Structural Analysis of Workflow Nets with Shared Resources

Kamel Barkaoui[1] and Laure Petrucci[2]

[1] CEDRIC-CNAM
292 rue St-Martin
F-75141 PARIS Cedex 03
barkaoui@cnam.fr

[2] CEDRIC-IIE
18, allée Jean Rostand
F-91025 EVRY Cedex
petrucci@iie.cnam.fr

Abstract. A *workflow* is the automation of *business processes* which describes activities in a business context. A *workflow management system* defines, creates and manages the execution of workflows. Petri nets have been shown to be a well-suited formalism to model and analyse business processes. The verification of *soundness* of a procedure, i.e. correct termination, using Petri nets, was considered in [vdA97]. In this paper, we extend this work by also taking into account resources shared by workflow procedures. We show how the soundness property can be proved efficiently by using structural Petri net techniques. These allow us to obtain parameterized results. Moreover, it leads to a tailoring of the system in order to enhance its performances.

1 Introduction

Workflow Management and *Business Process Reengineering* consider administrative tasks in large organisations, which interact and compete for shared resources. The key entity in such systems is the *business process* ([WFM96]). It consists in a set of linked activities which collectively realise a business objective or policy goal, normally within the context of an organisational structure defining functional roles and relationships. A single enactment of a process is a *case*. A *workflow* is the automation of a business process, in whole or part, during which documents, information or tasks are passed from one participant to another for action, according to a set of procedural rules. A *Worflow Management System* (WFMS for short) is a system that defines, creates and manages the execution of workflows through the use of software which is able to understand the process definition, interacts with workflow participants. It is possible to distinguish two different categories of workflow software which support respectively structured and unstructured processes. *Structured processes* have a fixed behaviour, and they will not change during the time. On the contrary, *unstructured processes* are susceptible to be influenced by external actions.

Numerous WFMS are nowadays available. Unfortunately, there are few theoretical formalisms and tools to deal with these. The need to model and analyse the correctnesss of workflow procedures has lead to use Petri nets as a formalism. They are a well-suited formalism allowing to capture the business processes concepts and structure, i.e. splitting, synchronisation, conflicts, parallel and sequential routing, conditions, ... Moreover, the representation of business processes in terms of Petri nets supports automated manipulation and formal verification techniques. In particular, this last point is extremely important as the numerous WFMS nowadays available ([BPR97], [OSS$^+$97], [Obe94]) generally offer only a simulation based partial verification. A significant work tackling the verification of workflow procedures using Petri nets theory was presented in [vdA97]. The soundness of a procedure, i.e. the correct termination (absence of dangling cases, deadlocks, livelocks, ...), can be checked in polynomial time under some conditions on the flow relation of the net model.

The work we present in this paper significantly extends these results. Indeed, we first take into account the general use of resources shared by workflow procedures. Second, we show how the soundness property, under this resource use constraint, can be proved efficiently by using structural Petri net techniques. The major advantage in using such techniques is to obtain parameterized results. Effectively, we charaterize families of sound models, where the number of cases and resources are considered as parameters. Another benefit of this approach is related to the performances of the whole WFMS. This is done by proposing another distribution of resources between tasks. It helps tailoring the workflow model, while preserving the soundness property.

2 Basic Notions of Petri Nets and Structural Analysis

In this section, we introduce the basic notions and notations used throughout this paper. We first define a Petri net.

Definition 1. *A* Petri net *is a tuple* $PN = < P, T, F, W >$ *where :*

(i) $P \neq \emptyset$ *is a set of* places *;*
(ii) $T \neq \emptyset$ *is a set of* transitions *;*
(iii) $F \subseteq P \times T \cup T \times P$ *is the* flow relation *;*
(iv) $W : P \times T \cup T \times P \to \mathbb{N} \wedge [W(x, y) = 0 \Leftrightarrow (x, y) \notin F]$ *is the* weight function.

In the following, we define the marking of a Petri net.

Definition 2. *A* marking *of a Petri net* PN *is a function* $M : P \to \mathbb{N}$*. The* initial marking *of* PN *is denoted by* M_0.

We then introduce the notations for pre-sets, post-sets and the incidence matrix.

Notation 1

$$\forall x \in P \cup T,\ {}^\bullet x = \{y \in P \cup T/\,(y, x) \in F\} \text{ and } x^\bullet = \{y \in P \cup T/\,(x, y) \in F\}$$

$$\forall (p, t) \in P \times T : C(p, t) = W(t, p) - W(p, t).$$

Then, we recall the firing rules.

Definition 3. *A transition $t \in T$ is* enabled *in a marking M (denoted by $M[t\rangle$) iff $\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)$.*

If transition t is enabled in marking M, it can be fired, *leading to a new marking M' such that: $\forall p \in P : M'(p) = M(p,t) + C(p,t)$. The firing is denoted by $M[t\rangle M'$.*

The set of all markings reachable from a marking M is denoted by $[M\rangle$.

