A Polynomial λ-Bisimilar Normalization for Reset Petri Nets*

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Abstract

Reset Petri nets extend Petri nets by allowing transitions to empty places, in addition with the usual adding of removing of constants. A Reset Petri net is *normalized* if the flow function over the Petri arcs (labeled with integers) and the initial state are valuated into $\{0,1\}$. In this paper, we give an efficient method to turn a general Reset Petri net into a λ-bisimilar normalized Reset Petri net. Our normalization preserves the main usually studied properties : *Boundedness*, *Reachability*, *t-Liveness* and *Language* (through a λ-labeling function). The main contribution is the improvement of the complexity : our algorithm takes a time in $O(\mathtt{size}(N)^2)$, for a Reset Petri net N, while other known normalizations require an exponential space and are presented for Petri nets only.

Key words: Concurrency, Verification, Petri Nets, Equivalences.

1 Introduction

Petri nets constitute one of the most powerful models among the analyzable models of parallel systems [Pet81, EN94]. General Petri nets allow transitions to remove from places, or to add to places, non negative integer constant numbers of tokens. *Reset Petri nets* [AK77, DFS98] are more powerful than Petri nets. They allow a transition either to remove/add constant numbers of tokens (this is performed with *Petri arcs*), or to *empty* some of the places (this is performed with *Reset arcs*). Normalized Reset Petri nets allow to remove/add at most one token by the Petri arcs.

Normalizations found in the literature are presented for Petri nets only and are not efficient. In fact, for a valuation v, $\Omega(v)$ items are created and this is exponential on the size, $\log v$, of v. The following list is not exhaustive and refer to Petri nets only. All of these normalizations preserve *boundedness*, *reachability* and *language* (most of the time through a λ-labeling function). Let $V(p)$ be the maximum among the initial value of the place p and the constants to be added to p or removed from p. In the normalization in [Pet81], each place p is transformed into a ring of $V(p)$ places and each valuation v of a transition is transformed into v arcs 1-valuated. Tokens are free to move all around the ring. In [nc94], [KM82] and [Sch85], the main idea is to create v levels of places and transitions for a valuation v. Levels are activated sequentially in order to achieve the adding/removing of the v tokens. In [Sif79], a queue of $V(p)$ tokens is managed for each place p. When v tokens are required, they are dequeued in parallel (one arc by token). In [KM82] the idea is to divide valuations over of the arcs and the initial marking by 2, until normalization. However, at each step, arcs have to be duplicated because cases "even" and "odd" appear. In [Pel92], the normalization preserves the language of the original Petri net without λ-transitions. Each place p is replaced with a set of $V(p)$ places. Valuation v is managed through all the possibilities to touch v over $V(p)$.

The motivation of our study is to give an efficient normalization (polynomial in time) that will be extendable to Reset Petri nets. The main idea is simple and, surprisingly, has not been exploited before. We use the *binary encoding* of a valuation v, instead of v itself. We then manipulate a linear number of items. In the previous approaches, firability is managed in a "hard-ware" fashion with an exponential

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number of transitions and places. Here, the power of calculus of the net is used to compute the firability of a transition. This leads to a normalization which has a time complexity in $O(\mathtt{size}(N)^2)$. A general net and its associated normalized net are λ-bisimilar and thus language is preserved through a λ-labeling function; *boundedness*, *reachability* and *t-liveness* are also preserved.

2 Reset Petri nets, definitions and complexities

Let $\mathbb{N}$ be the set of nonnegative integers, $\mathbb{N}^k$ $(k \geq 1)$ be the set of k-dimensional column vectors of elements in $\mathbb{N}$. For a vector $X \in \mathbb{N}^k$, $X(i)$ $(1 \leq i \leq k)$ is the i^{th} component of X. Let Σ be a finite alphabet and Σ^* be the set of all words over Σ. The length of a word $\sigma \in \Sigma^*$ is denoted by $|\sigma|$. The empty word is denoted by λ and $|\lambda| = 0$. The cardinal of a finite set S is denoted by $|S|$. The function $\mathtt{max} : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ returns the maximum between two integers.

