Weighted O-Minimal Hybrid Systems Are More Decidable Than Weighted Timed Automata!*

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Abstract. We consider weighted o-minimal hybrid systems, which extend classical o-minimal hybrid systems with cost functions. These cost functions are "observer variables" which increase while the system evolves but do not constrain the behaviour of the system. In this paper, we prove two main results: (*i*) optimal o-minimal hybrid games are decidable; (*ii*) the model-checking of WCTL, an extension of CTL which can constrain the cost variables, is decidable over that model. This has to be compared with the same problems in the framework of timed automata where both problems are undecidable in general, while they are decidable for the restricted class of one-clock timed automata.

1 Introduction

O-minimal hybrid systems. Hybrid systems are finite-state machines where each state is equipped with a continuous dynamics. In the last thirty years, formal verification of such systems has become a very active field of research in computer science. In this context, hybrid automata, an extension of timed automata [AD94], have been intensively studied [Hen95,Hen96], and decidable subclasses of hybrid systems have been identified like initialized rectangular hybrid automata [Hen96] or o-minimal hybrid automata. This latter model has been pointed out in [LPS00] as an interesting class of systems with very rich continuous dynamics, but limited discrete steps (at each discrete step, all variables have to be reset, independently from their initial values). Behaviours of such a system can be decoupled into continuous and discrete parts, properties of a global o-minimal system can thus be deduced directly from properties of the continuous parts of the system. This property and properties of o-minimal structures (see [vdD98] for an overview) are exploited in the word encoding techniques, which have been developed in [BMRT04] for (finitely) abstracting behaviours of the system. Using techniques based on this abstraction, reachability properties [BM05], reachability control properties [BBC06] have been proved decidable for o-minimal hybrid systems. This technique was also used in order to compute a (tight) exponential bound on the size of the coarsest finite bisimulation of *Pfaffian hybrid systems* (see [KV06]).

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Models for resource consumption. A research direction which has recently received substantial attention is the twist or extension of (decidable) models for representing more fairly interesting properties of embedded systems, for instance resource consumption. In that context, timed automata [AD94] have been extended with cost information bringing the model of weighted (or priced) timed automata [ALP01,BFH$^+$01]. A timed automaton is a finite automaton with clock variables (*i.e.* variables which increase as global time) that can be tested towards constants or reset. In the model of weighted timed automata, an extra cost variable is added, which is used as an *observer* variable (it does not constrain the behaviour of the system), evolving linearly while time elapses, and subject to discrete jumps when discrete transitions are taken. This model was appealing for expressing quantitative properties of real-time systems, which was concretized by the decidability of the optimal reachability problem (find the best way — in terms of cost — of reaching a given state) [ALP01,BFH$^+$01,BBBR06] together with the development of the tool Uppaal Cora [cor06], and then by the computability of the optimal mean-cost (find the best way for the system to have a "cost per time unit" as low as possible) [BBL04]. However, more involved properties like cost-optimal reachability control (find the minimum cost that can be ensured for reaching a given state, whatever does the environment in which the system is embedded) or WCTL model-checking (WCTL extends the branching-time temporal logic CTL with cost constraints on modalities [BBR04,BBR06]) have been proved undecidable for weighted timed automata with three clocks or more, see [BBR04,BBR05,BBM06]. Though both problems have recently been proved decidable for one-clock weighted timed automata [BLMR06,BLM07] these undecidability results are nevertheless disappointing, because the one-clock assumption is rather restrictive.

Our contributions. In this paper, we propose a natural extension of o-minimal hybrid systems with (definable) positive cost functions which increase while time progresses and which can be used in an optimization criterion, as in the case of weighted timed automata. It is worth noticing here that though the underlying system is o-minimal, this extended model, called *weighted o-minimal hybrid automaton*, is not o-minimal as we do not require that the cost is reset when a discrete transition is taken. However, we prove in this paper that the cost-optimal reachability control problem and the WCTL model-checking problem are both decidable for this class of systems. Because of the existing results on weighted timed automata, this is really a surprise, and makes o-minimal hybrid systems an analyzable, though powerful model. The decidability results of course partly rely on the word encoding techniques that we mentioned earlier, but also require refinements and involved techniques, specific to each of the two problems.

2 General Background

Let $\mathcal{M}$ be a structure. In this paper when we say that some relation, subset or function is *definable*, we mean it is first-order definable in the sense of the structure $\mathcal{M}$. A general reference for first-order logic is [Hod97]. We denote by

$\mathsf{Th}(\mathcal{M})$ the theory of $\mathcal{M}$. In the sequel we only consider structures $\mathcal{M}$ that are expansions of ordered groups containing two constant symbols, *i.e.* $\mathcal{M} = \langle M, +, 0, 1, <, \ldots \rangle$.

