi$$

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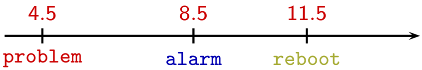
- Examples:




$$\mathbf{F}_{[5,6]}\,(\mathtt{alarm} \vee \mathtt{reboot})$$

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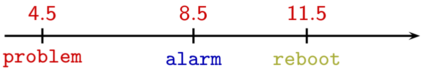
✘ 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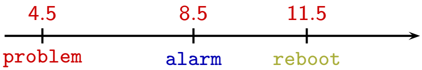


$\mathbf{F}_{=7}$ `reboot`

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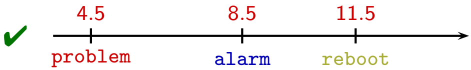


$\mathbf{F}_{=7}$ reboot

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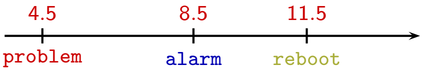


$$\mathbf{F}_{=4}\left(\mathbf{F}_{=3}\ \texttt{reboot}\right)$$

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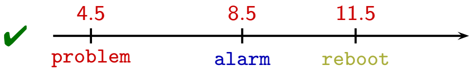


$\mathbf{F}_{=4}\,(\mathbf{F}_{=3}\ \mathtt{reboot})$

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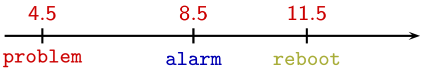


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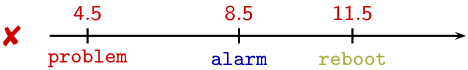


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# TPTL and MTL in the pointwise semantics

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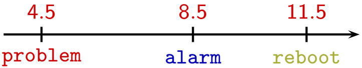


$x.\ (\mathtt{problem}\ \mathbf{U}\ (\mathtt{alarm} \wedge x \leq 5))$

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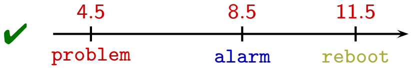


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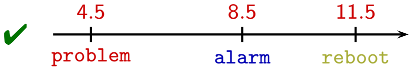


$$x.\ (\texttt{problem}\ \mathbf{U}\ (\texttt{alarm} \wedge x \leq 5)) \equiv \texttt{problem}\ \mathbf{U}_{\leq 5}\ \texttt{alarm}$$

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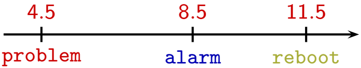


$x.\ \mathbf{F}\,(\texttt{alarm} \wedge \mathbf{F}\,(\texttt{reboot} \wedge x \leq 8))$

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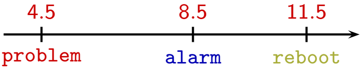
✔ 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$\texttt{running} \; \mathbf{U}_{\geq 5} \; \texttt{problem}$

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# TPTL and MTL in the continuous semantics

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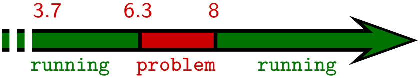


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# **TPTL** and **MTL** in the continuous semantics

- Syntax of **MTL**:

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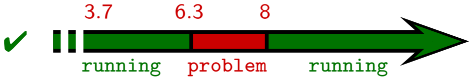


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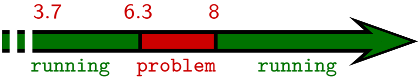
✔ 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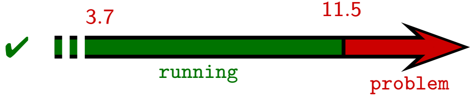


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# Outline of the talk

# Alur and Henzinger's conjecture

Clearly, **MTL** can be translated into **TPTL**:

$$\phi \ \mathbf{U}_I \ \psi \equiv x.\ \phi \ \mathbf{U} \left(\psi \wedge x \in I\right).$$

**Conjecture ([AH90])**

- **TPTL** is strictly more expressive than **MTL**,
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It turns out that the formula can be expressed in **MTL**:

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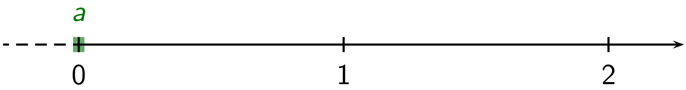


$$\mathbf{G} \quad a \Rightarrow \begin{cases} \mathbf{F}_{\leq 1}\, b \wedge \mathbf{F}_{[1,2]}\, c \\ \vee \\ \mathbf{F}_{\leq 1}\,(b \wedge \mathbf{F}_{\leq 1}\, c) \\ \vee \\ \mathbf{F}_{\leq 1}\,(\mathbf{F}_{\leq 1}\, b \wedge \mathbf{F}_{=1}\, c) \end{cases}$$

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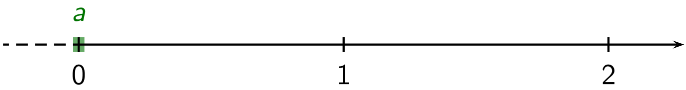


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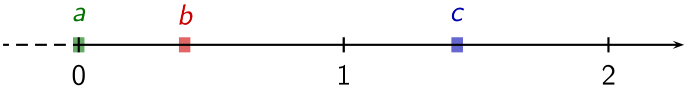


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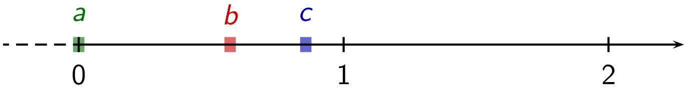


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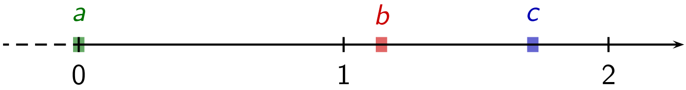


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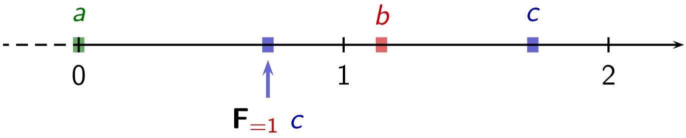


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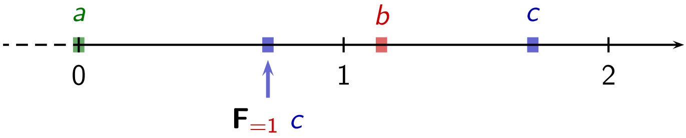


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Several remarks:

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Several remarks:

- of course, it does not mean that **TPTL** $\equiv$ **MTL**.
- the formulas are equivalent only for the continuous semantics.

# … but the result still holds!

We prove the following results:

**Theorem**

- **TPTL** *is* strictly more expressive than **MTL** for *both* semantics,
- the **TPTL** formula

$$\mathbf{G}\,(a \Rightarrow x.\ \mathbf{F}\,(b \wedge \mathbf{F}\,(c \wedge x \leq 2))$$

  *cannot* be expressed in **MTL** under the pointwise semantics,
- the **TPTL** formula

$$x.\ \mathbf{F}\,(a \wedge x \leq 1 \wedge (\neg\, b)\ \mathbf{U}\,(a \wedge x \geq 1))$$

  cannot be expressed in **MTL** under both semantics.

# How to prove expressiveness results?

How to prove that a **TPTL** formula $\phi$ can't be expressed in **MTL**?

Find two models $A$ and $B$ such that

- $A \models \phi$ and $B \not\models \phi$,
- any **MTL** formula is either true on both or false on both

Problem: if $A$ and $B$ differ at some point $t$, the modality $\mathbf{F}_{=t}$ permits to distinguish them... (*distinguishing power*)

Find two **families** of models $(A_i)_i$ and $(B_i)_i$ s.t.

- $A_i \models \phi$ and $B_i \not\models \phi$, for any $i$,
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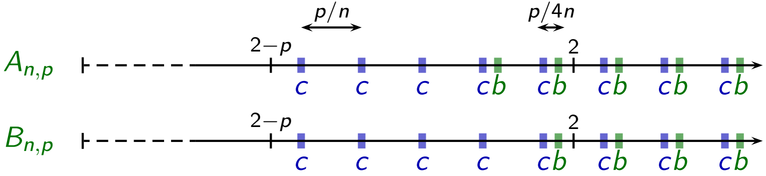
# Alur and Henzinger's formula in the pointwise semantics

- Let $\phi = x.\ \mathbf{F}\,(b \wedge \mathbf{F}\,(c \wedge x \leq 2))$
- Let $\psi$ be an **MTL** formula.
  - $n$ = temporal height of $\psi$ = maximum number of nested modalities in $\psi$,
  - $p$ = *granularity* of $\psi$ = inverse of the least common denominator of the constants appearing in $\psi$.