We define the classical properties checked for Petri nets.

Definition 4. *Let PN be a Petri net and M_0 its initial marking.*

(i) *a marking M_h is a* home state *iff $\forall M \in [M_0\rangle,\ M_h \in [M_0\rangle$;*
(ii) *(PN, M_0) is* reversible $\Leftrightarrow$ *M_0 is a home state ;*
(iii) *(PN, M_0) is* bounded $\Leftrightarrow \forall p \in P : [\exists k \in \mathbb{N} : \forall M \in [M_0\rangle,\ M(p) \leq k]$
$\Leftrightarrow [M_0\rangle$ *is finite ;*
(iv) *(PN, M_0) is* quasi-live $\Leftrightarrow \forall t \in T : \exists M \in [M_0\rangle,\ M[t\rangle$ *;*
(v) *(PN, M_0) is* deadlock-free $\Leftrightarrow \forall M \in [M_0\rangle : \exists t \in T,\ M[t\rangle$ *;*
(vi) *(PN, M_0) is* live $\Leftrightarrow \forall t \in T : [\forall M \in [M_0\rangle : \exists M' \in [M\rangle,\ M'[t\rangle]$ *;*
(vii) *PN is* structurally live $\Leftrightarrow [\exists M_0,\ (PN, M_0)$ *is live*$]$.

Definition 5. *A function $\nu : [M_0\rangle \to \mathbb{N}$ is a* norm *(strict) for a marking $M_h \in [M_0\rangle$ iff:*

(i) $\nu(M) = 0 \Leftrightarrow M = M_h$ *;*
(ii) $\forall M \in [M_0\rangle : [\nu(M) > 0 \Leftrightarrow \exists t \in T : M[t\rangle M' \wedge \nu(M') < \nu(M)]$.

In this paper, we use techniques from structure theory of Petri nets. Therefore, we introduce the basic notion of invariants.

Definition 6. *Let PN be a Petri net. An integer vector $f, f \neq 0$, indexed by P ($f \in \mathbb{Z}^P$) is a* place invariant *iff it satisfies ${}^t f \cdot C = 0$.*
The positive support *of f is the set of places $\|f\|^+ = \{p \in P : f(p) > 0\}$.*
The negative support *of f is the set of places $\|f\|^- = \{p \in P : f(p) < 0\}$.*
PN is conservative $\Leftrightarrow \exists f,\ f$ p-invariant, $\|f\|^+ = P$
$\Rightarrow \forall M_0,\ N$ *is bounded.*

A key concept in structural analysis is the siphon.

Definition 7. *Let PN be a Petri net and $S \subseteq P,\ S \neq \emptyset$. S is a* siphon *iff ${}^{\bullet}S \subseteq S^{\bullet}$. S is* minimal *iff it contains no other siphon as a proper subset.*

Now, we introduce the notion of controlled siphon.

Definition 8. *Let (PN, M_0) be a Petri net, and S a siphon of PN.*

(i) *S is* controlled *iff $\forall M \in [M_0\rangle, \exists p \in S : M(p) \geq \max_{p^{\bullet}} = \max_{t \in p^{\bullet}} W(p,t)$;*
(ii) *(PN, M_0) satisfies the* controlled-siphon property *(cs-property) iff each minimal siphon of PN is controlled.*

Two basic relations between liveness properties and the cs-property are stated in the following proposition.

Proposition 1. *Let (PN, M_0) be a Petri net. The following properties hold:*

(i) (PN, M_0) *live* $\Rightarrow (PN, M_0)$ *satisfies the cs-property ;*
(ii) (PN, M_0) *satisfies the cs-property* $\Rightarrow (PN, M_0)$ *is deadlock-free ;*

Two other properties useful to liveness analysis are recalled below.

Proposition 2. *Let (PN, M_0) be a Petri net and $M_h \in [M_0\rangle$. Then:*

(i) (PN, M_0) *is quasi-live under home state* $M_h \Rightarrow (PN, M_0)$ *is live under* M_h
(ii) M_h *is a home state* $\Leftrightarrow \exists$ *a norm for* M_h.

3 WorkFlow Nets

Van der Aalst showed in [vdA97] that Petri nets modelling business processes generally satisfy some typical properties. They always have two special places i and o, which correspond to the beginning and termination of the processing of a case. They are respectively source and sink places.