Definition 1 *[AK77]*

- *A* Reset Petri net *is a tuple $N =< P, T, F, m_0 >$ where P is a finite set of* places*, T is a finite set of* transitions*, $T \cap P = \emptyset$, F is a* flow function *such that :*

$$F : \begin{cases} (P \times T) \longrightarrow \mathbb{N} \cup P \\ (T \times P) \longrightarrow \mathbb{N} \end{cases}$$

and $m_0 \in \mathbb{N}^{|P|}$ is the initial marking*.*

- *A Reset Petri net is* normalized *if $m_0 \in \{0,1\}^{|P|}$ and* $F : \begin{cases} (P \times T) \longrightarrow \{0,1\} \cup P \\ (T \times P) \longrightarrow \{0,1\} \end{cases}$

- *A Petri net is a Reset Petri net such that $F \ : (P \times T) \cup (T \times P) \longrightarrow \mathbb{N}$.*

Remark 1 *normalized nets may be unbounded and they should not be mistaken for* 1-safe *nets, for which any reachable marking is in $\{0,1\}^{|P|}$.*

Let N be a Reset Petri net. A transition $t \in T$ is *firable* from a marking $m \in \mathbb{N}^{|P|}$, written $m \xrightarrow{t}$, if for any place $p \in P$, $F(p,t) \in \mathbb{N} \Rightarrow m(p) \geq F(p,t)$. Firing transition t from m leads to the marking m', this is written $m \xrightarrow{t} m'$, such that for any place $p \in P$: $\begin{cases} F(p,t) \in P \Rightarrow m'(p) = F(t,p) \\ F(p,t) \in \mathbb{N} \Rightarrow m'(p) = m(p) - F(p,t) + F(t,p) \end{cases}$

Firability is classically extended to sequences $\sigma \in T^*$. The *language* $L(N)$ is the set of all the sequences firable from the initial marking. A marking m' is *reachable* from m if there exists a sequence $\sigma \in T^*$ such that $m \xrightarrow{\sigma} m'$. The *reachability set* $RS(N,m)$ contains all the markings reachable from m.

A transition t is *quasi-live* if it is firable from a marking in $RS(N, m_0)$. A transition $t \in T$ is *live* if it is quasi-live from any marking in $RS(N, m_0)$. The *General Boundedness Problem* (GBP) is to determine whether $RS(N, m_0)$ is finite or not. The *Reachability Problem* (RP) is to determine whether $m \in RS(N, m_0)$. The *t-Liveness Problem* (t-LP) is to determine whether the transition t is live. The *General Liveness Problem* (GLP) is to determine whether all the transitions are live.

The problems above are decidable for Petri nets, but unfortunately require exponential space (see [EN94]). Reset Petri nets are strictly more powerful than Petri nets and can simulate counters machines [AK77, Duf98]. A consequence is that reachability [AK77] and boundedness [DFS98] are undecidable for Reset Petri nets.

Definition 2 *Let ${}^\bullet p = \{t \mid F(p,t) \neq 0\}$ and $p^\bullet = \{t \mid F(t,p) \neq 0\}$.*

Let $p \in P$, $V(p) = \mathtt{max}\{\, m_0(p), \mathtt{max}_{t \in T, F(p,t) \in \mathbb{N}}\, F(p,t), \mathtt{max}_{t \in T}\ F(t,p)\,\}$ and $V = \mathtt{max}_{p \in P}\ V(p)$.

Size of N. We encode N with two integer matrices $|P| \times |T|$ (one for $F(p,t) \in \mathbb{N}$ and the other for $F(t,p)$), one matrix $|P| \times |T|$ of booleans (for the Reset arcs) and one vector in $\mathbb{N}^{|P|}$ (for the initial marking). Any valuations inside the two integer matrices and the integer vector are upper-bounded by V and coded over $\Theta(\log V)$ bits. The size $|N|$ of a Reset Petri net N belongs to $\Theta(2|P|.|T|.\log V + |P|.|T| + |P|.\log V) = \Theta(|P|.|T|.\log V)$.

λ-bisimilarity and fusion : we use the λ-*bisimilarity* [GW89] notion to compare N and its associated normalized net, and to show that the two nets have a very similar behavior.