2.1 O-Minimality

Let us recall the definition of o-minimal structures [PS86] and give some examples of such structures. The reader interested in o-minimality should refer to [vdD98] for further results and an extensive bibliography on this subject. In the sequel of the paper we focus on o-minimal structures with a decidable theory in order to obtain decidability and computability results.

Definition 1. *A totally ordered structure* $\mathcal{M} = \langle M, <, \ldots \rangle$ *is* o-minimal *if every definable subset of* M *is a finite union of points and open intervals (possibly unbounded).*

Example 1. Examples of o-minimal structures are the ordered group of rationals $\langle \mathbb{Q}, <, +, 0, 1 \rangle$, the ordered field of reals $\langle \mathbb{R}, <, +, \cdot, 0, 1 \rangle$, the field of reals with restricted pfaffian functions and the exponential function [Wil96].

2.2 O-Minimal Dynamical Systems

In this subsection we define the notion of *o-minimal dynamical systems*.

Definition 2 (O-minimal dynamical system). *An* o-minimal dynamical system *is a pair* $(\mathcal{M}, \gamma)$ *where:*

- $\mathcal{M} = \langle M, +, 0, 1, <, \ldots \rangle$ *is an o-minimal expansion of an ordered group,*
- $\gamma : V_1 \times V \to V_2$ *is a function definable in* $\mathcal{M}$ *(where* $V_1 \subseteq M^{k_1}$, $V \subseteq M$, *and* $V_2 \subseteq M^{k_2}$*).*[1] *The function* γ *is called the dynamics of the system.*

Classically, when M is the set of the real numbers, we see V as the time, $V_1 \times V$ as the space-time, V_2 as the (output) space and V_1 as the input space. We keep this terminology in the more general context of a structure $\mathcal{M}$.

Example 2. We can view the continuous dynamics of *timed automata* [AD94] as an o-minimal dynamical system. In this case, we have that $\mathcal{M} = \langle \mathbb{R}, <, +, 0, 1 \rangle$ and the dynamics $\gamma : (\mathbb{R}^+)^n \times \mathbb{R}^+ \to (\mathbb{R}^+)^n$ is defined by $\gamma(x_1, ..., x_n, t) = (x_1 + t, ..., x_n + t)$.

We define a transition system associated with the dynamical system, this definition is an adaptation to our context of the classical *continuous transition system* in the case of hybrid systems (see [LPS00] for example).

Definition 3. *Given* $(\mathcal{M}, \gamma)$ *a dynamical system, we define a* transition system $T_\gamma = (Q, \Sigma, \to_\gamma)$ associated with $(\mathcal{M}, \gamma)$ *by: the set* Q *of states is* V_2*; the set* Σ *of events is* $M^+ = \{m \in M \mid m \geq 0\}$*; the transition relation* $y_1 \xrightarrow{\tau}_\gamma y_2$ *is defined by:* $\exists x \in V_1,\ \exists t_1, t_2, \in V \text{ s.t. } t_1 \leq t_2,\ \gamma(x, t_1) = y_1,\ \gamma(x, t_2) = y_2 \ \text{ and } \ \tau = t_2 - t_1$.

[1] We use these notations in the rest of the paper.

2.3 O-Minimal Hybrid Systems and Games

In this subsection, we define o-minimal hybrid systems and games

Definition 4 (O-minimal hybrid systems). *Let* $\mathcal{M} = (M, +, 0, 1, <, \cdots)$ *be an o-minimal structure. An* $\mathcal{M}$-hybrid system *(or simply* o-minimal hybrid system*)* $\mathcal{H}$ *is a tuple* $(Q, \Sigma, \delta, \gamma)$ *where* Q *is a finite set of locations,* Σ *is a finite set of actions,* δ *consists in a finite number of transitions* $(q, g, a, R, q') \in Q \times 2^{V_2} \times \Sigma \times 2^{V_2} \times Q$ *where the* guard g *and the* reset R *are definable in* $\mathcal{M}$*, and* γ *maps every location* $q \in Q$ *to a dynamic* $\gamma_q : V_1 \times V \to V_2$ *definable in* $\mathcal{M}$*.*

An $\mathcal{M}$-hybrid system $\mathcal{H} = (Q, \Sigma, \delta, \gamma)$ defines a *mixed transition system* $T_{\mathcal{H}} = (S, \Gamma, \to)$ where:

- the set S of states is $Q \times V_2$;
- the set Γ of labels is $M^+ \cup \Sigma$;
- the transition relation $(q, y) \xrightarrow{e} (q', y')$ is defined when:
  - $e \in \Sigma$ and there exists $(q, g, e, R, q') \in \delta$ with $y \in g$ and $y' \in R(y)$, or
  - $e \in M^+$, $q = q'$, and $y \xrightarrow{e}_{\gamma_q} y'$ where γ_q is the dynamic in location q.