Definition 9. *A Petri net PN is a* WF-net *iff:*

(i) *PN has two special places: i and o. Place i is a source place: ${}^\bullet i = \emptyset$. Place o is a sink place: $o^\bullet = \emptyset$.*
(ii) *If we add a transition t^* to PN, connecting place o with i, i.e. ${}^\bullet t^* = \{o\}$ and $t^{*\bullet} = \{i\}$, the Petri net $\overline{PN}$ obtained is strongly connected. $\overline{PN}$ is called the* augmented net *of PN.*

A key property of workflow procedures is the *soundness property.* It states that, for any case, the procedure will terminate eventually, and at the moment the procedure terminates, there is a token in place o and all other places are empty. This is a slight extension of the soundness definition given in [vdA97] which considers only one case.

Definition 10. *A WF-net $(PN, n.i)$, n being the number of cases to process, is* sound *iff:*

(i) $\forall M \in [n.i\rangle,\ n.o \in [M\rangle$ *;*
(ii) $\forall M \in [n.i\rangle : M(o) \geq n \Rightarrow M = n.o$ *;*
(iii) $\forall t \in T, \exists M \in [n.i\rangle : M[t\rangle$.

It was proved in [vdA97] that the soundness of a WF-net (PN, i) is equivalent to the liveness plus boundedness of the augmented net $(\overline{PN}, i)$. One can easily extend this property to n cases.

Proposition 3. *A WF-net $(PN, n.i)$ is sound iff $(\overline{PN}, n.i)$ is live and bounded, with $W(o, t^*) = W(t^*, o) = n$.*

Proof. The proof is similar to the one in [vdA97].

$\Leftarrow$: Let us suppose that $(\overline{PN}, n.i)$ is live and bounded. As $(\overline{PN}, n.i)$ is live, $\forall M \in [n.i\rangle : \exists M' \in [M\rangle,\ M'[t^*\rangle$.

Thus, $\forall M \in [n.i\rangle : \exists M' \in [M\rangle,\ M'(o) \geq n$.

Let us suppose that $M' = n.o + M'',\ M'' \neq 0$. Then $M'[t^*\rangle n.i + M''$, which contradicts the boundedness hypothesis. Therefore $M' = n.o$ and conditions (i), (ii) of definition 10 are ensured. Condition (iii) is guaranteed by liveness.

$\Rightarrow$: Let us assume that $(PN, n.i)$ is sound.

We first prove that $(\overline{PN}, n.i)$ is bounded. Suppose that $(PN, n.i)$ is not bounded. Then $\exists M_1 \in [n.i\rangle : \exists M_2 \in [M_1\rangle,\ M_2 > M_1$.

As $(PN, n.i)$ is sound, we know, by definition 10.(i) that $\exists \sigma \in T^* : M_1[\sigma\rangle n.o$. Thus, $\exists M,\ M_2[\sigma\rangle M : M > n.o$. This contradicts the soundness hypothesis (definition 10.(ii)). Thus $(PN, n.i)$ is bounded and therefore $(\overline{PN}, n.i)$ is bounded.

We now prove that $(\overline{PN}, n.i)$ is live. As $(PN, n.i)$ is sound, from definition 10.(i) $\forall M \in [n.i\rangle : n.o \in [M\rangle$. Then, by firing t^*, we obtain: $\forall M \in [n.i\rangle : n.i \in [M\rangle$, i.e. $n.i$ is a home state of $(\overline{PN}, n.i)$. Using definition 10.(iii), we conclude that all transitions are quasi-live and thus, by proposition 2.(i) that $(\overline{PN}, n.i)$ is live. $\diamond$

For a Free-Choice WF-net, proposition 3 can be checked in polynomial time using algorithms based either on the rank theorem ([KB92]) or on Commoner's property ([BM92]). This is due to the fact that checking soundness for one case is equivalent to checking it for any number of cases, as liveness is monotonic for Free Choice nets, contrary to the general case. However, a structural necessary and sufficient liveness condition for Asymmetric Choice nets was presented in [BP96]. This condition, namely *cs-property*, generalizes Commoner's property.

In [vdA97], it is said that most (all but one) of the existing WFMS use constructs corresponding to free-choice nets. In our opinion, non free-choice synchronization patterns can be encountered in practice.

4 Structural Soundness of WF-Nets

Our aim is to consider the number of cases as a parameter, and to monitor the number of cases which can be processed simultaneously, such that the soundness property is satisfied. As in our model liveness is not monotonic, the techniques used for Free Choice nets are not appropriate. Therefore, we introduce the notion of structural soundness.

Definition 11. *A WF-net PN is* structurally sound *iff $\exists n$ such that $(PN, n.i)$ is sound.*

In order to prove structural soundness of a model, we will use the cs-property. This technique will also allow us, when adding resources, to enhance their use.

In the general case, the cs-property guarantees the absence of deadlock apart from the final marking $M = n.o$. But the soundness is not ensured. In the particular case of Asymmetric Choice WF-nets (including Free Choice WF-nets), the soundness is verified iff the augmented net is bounded and satisfies the cs-property. Moreover, it is possible to characterise a set of initial markings for which soundness is guaranteed.