Definition 3

- *A* labeled Reset Petri net (N, Σ, l) *is a triplet such that N is a Reset Petri net, Σ is a finite alphabet and $l : T^* \to \Sigma^*$ is a labeling function.*

- *Two labeled Reset Petri nets (N_1, Σ_1, l_1) and (N_2, Σ_2, l_2) are λ-bisimilar if there exists a relation $\mathcal{R} \subseteq \mathbb{N}^{|P_1|} \times \mathbb{N}^{|P_2|}$ such that the following conditions hold :*

  1. $m_{0_1} \; \mathcal{R} \; m_{0_2}$.
  2. *For any markings $m_1 \in \mathbb{N}^{|P_1|}, m_2 \in \mathbb{N}^{|P_2|}$ such that $m_1 \; \mathcal{R} \; m_2$:*
     - *For any sequence $s_1 \in T_1^*$ such that $m_1 \xrightarrow{s_1} m_1'$, there exists a sequence $s_2 \in T_2^*$ such that $m_2 \xrightarrow{s_2} m_2'$, $l_1(s_1) = l_2(s_2)$ and $m_1' \; \mathcal{R} \; m_2'$.*
     - *For any sequence $s_2 \in T_2^*$ such that $m_2 \xrightarrow{s_2} m_2'$, there exists a sequence $s_1 \in T_1^*$ such that $m_1 \xrightarrow{s_1} m_1'$, $l_2(s_2) = l_1(s_1)$ and $m_1' \; \mathcal{R} \; m_2'$.*

At last, the operator of *fusion by transitions* of two nets is used as a tool in the description of our method. The fusion by transitions merges two Reset Petri nets N_1 and N_2 into a new one N_3 over a couple of transitions, let us say t_1 and t_2, as illustrated in figure 1. Fusion is denoted by $(N_1 \underset{(t_1,t_2)}{||} N_2)$; it may be easily extended to a set a couples of transitions and to labeled nets.

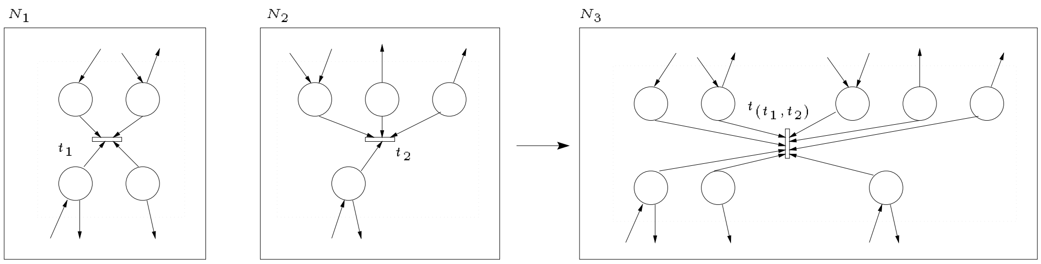

Figure 1: Fusion of two Reset Petri nets over t_1 and t_2 : $(N_1 \underset{(t_1,t_2)}{||} N_2) = N_3$

3 A $O(|N|^2)$ algorithm for normalization

For sake of simplicity, we first build a net admitting 0, 1 and 2 as valuations. The main idea is to use the binary encoding of an integer v which requires $O(\log v)$ bits. Valuations found in the net to be normalized are translated into their binary coding and a constant number of places, transitions and/or arcs is associated to each significant bit.

3.1 From v-valuations to 2-valuations

Definition 4 *Let $N =< P, T, F, m_0 >$ be a Reset Petri net. $K(p)$ is the length of the maximum of valuations relative to a place p : $K(p) = \lfloor V(p) \rfloor + 1$. The function $B_k(n)$, for $k \in \mathbb{N}$, takes an integer n*