We will also need more precise notions of transitions. When $(q, y) \xrightarrow{\tau} (q, y')$ with $\tau \in M^+$, this is due to some choice of $(x, t) \in V_1 \times V$ such that $\gamma_q(x, t) = y$. We say that $(q, x, t, y) \xrightarrow{\tau} (q', x', t', y')$ if $q = q'$, $x = x'$, $t' = t + \tau$, $\gamma_q(x, t) = y$ and $\gamma'_q(x', t') = y'$. We say that an action $(\tau, a) \in M^+ \times \Sigma$ is enabled in a state (q, x, t, y) if there exists (q', x', t', y') and (q'', x'', t'', y'') such that $(q, x, t, y) \xrightarrow{\tau} (q', x', t', y') \xrightarrow{a} (q'', x'', t'', y'')$. We then write $(q, x, t, y) \xrightarrow{\tau, a} (q'', x'', t'', y'')$. We note $\mathsf{Enb}(q, x, t, y)$ the set of actions enabled in (q, x, t, y).

A *run* of $\mathcal{H}$ is a(n) (in)finite sequence $\varrho = (q_0, x_0, t_0, y_0) \xrightarrow{\tau_1, a_1} (q_1, x_1, t_1, y_1) \cdots$ A *position* π along ϱ is a pair $(i, \tau) \in \mathbb{N} \times M^+$ such that $\tau \leq \tau_{i+1}$. We define $\varrho[(i, \tau)] = (q_i, \gamma_{q_i}(x_i, t_i + \tau))$. Let us notice that the positions of a given run are totally ordered in a natural way. If ϱ is finite we define $last(\varrho) = (q_n, x_n, t_n, y_n)$. We note $\mathsf{Runs}_f(\mathcal{H})$ the set of finite runs in $\mathcal{H}$.

An interesting tool to study hybrid systems is the *(time-abstract) bisimulation*[2] (see [Hen95]). One of the main results concerning o-minimal hybrid systems is that they admit finite bisimulation quotient. This result has been first proved in [LPS00], it was reproved in [Dav99] in a more topological way, amongst a lot of other interesting results. In [Bri06a], the existence of finite bisimulations for o-minimal hybrid systems is proved by means of the *suffix partition*, a technique initiated in [BMRT04,BM05,Bri06b].

Theorem 5. ([Bri06a, Theorem 12.6.14])
Let $\mathcal{H}$ *be an o-minimal hybrid system and* $\mathcal{P}_{\mathcal{H}}$ *be a finite and definable partition of* $Q \times V_2$ *respecting the guards and the resets of* $\mathcal{H}$*. There is a finite and definable partition, noted* $\mathsf{Suf}(\mathcal{P}_{\mathcal{H}})$*, inducing a bisimulation on* $\mathcal{H}$*.*

[2] Two systems are time-abstract bisimilar whenever they can both do the same actions and wait some delay.

O-minimal hybrid systems are models for *closed systems*, where all transitions are controlled. If we want to consider *open systems* where we distinguish between the actions of a *controller* and of an *environment*, we need to consider games on o-minimal hybrid systems. We are now going to define this notion.

Definition 6 (O-minimal game). *Let $\mathcal{M} = (M, +, 0, 1, <, \cdots)$ be an o-minimal structure. An $\mathcal{M}$-*game *(or simply an* o-minimal game*) is a tuple $(Q, \mathsf{Goal}, \Sigma, \delta, \gamma)$ where $\mathsf{Goal} \subseteq Q$ is a subset of winning locations, $(Q, \Sigma, \delta, \gamma)$ is an $\mathcal{M}$-hybrid system and Σ is partitioned into two subsets Σ_c and Σ_u corresponding to* controllable *and* uncontrollable *actions.*

Let $\mathcal{H}$ be an o-minimal game. The game is played by two players, the *controller* and the *environment*; the goal of the controller is to reach a winning state whatever the environment does. In every state s, the controller picks a delay τ and an action $a \in \Sigma_c$ such that there is a transition $s \xrightarrow{\tau,a} s'$. The environment has two choices:

- either it waits τ and executes a transition $s \xrightarrow{\tau,a} s'$ proposed by the controller,
- or it waits τ', $0 \leq \tau' \leq \tau$, and executes a transition $s \xrightarrow{\tau',u} s''$ with $u \in \Sigma_u$.

The game then evolves to a new state (according to the choice of the environment) and the two players proceed to play as before.

We will now formalize the semantic through the concept of *strategy*.