Now, we will characterise a more general class, abstracting from circuits which can be performed by a case. In fact, WF-nets with circuits can be changed into WF-nets without circuits, using transformations presented in [vdA97]. From the soundness property, one can easily see that $n.o$ is a home state of $(PN, n.i)$. Using property 2.(ii), there exists a norm for $n.o$. In the particular case of circuit-free nets, this norm can easily be built, as shown in the proof of theorem 1, where the soundness property is structurally characterized.

Definition 12. *A net PN is a* Circuit Free WF-net *(CFWF) iff PN is a WF-net and has no circuit.*

For a bounded and quasi-live CFWF, the cs-property is a necessary and sufficient soundness condition. One can consider easily that boundedness and quasi-liveness properties are minimal requirements.

Theorem 1. *Let PN be a CFWF. PN is structurally sound iff $\exists n$ such that $(\overline{PN}, n.i)$ is bounded, quasi-live and satisfies the cs-property.*

Proof. $\Rightarrow$: Let us suppose that PN is structurally sound. This means (definition 11) that $\exists n$ such that $(PN, n.i)$ is sound. From proposition 3, $(\overline{PN}, n.i)$ is live and bounded, in particular bounded and quasi-live. Moreover, from proposition 1.(i), it satisfies the cs-property.

$\Leftarrow$: Let us suppose that $\exists n$ such that $(\overline{PN}, n.i)$ is bounded, quasi-live and satisfies the cs-property. We will first exhibit a norm ν for marking $n.o$. We construct function ν as follows: we number the places in reverse topological order, i.e. place o is numbered 0, place i has the highest number, and the other places are such that a successor p' of a place p in the graph of the Petri net has a lower number than place p. This can be done due to the absence of cycles. We call num this numbering function. Then we define $\forall M : \nu(M) = \sum_{p \in P} M(p)num(p)$.

We now prove that ν is a norm for $n.o$. By construction, $\nu(M) = 0 \Leftrightarrow M = x.o$. If $x < n$, then $\nexists t \in T : M[t\rangle$. As $(\overline{PN}, n.i)$ satisfies the cs-property, we deduce from proposition 1.(ii) that $(\overline{PN}, n.i)$ is deadlock-free, i.e. $\forall M \in [n.i\rangle :$ $\exists t \in T,\ M[t\rangle$. Thus, there is a contradiction. If $x > n$, $x.o[t^*\rangle M' > n.i$. This contradicts the boundedness hypothesis. Thus, we have proved condition (i) of definition 5.

Let us suppose that $\nu(M) > 0$ for a marking M. As $(\overline{PN}, n.i)$ is deadlock-free, $\exists t : M[t\rangle M'$. If $t \neq t^*$, by construction of ν, $\nu(M') < \nu(M)$. Otherwise $(t = t^*)$, as $\nu(M) > 0$ and $\nu(M) = 0 \Leftrightarrow M = n.o$ (already proven), marking M must have the form $M = n.o + M''$ with $M'' \neq 0$. Then $M[t^*\rangle n.i + M'' > n.i$, which contradicts the boudedness hypothesis. Thus, $\Rightarrow$ of definition 5.(ii) is satisfied.

Let us now suppose that $\exists t \in T : M[t\rangle M' \wedge \nu(M') < \nu(M)$. The construction of function ν is such that $\forall M : \nu(M) \geq 0$. Then $\nu(M) > \nu(M') \geq 0$. Thus, $\Leftarrow$ of definition 5.(ii) is satisfied.

We deduce from all this that we found a norm function ν for $n.o$.

From proposition 2.(ii), $n.o$ is a home marking. Then, as $n.o[t^*\rangle n.i$, $n.i$ also is. As $(\overline{PN}, n.i)$ is quasi-live and its initial state is a home state, it is live. From proposition 3, $(PN, n.i)$ is sound. $\diamond$

In some subclasses of Petri nets, these last conditions can be lessened.

Definition 13. *A Petri net PN is an* Asymmetric Choice net *iff:*

$$\forall (p, q) \in P \times P : [p^\bullet \cap q^\bullet \neq \emptyset \Rightarrow p^\bullet \subseteq q^\bullet \vee q^\bullet \subseteq p^\bullet].$$

Corollary 1. *Let PN be an Asymmetric Choice CFWF-net. PN is structurally sound iff $\exists n$ such that $(\overline{PN}, n.i)$ is bounded and satisfies the cs-property.*

Proof. For Asymmetric Choice nets, the cs-property is a necessary and sufficient liveness condition ([BP96]). The result follows from the fact that liveness implies quasi-liveness and from theorem 1. $\diamond$

In the next section, we will add resources to sound nets in order to study the adequation between the number of resources available and the number of cases to be handled.