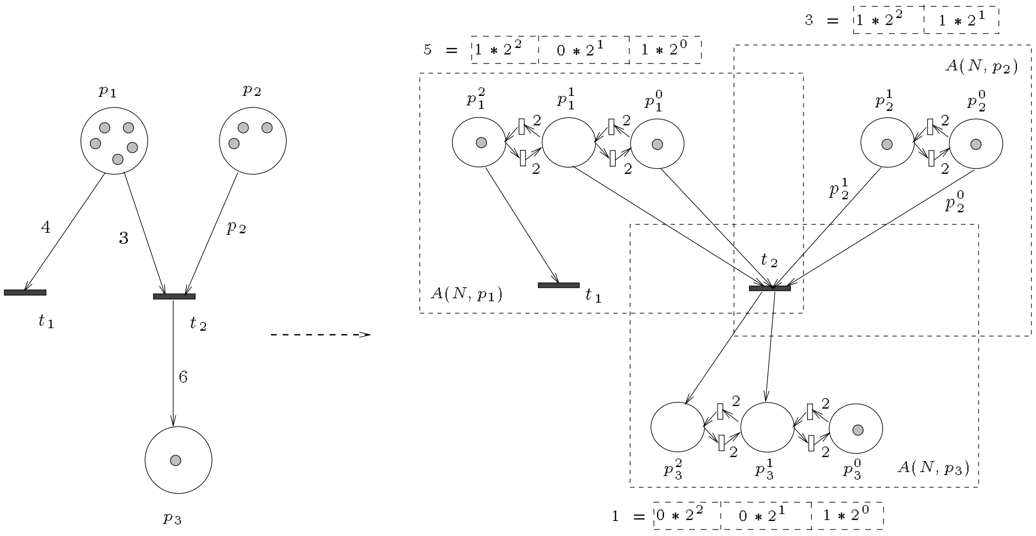

Figure 2: from v-valuations to 2-valuations

such that ($\lfloor \log n \rfloor \leq k-1$) in input and returns a vector of bits of dimension k which is the binary coding of n, with leading 0 if needed.

We index the vector output of $B_k(n)$ from right to left starting from 0 (which means from $k-1$ downto 0). For instance, $23 = 10111_2$ and thus $B_7(23) = (0,0,1,0,1,1,1) = B$ with $B(0) = 1$, ..., $B(6) = 0$.

Informal description of the construction (see figure 2). In input, we have a Reset Petri net $N =< P, T, F, m_0 >$. Note that the arc from place p_2 to transition t_2 is a Reset arc, labeled with p_2. In output, we obtain a 2-valuated labeled net $N' =< P', T', F', m_0', T, l' >$ with $l' : T'^* \to T^*$.

1. **Building of nets associated to places of** N **:** we associate a net $A(N, p_j)$ ("A" stands for "associated net") to each place $p_j \in P$. In our example, associated nets are framed with dashed lines. An associated net $A(N, p_j)$ has $K(p_j)$ places, $2.(K(p_j) - 1)$ local λ-transitions (unlabeled on the figure), $|{}^\bullet p_j| + |p_j^\bullet|$ non λ-transitions and at most $(|{}^\bullet p_j| + |p_j^\bullet|).K(p_j)$ arcs.

   (a) $K(p_j)$ is calculated in first. For instance, $K(p_1) = 3$ since $Max(p_1) = \max(m_0(p_1), F(p_1, t_1), F(p_1, t_2)) = \max(5, 4, 3) = 5$ and $\lfloor \log 5 \rfloor + 1 = 3$.

   (b) **Places** of $A(N, p_j)$ are labeled from p_j^0 to $p_j^{K(p)-1}$ (and drawn from right to left).

   (c) **Initial marking** of $A(N, p_j)$ is $B_{K(p_j)}(m_0(p_j))$. We read a marking from $p_j^{(K(p_j)-1)}$ downto p_j^0. For instance, the initial marking of $A(N, p_3)$ is $(0, 0, 1)$ since $m_0(p_3) = 1$, $K(p_3) = 3$ and $B_3(1) = (0, 0, 1)$.

   (d) **λ-transitions** of $A(N, p_j)$ are of two kinds. The *spreading transitions*, remove two tokens from a place p_j^i and adds one token into the place p_j^{i+1} at its left ($0 \leq i \leq k(p_j) - 2$). The *grouping transitions*, perform the reverse operation (by default, unlabeled arcs of the figure are labeled with 1 by F).