$\rightarrow$ we assume that the constants of $\psi$ are multiples of $p$.

We build the following two families of models:



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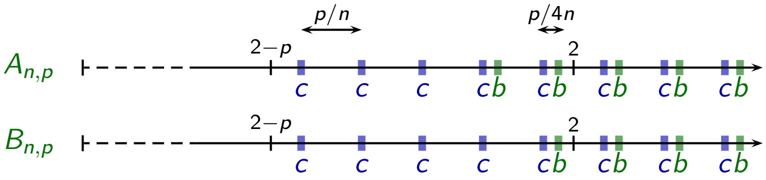


**Lemma**

- $$A_{n,p}, 0 \models \phi \quad \text{and} \quad B_{n,p}, 0 \not\models \phi$$
- For any formula $\psi$ of **MTL** with temporal height $n$ and granularity $p$,

$$A_{n+3,p}, 0 \models \psi \;\Leftrightarrow\; B_{n+3,p}, 0 \models \psi$$

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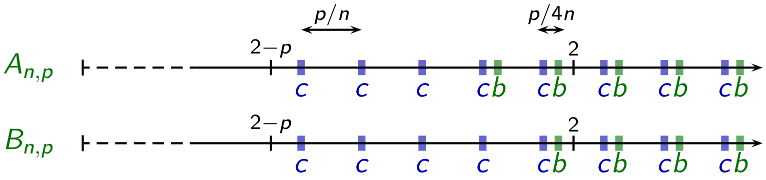


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# Results for the pointwise semantics

**Theorem**

Under the pointwise semantics:

- **TPTL** is strictly more expressive than **MTL**,
- **MTL+Past** is strictly more expressive than **MTL**,
- **MITL+Past** is strictly more expressive than **MITL**.

Nota: **MITL** is the fragment of **MTL** where punctuality is not allowed, *i.e.*, where timing constraints cannot be singular.

# A new formula for the continuous semantics

- Let $\phi = x.\ \mathbf{F}\,(a \wedge x \leq 1 \wedge \mathbf{G}\,(x \leq 1 \Rightarrow \neg\, b))$

We build the following two families of models:

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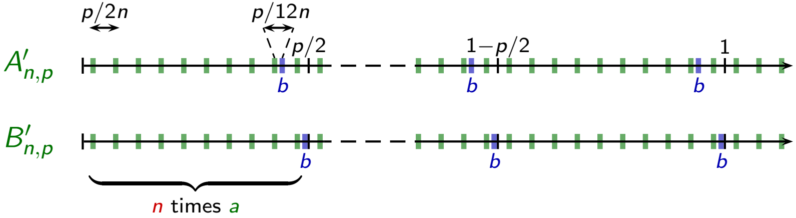

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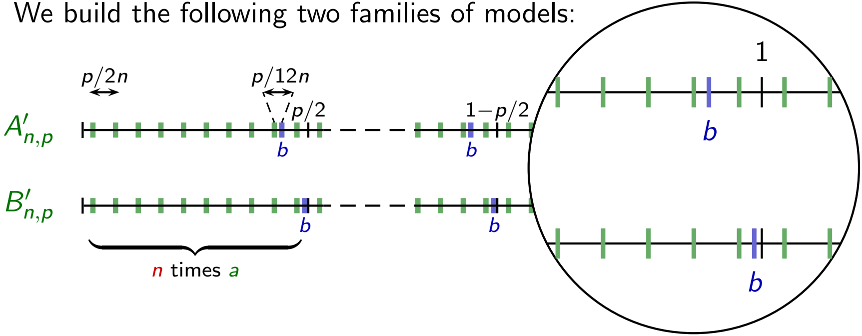