5 Coping with Shared Resources

In this section, we first introduce the model of WF-nets with resources and combine them into a system where they share resources. Secondly, we show how to perform analysis of structural soundness.

5.1 Modelling Business Processes Competing for Shared Resources

We introduce the notion of WF-net with resources. It is basically a WF-net plus a set of places modelling the resources. We demand the WF-net (without resources) to be sound, and the resources to be preserved by the net, i.e. a resource requested will eventually be released and a resource released has previously been requested. Several resources can be requested/released at a same time. The resource preservation can be expressed by a place invariant of the system (definition 14.(v)). One can note that the subnet associated with this invariant is not necessarily a state machine.

Definition 14. *A* WFR-net *is a tuple $PNR = < P \cup P_R, T, F \cup F_R, W \cup W_R >$ where:*

(i) $PN = < P, T, F, W >$ *is a* structurally sound *WF-net.*
(ii) $P_R \neq \emptyset \wedge P \cap P_R = \emptyset$ *(set of resources)*
(iii) $F_R \subseteq (P_R \times T) \cup (T \times P_R)$ *(flow relation for resources)*
(iv) $\forall u \in F_R, W_R(u) \geq 1$ *(resource use)*
(v) $\forall r \in P_R, \exists f_r \geq 0 : {}^t f_r \cdot C = 0 \;\wedge \|f_r\| \cap P_R = \{r\}$ *(resource preservation)*

Then, we can compose several WFR-nets into a system where they share resources. This is obtained by fusion of the places representing the shared resources.

Definition 15. *A* WFRS *is recursively defined. A WFR-net is a WFRS.*

Let $PN_i = < P_i \cup P_{Ri}, T_i, F_i, W_i >, i \in \{1, 2\}$, be two WFRS such that $P_1 \cap P_2 = T_1 \cap T_2 = \emptyset$. We denote the set of shared resources by $P_{R1R2} = P_{R1} \cap P_{R2}$. The net $PN = PN_1 \circ PN_2$ resulting of the fusion of nets PN_1 and PN_2 over the set P_{R1R2} is a WFRS.

Definition 16. *Let N be a WFRS. We denote by $\overline{N}$ the generalized Petri net constructed as N, where the WF-nets components (definition 14.(i)) are replaced by the corresponding augmented WF-nets.*

One can note that, compared to other classes presented in the literature, WFRS extend S^4R-nets ([BBA96]), which are a generalisation of S^3R-nets ([ECM95]) proposed to cope with deadlocks in flexible manufacturing systems.

5.2 Structural Analysis

The definition of structural soundness (definition 11) can easily be extended to WFRS.

Lemma 1. *Let N be a WFRS, and S a minimal siphon of $\overline{N}$. There exists an initial marking M_0 under which S is controlled.*

Proof. Let N be a WFRS and S a minimal siphon of $\overline{N}$.

Let us first suppose that $S \cap \bigcup P_{Ri} = \emptyset$. By construction, $\exists PN_i : S \subseteq P_i$. As PN_i is structurally sound, there exists an initial marking under which S is controlled.

Let us now consider the complementary case: $S \cap \bigcup P_{Ri} \neq \emptyset$. We suppose that siphon S is not controlled. We denote by $f(r)$ a flow of minimal support associated with a resource r, and by $f(p)$ a flow of minimal support associated with p in its WF-net. Let:

$$g_S = \sum_{r \in S \cap \bigcup P_{Ri}} f(r)$$

$$Out(S) = \|g\| \setminus S$$

$$h_S = \sum_{p \in Out(S)} f(p)$$

$$\lambda_S = \max_{p \in Out(S) \cap \|h_S\|} g(p)$$

$$z_S = g_S - \lambda_S . h_S$$

Siphon S is controlled as soon as:

$${}^t z_S \cdot M_0 > \sum_{p \in S} z_s(p).(\max_{p^\bullet} -1)$$

Therefore, there exists a marking under which S is controlled. ◇

The following property holds for any WFRS.

Lemma 2. *Let N be a structurally sound WFRS. There exists an initial marking M_0 such that $(\overline{N}, M_0)$ satisfies the cs-property.*

Proof. Let N be a structurally sound WFRS. If it does not satisfy the cs-property, it cannot be live (proposition 1.(i)). Thus, it cannot be sound. $\diamond$

In theorem 2, we extend lemma 2 in the particular case where the components PN_i are circuit free.

Theorem 2. *Let N be a WFRS where the PN_i are CFWF. N is structurally sound iff there exists an initial marking M_0^* under which $(\overline{N}, M_0^*)$ is bounded, quasi-live and satisfies the cs-property.*

Proof. $\Leftarrow$: The cs-property is ensured by lemma 2. The boundedness and quasi-liveness are deduced from soundness.