   (e) **Non λ-transitions** of $A(N, p_j)$ correspond to original transitions of N and have the same name via l'. For each transition t that resets p_j, we put in the 2-valuated net $K(p_j)$ arcs $< p_j^i, t >$ labeled with p_j^i ($0 \leq i \leq K(p_j) - 1$). For instance $F(p_2, t_2)$ gives two arcs in $A(N, p_2)$: $< p_2^1, t_2 >$ that resets P_2^1 and $< p_2^0, t_2 >$ that resets p_2^0.
   For each transition t in input of a place $p \in P$ such that $F(p_j, t) \in \mathbb{N}$, we evaluate $B = B_{K(p_j)}(F(t, p_j))$. New arcs $< p_j^i, t >$ valuated with 1 are added in N' if and only if $B(i) = 1$ ($0 \leq i \leq K(p_j) - 1$). For instance $F(t_2, p_3)$ is turned into 2 arcs in N', i.e. $< t_2, p_3^2 >$ and $< t_2, p_3^1 >$, since $B_{K(p_3)}(F(t_2, p_3)) = B_3(6) = (1, 1, 0)$.

For each transition t output of p_j in N, we evaluate $B = B_{K(p_j)}(F(p_j,t))$. New arcs $< t, p_j^i >$ valuated with 1 are added in N' if and only if $B(i) = 1$ $(0 \leq i \leq K(p_j) - 1)$. For instance, $F(p_1,t_2)$ is turned into 2 arcs in N' i.e. $< p_1^0, t_2 >$ and $< p_1^1, t_2 >$ since $B_{K(p_1)}(F(p_1,t_2)) = B_3(3) = (0,1,1)$.

2. **Fusion of associated nets:** the final net is obtained by fusion of all the $A(N,p)$ over the visible transitions : $N' = A(N,p_1) \underset{t\in T}{||} A(N,p_2) \underset{t\in T}{||} \dots \underset{t\in T}{||} A(N,p_{|P|})$.

Firing a λ-transition of an $A(N,p)$ net modifies the marking of $A(N,p)$ only. Firing a non λ-transition may modify the whole marking of the normalized net.

There exists a narrow relation between the reachability set of a general net and the reachability set of its associated normalized net. This relation is given by the function φ_k. Let $k \geq 1$, $\varphi_k : \mathbb{N}^k \rightarrow \mathbb{N}$ is defined by : $\varphi_k(m) = \sum_{i=1}^{k} 2^{i-1}.m(i)$ where $m \in \mathbb{N}^k$. There are two major points to underline. Firstly, for any net $A(N,p)$ and any marking m, the value of $\varphi_{K(p)}(m)$ is preserved with the firing of λ-transitions and moreover, any marking m' such as $\varphi_{K(p)}(m') = \varphi_{K(p)}(m)$ is reachable from m. This last characteristic is necessary to well-place the tokens in order to fire a non λ-transition.

Example (figure 2): in N, t_2 is firable since p_1 contains at least 3 tokens (and Reset arcs are not constraining). In N' the transition labeled with t_2 is not immediately firable since p_1^1 has no token. However the grouping transition from p_1^2 to p_1^1 is firable and has for consequence to add 2 tokens in p_1^1, allowing thus the firing of t_2.

Secondly, the function φ_k is such that for all $m, m' \in \mathbb{N}^K$, $\varphi_k(m+m') = \varphi_k(m) + \varphi_k(m')$ and $m \geq m' \Rightarrow \varphi_k(m - m') = \varphi_k(m) - \varphi_k(m')$. This means that coherence between N and N' is preserved when a non λ-transition (associated to a transition of T) is fired in N'.

Let us now work with the whole 2-valuated net. To each marking reachable in N, we can associate a set of markings reachable in N'.