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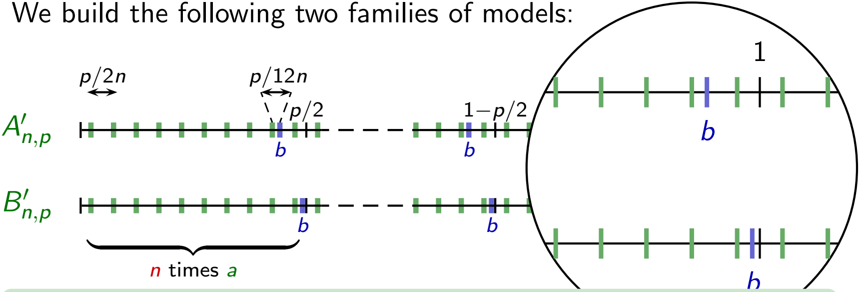


**Lemma**

- $$A'_{n,p}, 0 \models \phi \quad \text{and} \quad B'_{n,p}, 0 \not\models \phi$$
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# Our results

**Theorem**

Under both semantics:

- **TPTL** is strictly more expressive than **MTL**,
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- **MITL+Past** is strictly more expressive than **MITL**.

# Outline of the talk

# Existential fragment of TPTL

$$\mathbf{TPTL_F} \ni \phi ::= p \mid \neg p \mid x \sim c \mid \phi \vee \phi \mid \phi \wedge \phi \mid \mathbf{F}\ \phi \mid x.\ \phi$$

For instance:

$$x.\mathbf{F}\,(\ a \wedge x \geq 1 \wedge \mathbf{F}\,(\ b \wedge x \leq 3\ ) \wedge y.\mathbf{F}\,(\ \neg a \wedge x \leq 3 \wedge y > 1\ )\ )$$

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$z_0 = 0$

# Existential fragment of **TPTL**

$$\mathbf{TPTL_F} \ni \phi ::= p \mid \neg p \mid x \sim c \mid \phi \vee \phi \mid \phi \wedge \phi \mid \mathbf{F}\,\phi \mid x.\,\phi$$

For instance:

$$x.\mathbf{F}\,(\,\boxed{a \wedge x \geq 1 \wedge \mathbf{F}\,(\,b \wedge x \leq 3\,) \wedge y.\mathbf{F}\,(\,\neg a \wedge x \leq 3 \wedge y > 1\,)}\,)$$

$z_1$

$$z_0 = 0 \qquad \begin{array}{l} z_1 - z_0 > 0 \\ z_1 - z_0 \geq 1 \\ \pi, z_1 \models a \end{array}$$

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For instance:

$$x.\mathbf{F}\,(\,a \wedge x \geq 1 \wedge \mathbf{F}\,(\underbrace{b \wedge x \leq 3}_{z_2}) \wedge y.\mathbf{F}\,(\,\neg a \wedge x \leq 3 \wedge y > 1\,)\,)$$

$$z_0 = 0 \qquad \begin{array}{l} z_1 - z_0 > 0 \\ z_1 - z_0 \geq 1 \\ \pi, z_1 \models a \end{array} \qquad \begin{array}{l} z_2 - z_1 > 0 \\ z_2 - z_0 \leq 3 \\ \pi, z_2 \models b \end{array}$$

# Existential fragment of **TPTL**

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$$x.\mathbf{F}\,(\ a \wedge x \geq 1 \wedge \mathbf{F}\,(\ b \wedge x \leq 3\ ) \wedge y.\mathbf{F}\,(\ \boxed{\neg a \wedge x \leq 3 \wedge y > 1}\ )\ )$$

$z_3$

$$z_0 = 0$$

$$z_1 - z_0 > 0$$
$$z_1 - z_0 \geq 1$$
$$\pi, z_1 \models a$$

$$z_2 - z_1 > 0$$
$$z_2 - z_0 \leq 3$$
$$\pi, z_2 \models b$$

$$z_3 - z_1 > 0$$
$$z_3 - z_0 \leq 3$$
$$z_3 - z_1 > 1$$
$$\pi, z_3 \models \neg a$$

# Existential fragment of **TPTL**

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$z_3$

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$$\pi, z_1 \models a$$
$$\pi, z_2 \models b$$
$$\pi, z_3 \models \neg a$$

DBM:

$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} =0 & & \leq 3 & \leq 3 \\ \leq -1 & =0 & & \\ & <0 & =0 & \\ & < -1 & & =0 \end{pmatrix} \end{array}$$

# Existential fragment of TPTL

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$\pi, z_2 \models b$

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DBM:

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$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} = 0 & < 3 & \leq 3 & \leq 3 \\ \leq -1 & = 0 & \leq 2 & \leq 2 \\ < -1 & < 0 & = 0 & < 2 \\ < -2 & < -1 & < 1 & = 0 \end{pmatrix} \end{array}$$

$z_3 \leq z_2$ or $z_2 \leq z_3$?