$\Rightarrow$: Let N be a WFRS where the PN_i are CFWF. Let us suppose that there exists an initial marking M_0^* under which $(\overline{N}, M_0^*)$ is bounded, quasi-live and satisfies the cs-property. We will now prove that $(\overline{N}, M_0^*)$ is live. To do that, we proceed as in the proof of theorem 1, i.e. we will exhibit a norm ν_R. This norm ν_R is an extension of norm ν where the resources are numbered 0. The proofs of the properties of a norm are similar to those in theorem 1, taking also into account resource preservation (definition 14.(v)). $\diamond$

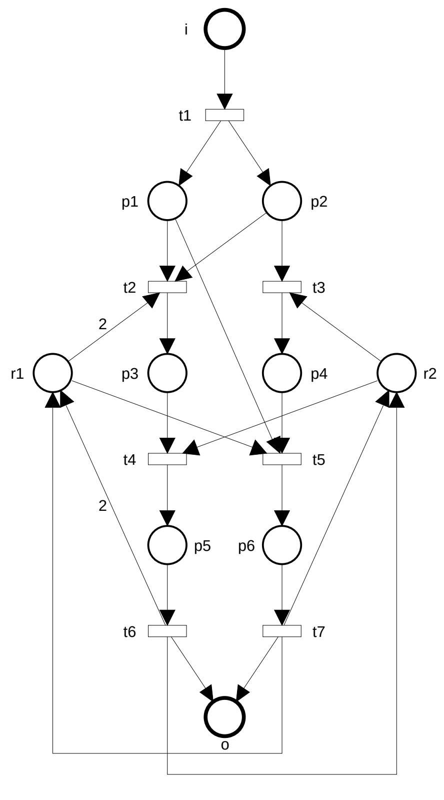

Fig. 1. A Circuit-Free WFRS.

We will now apply theorem 2 to the net of figure 1. The minimal siphons of the augmented net are:

$S_1 = \{i, p1, p3, p5, p6, o\}$
$S_2 = \{i, p2, p3, p4, p5, p6, o\}$
$S_3 = \{p3, p5, p6, r1\}$
$S_4 = \{p4, p5, p6, r2\}$
$S_5 = \{r1, r2, p5, p6\}$
$S_6 = \{i, p1, p5, p6, o, r2\}$

The WF-net composing the WFRS is structurallay sound: it is live and bounded for e.g. $M_0(i) = 1$. Thus, the two siphons without resource places, S_1 and S_2 are controlled as soon as $M_0(i) > 0$. Siphons S_3 and S_4 are the support of positive flows. Hence they are invariant-controlled. The control condition for $S3$ is $M_0(r1) > 1$, and for S_4, $M_0(r2) > 0$. We now have to examine siphons S_5 and S_6 more in detail in order to calculate their control condition.

$$\begin{aligned} g_{S_5} &= f(r1) + f(r2) \\ &= r1 + 2.p3 + 2.p5 + p6 + r2 + p4 + p5 + p6 \\ &= r1 + r2 + 2.p3 + p4 + 3.p5 + 2.p6 \end{aligned}$$

$Out(S_5) = \{p3, p4\}$

$$\begin{aligned} h_{S_5} &= f(p3) + f(p4) \\ &= i + p1 + p3 + p5 + p6 + o + i + p2 + p3 + p4 + p5 + p6 + o \\ &= 2.i + p1 + p2 + 2.p3 + p4 + 2.p5 + 2.p6 + 2.o \end{aligned}$$

$\lambda_{S_5} = 2$

$$\begin{aligned} z_{S_5} &= r1 + r2 + 2.p3 + p4 + 3.p5 + 2.p6 \\ &\quad -4.i - 2.p1 - 2.p2 - 4.p3 - 2.p4 - 4.p5 - 4.p6 - 4.o \\ &= r1 + r2 - 4.i - 2.p1 - 2.p2 - 2.p3 - p4 - p5 - 2.p6 - 4.o \end{aligned}$$

S_5 is controlled as soon as $M_0(r1) + M_0(r2) - 4.M_0(i) > 1$.

$$\begin{aligned} g_{S_6} &= f(r2) \\ &= r2 + p4 + p5 + p6 \end{aligned}$$

$Out(S_6) = \{p4\}$

$$\begin{aligned} h_{S_6} &= f(p4) \\ &= i + p2 + p3 + p4 + p5 + p6 + o \end{aligned}$$

$\lambda_{S_6} = 1$

$$\begin{aligned} z_{S_6} &= r2 + p4 + p5 + p6 - i - p2 - p3 - p4 - p5 - p6 - o \\ &= r2 - i - p2 - p3 - o \end{aligned}$$

S_6 is controlled as soon as $M_0(r2) - M_0(i) > 0$.