Definition 5 *Let N be a Reset Petri net, N' its 2-valuated net, $m \in \mathbb{N}^{|P|}$ and $m' \in \mathbb{N}^{|P'|}$. The function $Sub(m',p)$ returns the submarking of m' that corresponds to $A(N,p)$. The relation $\mathcal{R}$ is defined as :*

$$m \,\mathcal{R}\, m' \Leftrightarrow \forall p \in P,\ m(p) = \varphi_{K(p)}(Sub(m',p))$$

Example (figure 2): the initial marking of N' is $m'_0 = (1,0,1,1,1,0,0,1)$ with $Sub(m'_0,p_1) = (1,0,1)$, $Sub(m'_0,p_2) = (1,1)$ and $Sub(m'_0,p_3) = (0,0,1)$. We have $(5,3,1) \,\mathcal{R}\, m'_0$, where $(5,3,1) = m_0$, since $\varphi_{K(p_1)}((1,0,1)) = 5$, $\varphi_{K(p_2)}((1,1)) = 3$ and $\varphi_{K(p_3)}((0,0,1)) = 1$. Now, in N we have $(5,3,1) \xrightarrow{t_3} (2,0,7)$. In N' the associated sequence of firing is :

$$(1,0,1,1,1,0,0,1) \xrightarrow{\lambda} (0,2,1,1,1,0,0,1) \xrightarrow{t_3} (0,1,0,0,0,1,1,1)$$

and one may verify that $(2,0,7) \,\mathcal{R}\, (0,1,0,0,0,1,1,1)$.

Theorem 1 *Let N be a Reset Petri net and N' its associated 2-valuated net, N and N' are λ-bisimilar through $\mathcal{R}$.*

The proof is straightforward from the construction of the associated nets. From Theorem 1 and the properties of $\mathcal{R}$, we can state that $L(N) = l'(L(N'))$; N bounded $\Leftrightarrow$ N' bounded; $m \in RS(N,m_0) \Leftrightarrow \exists m' \in RS(N',m'_0)$ such that $\mathcal{R}(m,m')$; t live in $N \Leftrightarrow t$ live in N'.

3.2 From 2-valuations to normalization and complexity

Figure 3 gives an informal presentation of the final step which leads to the normalization preserving the properties given above. The complexity of the normalization is computed as follows : let

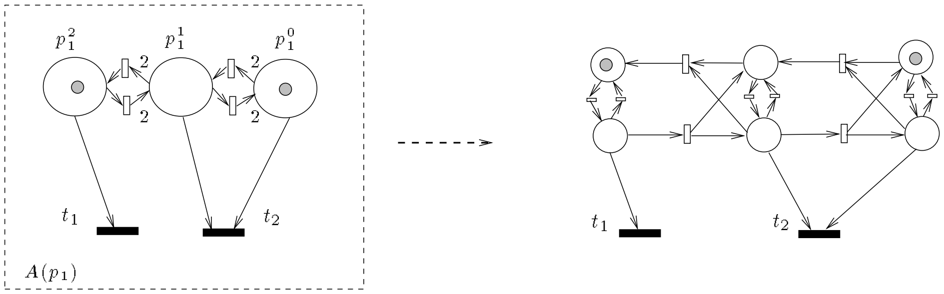

Figure 3: from 2-valuations to the normalized net

$N'' =< P'', T'', F'', m_0'' >$ be the final normalized net obtained from N. The size of the original net is in $\Theta(|P|.|T|.\log V)$. For each $p \in P$, $A(N,p)$ contains $2.K(p)$ places and less than $4.K(p)$ λ-transitions. Then we have $|P''| \in O(|P|.\log V)$ and $|T''| \in O(|P|.\log V + |T|)$. As F'' is normalized, its size is in $O(|P''|.|T''|)$ leading to a final size for N'' in $O(|N|^2)$. Now, for didactic reasons, we have chosen to present the algorithm via the fusion of associated nets, but the construction may be done directly. In this case, the time of construction is linear in $|N''|$ and is thus in $O((|P|.\log V).(|P|.\log V + |T|))$.

Remark 2 *Suppose that in the original net we have $O(|P|) = O(|T|/\log V)$, then for some constant C_1 the size of N'' is less than : $C_1.(|P|.\log V).(|T| + |T|) \in O(|N|)$. Suppose now that $O(|P|) = O(|T|)$ then $|N''| \in \log V.|N|$.*

Theorem 2

- *Let N be a Reset petri net and N'' its associated normalized Reset Petri net, N and N'' are λ-bisimilar.*

- *The time-complexity of the normalization of a Reset Petri net N is $O(|N|^2)$.*

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