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$\pi, z_1 \models a$
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DBM:

$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} = 0 & < 3 & \leq 3 & \leq 3 \\ \leq -1 & = 0 & \leq 2 & \leq 2 \\ < -1 & < 0 & = 0 & \leq 0 \\ < -2 & < -1 & < 1 & = 0 \end{pmatrix} \end{array}$$

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DBM:

$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} = 0 & < 3 & \leq 3 & \leq 3 \\ \leq -1 & = 0 & \leq 2 & \leq 2 \\ < -1 & < 0 & = 0 & < 2 \\ < -2 & < -1 & \leq 0 & = 0 \end{pmatrix} \end{array}$$

$z_3 \leq z_2$ or $z_2 \leq z_3$?

# Existential fragment of **TPTL**

$$\pi, z_1 \models a$$
$$\pi, z_2 \models b$$
$$\pi, z_3 \models \neg a$$

$$\begin{array}{c c}
 & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\
\begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} =0 & <3 & \leq 3 & \leq 3 \\ \leq -1 & =0 & \leq 2 & \leq 2 \\ < -1 & <0 & =0 & <2 \\ < -2 & < -1 & \leq 0 & =0 \end{pmatrix}
\end{array}$$

# Existential fragment of **TPTL**

$\pi, z_1 \models a$
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$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} = 0 & < 3 & \leq 3 & \leq 3 \\ \leq -1 & = 0 & \leq 2 & \leq 2 \\ < -1 & < 0 & = 0 & < 2 \\ < -2 & < -1 & \leq 0 & = 0 \end{pmatrix} \end{array}$$

Region decomposition

# Existential fragment of **TPTL**

$$\pi, z_1 \models a \qquad \pi, z_2 \models b \qquad \pi, z_3 \models \neg a$$

$$\begin{array}{c|cccc} & z_0 & z_1 & z_2 & z_3 \\ \hline z_0 & =0 & <3 & \leq 3 & \leq 3 \\ z_1 & \leq -1 & =0 & \leq 2 & \leq 2 \\ z_2 & <-1 & <0 & =0 & <2 \\ z_3 & <-2 & <-1 & \leq 0 & =0 \end{array}$$

Region decomposition

$$\pi, z_1 \models a \qquad \pi, z_2 \models b \qquad \pi, z_3 \models \neg a$$

$$\begin{array}{c|cccc} & z_0 & z_1 & z_2 & z_3 \\ \hline z_0 & =0 & <2 & <3 & <3 \\ z_1 & <-1 & =0 & =1 & <2 \\ z_2 & <-2 & =-1 & =0 & <1 \\ z_3 & <-2 & <-1 & <0 & =0 \end{array}$$

# Existential fragment of **TPTL**

$$\begin{aligned} \pi, z_1 &\models a \\ \pi, z_2 &\models b \\ \pi, z_3 &\models \neg a \end{aligned}
\qquad
\begin{array}{c c}
 & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\
\begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} &
\begin{pmatrix} =0 & <2 & <3 & <3 \\ <-1 & =0 & =1 & <2 \\ <-2 & =-1 & =0 & <1 \\ <-2 & <-1 & <0 & =0 \end{pmatrix}
\end{array}$$

# Existential fragment of **TPTL**

$$\pi, z_1 \models a$$
$$\pi, z_2 \models b$$
$$\pi, z_3 \models \neg a$$

$$\begin{array}{c c} & \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} & \begin{pmatrix} =0 & <2 & <3 & <3 \\ <-1 & =0 & =1 & <2 \\ <-2 & =-1 & =0 & <1 \\ <-2 & <-1 & <0 & =0 \end{pmatrix} \end{array}$$