To conclude, the net of figure 1 is controlled as soon as the following inequalities are satisfied:

$$M_0(r1) > 0,\ M_0(r2) > 0,\ M_0(i) > 0,$$

$$M_0(r1) + M_0(r2) - 4.M_0(i) > 1,\ M_0(r2) - M_0(i) > 0$$

In the next section, we show how to enhance the performances of the system by allowing more cases to enter the system and still preserve the structural soundness property.

6 Enhancing Performances of WFRS

The control we have up to now is global, and a thinner control can only improve the concurrency, i.e. increase the number of concurrent cases in the system. The basic idea is to dissociate the control of the siphons from the input places of the WF-nets constituting the WFRS. In practice, these places can be considered as part of the environment.

The set of minimal siphons of a given WFRS can be partitionned into 3 classes. The first class (type 1) contains the minimal siphons without resource places. They are controlled since the WF-nets constituting the WFRS are sound. The second class (type 2) contains those which include resources and are invariant-controlled in the sense of [BP96]. The last class (type 3) contains the minimal siphons including resource places but not invariant-controlled.

We associate, with each siphon S of type 3, a local control place C_S with:

$$C_S{}^\bullet = {}^\bullet Out(S),\ {}^\bullet C_S = Out(S)^\bullet$$

$$\forall p \in Out(S),\ \forall t \in {}^\bullet p,\ \forall t' \in p^\bullet : W(C_S, t) = W(t', C_S) = g(p)$$

One can easily avoid self-loops introduced by the flow relation restricted to C_S, since this operation preserves the invariant and thus the future control. Adding place C_S has created a new flow:

$$f(C_S) = C_S + \sum_{p \in Out(S)} g(p).p$$

Let $z_{C_S} = g_S - f(C_S)$. For siphon S to be controlled, we must have:

$${}^t z_{C_S} \cdot M_0 > \sum_{p \in S} z_{C_S}(p).(\max_{p^\bullet} -1)$$

New control places behave like resources, i.e. they satisfy the resource preservation condition of definition 14.(v). Hence the net with these new control places is a WFRS.

Let us now consider the simple example in figure 2 without the grey part (place C). The initial marking is parameterized by $M_0(i)$, $M_0(r1)$ and $M_0(r2)$. The initial marking is $M_0 = M_0(i).i + M_0(r1).r1 + M_0(r2).r2$. Our example presents one minimal siphon of each of the 3 types:

$S_1 = \{i, p1, p2, p3, p4, p5, p6, o\}$ (type 1)
$S_2 = \{r2, p2, p3, p5\}$ (type 2)
$S_3 = \{r1, p2, p3, p4, p5, p6\}$ (type 3)

Siphons S_1 and S_2 are the support of positive flows. S_1 is controlled as soon as $M_0(i) > 0$, S_2 is controlled for $M_0(r2) > 0$.

We now consider siphon S of type 3 for which the control is not guaranteed. We use the same notations for flows associated with places and for the calculus of the cs-property as in the proof of lemma 1. For S_3 in our example:

$g_{S_3} = f(r1)$
$\quad = r1 + 2.p1 + 4.p2 + 3.p3 + 5.p4 + 3.p5 + 3.p6$

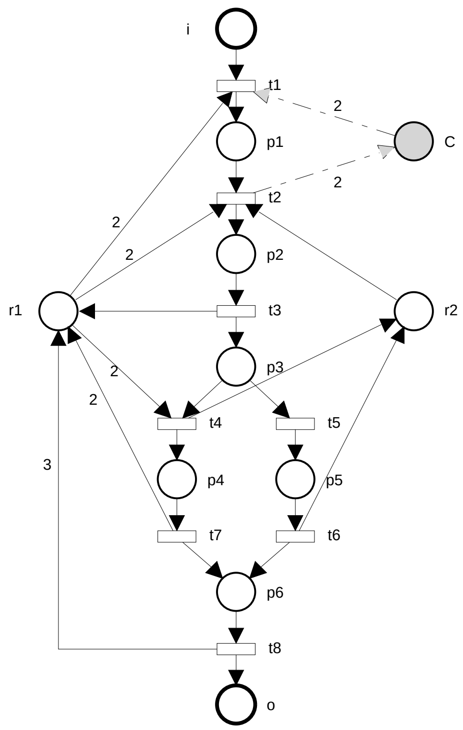

Fig. 2. A Free Choice Circuit-Free WFRS.

$Out(S_3) = \{p1\}$
$h_{S_3} = f(p1)$
$\quad = i + p1 + p2 + p3 + p4 + p5 + p6 + o$
$\lambda_{S_3} = 2$
$z_{S_3} = r1 + 2.p1 + 4.p2 + 3.p3 + 5.p4 + 3.p5 + 3.p6$
$\quad -2.i - 2.p1 - 2.p2 - 2.p3 - 2.p4 - 2.p5 - 2.p6 - 2.o$
$\quad = r1 - 2.i + 2.p2 + p3 + 3.p4 + p5 + p6 - 2.o$

Thus, S_3 is controlled as soon as $M_0(r1) - 2.M_0(i) > 1$. We conclude, using theorem 2, that this net is structurally sound for any initial marking $M_0 = M_0(i).i + M_0(r1).r1 + M_0(r2).r2$ such that:

$$M_0(i) > 0,\ M_0(r2) > 0,\ M_0(r1) > 2.M_0(i) + 1$$

Hence, for example if $M_0(r1) = 7$, $M_0(r2) = 1$ then $M_0(i) \leq 2$, i.e. at most 2 cases can be simultaneously processed.

We now want to enhance the performances of the example of figure 2. S_3 is the only siphon of type 3. Thus, we will control it more locally. Since $Out(S_3) = \{p1\}$, the associated local control place C (see figure 2) satisfies: $C^\bullet = \{t1\}$, ${}^\bullet C = \{t2\}$, $W(C, t1) = W(t2, C) = 2$. We have: $f(C) = C + 2.p1$. Then:

$z_C = g_{S_3} - f(C)$
$\quad = r1 + 4.p2 + 3.p3 + 5.p4 + 3.p5 + 3.p6 - C$

Siphon S_3 is controlled for any marking satisfying $M_0(r1) - M_0(C) > 1$.

For example, if $M_0(r1) = 7$, $M_0(r2) = 1$, we must have $M_0(C) \leq 5$. The introduction of place C adds only one siphon $S_4 = \{C, p1\}$, which is of type 2, and thus controlled when $M_0(C) > 1$. The new WFRS is structurally sound for the initial marking we have exhibited. Without place C, we could only handle 2 cases at a time. Now, we can process 3 cases simultaneously. Thus the throughput, resources use and parallelism are better with a local control than with a global one. One could object that this new place can add siphons of type 3. If there are new uncontrolled siphons, they can be handled in the same manner, either locally or globally. It was proved in [Bar97] that this iterative process necessarily stops: we eventually reach a step where the role of the control place to be added can be played by an already existing one.

7 Conclusion

Many researchers have investigated properties related to the soundness property. In this work, we have shown that, for an important subclass of WF-nets sharing resources, called WFRS, the soundness property can be structurally characterized. The technique presented has a great advantage compared to other approaches based on the computation of the reachability set. Indeed, if the initial marking, i.e. the number of cases involved and resources availability, is modified, the soundness checking requires only to compute the initial markings of the control places.

We are currently applying our method in the workflow of an hospital ([Car97]) in order to enhance the performances of an operating theatre block by performing reengineering. This can be achieved by introducing a balance between specialisation and generalisation or between centralisation and decentralisation of resource classes (human and material) of the system.

References

[Bar97] K. Barkaoui. Control systems design for automated manufacturing systems. Technical report, CEDRIC, 1997.

[BBA96] K. Barkaoui and I. Ben Abdallah. Analysis of a resource allocation problem in FMS using structure theory of Petri nets. In *Proceedings of the* 1^{st} *International Workshop on FMS and Petri Nets*, Osaka, June 1996.

[BM92] K. Barkaoui and M. Minoux. A polynomial-time graph algorithm to decide liveness of some basic classes of bounded Petri nets. *LNCS*, 616, June 1992.

[BP96] K. Barkaoui and J-F. Pradat Peyre. On liveness and controlled siphons in Petri nets. *LNCS*, 1091, June 1996.

[BPR97] *BPR tools description.* http://www-a5.igd.fhg.de/reuse-m/bpr-description.html, 1997.

[Car97] P. Carvalho. Modelling, by queuing network, of hospital operating theatre block. Mémoire d'ingénieur CNAM, March 1997.

[ECM95] J. Ezpeleta, J.M. Colom, and J. Martinez. A Petri net based deadlock prevention policy for flexible manufacturing systems. *IEEE Transactions on Robotics and Automation*, 11(2), April 1995.

[KB92] P. Kemper and F. Bause. An efficient polynomial-time algorithm to decide liveness and boundedness of free-choice nets. *LNCS*, 616, 1992.

[Obe94] A. Oberweis. Workflow management in software engineering. In *Proceedings of the* 2^{nd} *International Conference on Concurrent Engineering and Electronic Design Automation*, Bournemouth, April 1994.

[OSS+97] A. Oberweis, R. Schätzle, W. Stucky, W. Weitz, and G. Zimmermann. INCOME/WF - A Petri net based approach to workflow management. In *Wirtscheftsinformatik'97*. Springer-Verlag, 1997.

[vdA97] W. M. P. van der Aalst. Verification of workflow nets. *LNCS*, 1248, June 1997.

[WFM96] *Workflow Management Coalition Specification: terminology and glossary.* http://www.aiai.ed.ac.uk/WfMC/DOCS/glossary/glossary.html, 1996.