Time shift

# Existential fragment of TPTL

$$\pi, z_1 \models a \qquad \pi, z_2 \models b \qquad \pi, z_3 \models \neg a$$

$$\begin{array}{c} \\ z_0 \\ z_1 \\ z_2 \\ z_3 \end{array} \begin{array}{c} \begin{array}{cccc} z_0 & z_1 & z_2 & z_3 \end{array} \\ \begin{pmatrix} =0 & <2 & <3 & <3 \\ <-1 & =0 & =1 & <2 \\ <-2 & =-1 & =0 & <1 \\ <-2 & <-1 & <0 & =0 \end{pmatrix} \end{array}$$

Time shift

$$\pi, z_1' \models \mathbf{F}_{=1}\ a \qquad \pi, z_2' \models \mathbf{F}_{=2}\ b \qquad \pi, z_3' \models \mathbf{F}_{=2}\ \neg a$$

$$\begin{array}{c} \\ z_0' \\ z_1' \\ z_2' \\ z_3' \end{array} \begin{array}{c} \begin{array}{cccc} z_0' & z_1' & z_2' & z_3' \end{array} \\ \begin{pmatrix} =0 & <1 & <1 & <1 \\ <0 & =0 & =0 & <1 \\ <0 & =0 & =0 & <1 \\ <0 & <0 & <0 & =0 \end{pmatrix} \end{array}$$

# Existential fragment of TPTL

$$\pi, z_1 \models a \qquad \pi, z_2 \models b \qquad \pi, z_3 \models \neg a$$

$$\begin{array}{c|cccc} & z_0 & z_1 & z_2 & z_3 \\ \hline z_0 & =0 & <2 & <3 & <3 \\ z_1 & <-1 & =0 & =1 & <2 \\ z_2 & <-2 & =-1 & =0 & <1 \\ z_3 & <-2 & <-1 & <0 & =0 \end{array}$$

Time shift

$$\pi, z_1' \models \mathbf{F}_{=1}\ a \qquad \pi, z_2' \models \mathbf{F}_{=2}\ b \qquad \pi, z_3' \models \mathbf{F}_{=2}\ \neg a$$

$$\begin{array}{c|cccc} & z_0' & z_1' & z_2' & z_3' \\ \hline z_0' & =0 & <1 & <1 & <1 \\ z_1' & <0 & =0 & =0 & <1 \\ z_2' & <0 & =0 & =0 & <1 \\ z_3' & <0 & <0 & <0 & =0 \end{array}$$

eq. to $\quad 0 = z_0' < z_1' = z_2' < z_3' < 1$

# Existential fragment of **TPTL**

$$\pi, z_i \models \phi_i \qquad 0 < z_h < z_{h+1} < \cdots < z_{h+k} < r \qquad (1)$$

We inductively (on $k$) build an **MTL** formula $\Psi_{[h..h+k,r]}$ s.t.

$$(1) \Leftrightarrow \pi \models \Psi_{[h..h+k,r]}.$$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

- $\Psi_{[1..1,r]} = \mathbf{F}_{<r}\,(\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b)$.
- $\Psi_{[2..2,r]} = \mathbf{F}_{<r}\,(\mathbf{F}_{=2}\ \neg a)$.

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

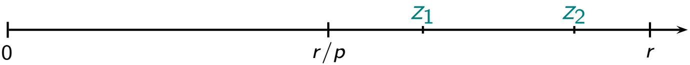
– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:



# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

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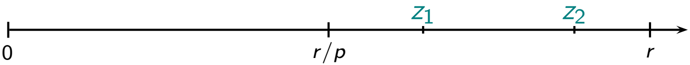
– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:




- $\Psi^1_{[1..2,r]} = \mathbf{F}_{[0,\frac{r}{p})} \left(\mathbf{F}_{=\frac{r}{p}}\ \phi_2 \wedge \Psi_{[1..1,\frac{r}{p}]}\right).$

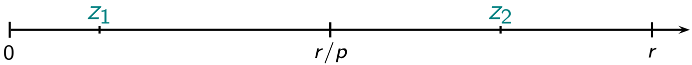
  $= \mathbf{F}_{[0,\frac{r}{p})} \left(\mathbf{F}_{=\frac{r}{2}} (\mathbf{F}_{=2}\ \neg a) \wedge \mathbf{F}_{<\frac{r}{2}} (\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b)\right)$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:

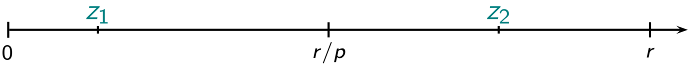


# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:




- $\Psi^2_{[1..2,r]} = \Psi_{[1..1],\frac{r}{p}} \wedge \mathbf{F}_{=\frac{r}{p}}\ \Psi_{[2..2,r-\frac{r}{p}]}$
  $= \mathbf{F}_{<\frac{r}{2}}\ (\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b) \wedge \mathbf{F}_{=\frac{r}{2}}\ \mathbf{F}_{<\frac{r}{2}}\ (\mathbf{F}_{=2}\ \neg a)$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

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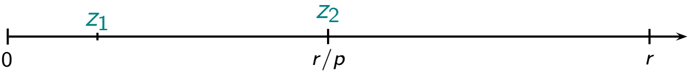
– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:




- $\Psi^3_{[1..2,r]} = \mathbf{F}_{<\frac{r}{2}}\,(\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b) \wedge \mathbf{F}_{=\frac{r}{2}}\,(\mathbf{F}_{<\frac{r}{2}}\,(\mathbf{F}_{=2}\ \neg a))$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$

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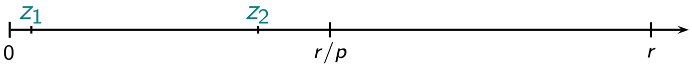
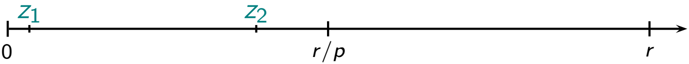


# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$
$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg a$$

– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:

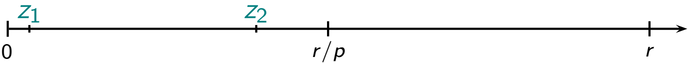



- $$\Psi^4_{[1..2,r]} = \mathbf{F}_{<\frac{r}{p}}\ \Big(\phi_1 \wedge \mathbf{F}_{<\frac{r}{p}}\ \phi_2\Big)$$
  $$= \mathbf{F}_{<\frac{r}{2}}\ \Big(\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \wedge \mathbf{F}_{<\frac{r}{2}}\ (\mathbf{F}_{=2}\ \neg a)\Big)$$

# Existential fragment of **TPTL**

$$\pi, z_1 \models \mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \qquad\qquad 0 < z_1 < z_2 < r$$
$$\pi, z_2 \models \mathbf{F}_{=2}\ \neg\, a$$

– $\Psi_{[1..2,r]}$ is the conjunction of the following possibilities:




- $$\Psi^4_{[1..2,r]} = \mathbf{F}_{<\frac{r}{p}} \left(\phi_1 \wedge \mathbf{F}_{<\frac{r}{p}}\, \phi_2\right)$$
  $$= \mathbf{F}_{<\frac{r}{2}} \left(\mathbf{F}_{=1}\ a \wedge \mathbf{F}_{=2}\ b \wedge \mathbf{F}_{<\frac{r}{2}}\, (\mathbf{F}_{=2}\ \neg\, a)\right)$$

**Theorem**

The resulting **MTL** formula is equivalent to the original $\mathbf{TPTL_F}$ one, and has size exponential.

# An interesting corollary

This transformation also yields the following result:

**Theorem**

Under both the interval-based and the pointwise semantics, satisfiability of a $\mathbf{TPTL_F}$ formula is decidable, and is NP-complete.

# Conclusion

- Many new expressiveness results:
  - TPTL *is* more expressive than **MTL**,
    (this result extends to the branching-time case)
  - Past-time modalities *do* add expressive power to timed logics,
  - The expressive power depends on the underlying semantics,
- Decidability of $\mathbf{TPTL_F}$ , and translation into **MTL**.

- Recent result: the formula

$$\mathbf{G}\,(a \Rightarrow x.\ \mathbf{F}\,(b \wedge \mathbf{F}\,(c \wedge x \leq 2)))$$

cannot be expressed in **MITL**, while it does not express a "punctual" property.

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