Definition 7 (Strategy). *A (controller)* strategy[3] *is a partial function λ from $\mathsf{Runs}_f(\mathcal{H})$ to $M^+ \times \Sigma_c$ such that for all runs ϱ in $\mathsf{Runs}_f(\mathcal{H})$, if $\lambda(\varrho)$ is defined, then $\lambda(\varrho)$ is enabled in last(ϱ).*

Intuitively, the strategy tells what needs to be done for controlling the system: at each instant it tells how much time we need to wait and which controllable action needs to be done after this delay. Note that even when the environment follows the controller's choice, it has to choose between several edges, each one labeled by the action given by the strategy (because the original game is not supposed to be deterministic).

Let $\varrho = (q_0, x_0, t'_0, y_0) \xrightarrow{\tau_1,a_1} \ldots$ be a run, and set for every i, ϱ_i the prefix of ϱ ending at position $(i, 0)$. The run ϱ is said to be *consistent with a strategy* λ when for all i, if $\lambda(\varrho_i) = (\tau, a)$ then either $\tau_{i+1} = \tau$ and $a_{i+1} = a$, or $\tau_{i+1} \leq \tau$ and $a_{i+1} \in \Sigma_u$. We denote by $\mathsf{Outcome}(s, \lambda)$ the set of runs starting from a state s of the (output) space consistent with the strategy λ. A run $\varrho = (q_0, x_0, t_0, y_0) \xrightarrow{\tau_1,a_1} \ldots \xrightarrow{\tau_p,a_p} (q_p, x_p, t_p, y_p)$ is said *winning* if $q_i \in \mathsf{Goal}$ for some i. A run ϱ is said *maximal* with respect to a strategy λ if it is infinite or if $\lambda(\varrho)$ is not defined. A strategy λ is *winning from a state* (q, y) if for all (x, t) such that $\gamma(x, t) = y$, all maximal runs starting in (q, x, t, y) compatible with λ are winning.

[3] In the context of control problems, a strategy is also called a *controller*.

3 Weighted O-Minimal Hybrid Systems and Games

In this section, we define the weighted o-minimal hybrid systems and games, which extend the two models of the previous section with cost functions. These cost functions give a quantitative information on the behaviours of the systems, which allows to give a measure of the performance of the system. These models are respectively inspired by the model of weighted (resp. priced) timed automata [ALP01,BFH+01] and the model of weighted (resp. priced) timed games [ABM04,BCFL04].

3.1 Definitions

We consider cost functions which are **non-negative** and **time-non-decreasing**. Note that cost functions in weighted timed automata [ALP01,BFH+01] satisfy these hypotheses.

A non-negative and time-non-decreasing cost function is a definable function $\mathsf{Cost} : Q \times V_1 \times V \times M^+ \rightarrow M^+$ such that for all $q \in Q$, $x \in V_1$, $t \in V$ and $\tau_1,\ \tau_2 \in M^+$ with $\tau_1 \leq \tau_2$ we have that $\mathsf{Cost}(q, x, t, \tau_1) \leq \mathsf{Cost}(q, x, t, \tau_2)$.

Definition 8 (Weighted o-minimal hybrid system and game). *An $\mathcal{M}$-*weighted hybrid system (resp. game) *is an $\mathcal{M}$-hybrid system (resp. game) $\mathcal{H} = (Q, \Sigma, \delta, \gamma)$ with a definable non-negative and time-non-decreasing cost function* Cost.

The semantics of an o-minimal weighted hybrid system (resp. game) is the one of the underlying o-minimal hybrid system (resp. game). Hence, the cost function does not affect the behaviours of the system; it gives for every single step of the system a non-negative value, which represents the cost of evolving following that step. It naturally extends to a run in the system: let $\varrho = (q_0, x_0, t_0, y_0) \xrightarrow{\tau_1, a_1} \ldots \xrightarrow{\tau_p, a_p} (q_p, x_p, t_p, y_p)$ be a finite run in $\mathcal{H}$. The *cost of* ϱ is $\mathsf{Cost}(\varrho) = \sum_{i=1}^{p} \mathsf{Cost}(q_{i-1}, x_{i-1}, t_{i-1}, \tau_i)$.

Let us give an example of weighted o-minimal hybrid system.

Example 3 ([BLM07]). The weighted o-minimal hybrid system of Figure 1 models a never-ending process of repairing problems. The repair of a problem has a certain cost, captured in the model by the cost function Cost. As soon as a problem occurs (modeled by the *pb* transition) the value of the cost grows with rate 1, until actual repair is taking place by one of the transitions rp_1 (cheap but long repair) or rp_2 (expensive but quick repair). At most 24 time units after the occurrence of a problem it will have been repaired one way or another.

3.2 Related Problems and Results

In this subsection we define the two problems we are interested in: the *cost-optimal control problem* and the WCTL *model-checking problem*.

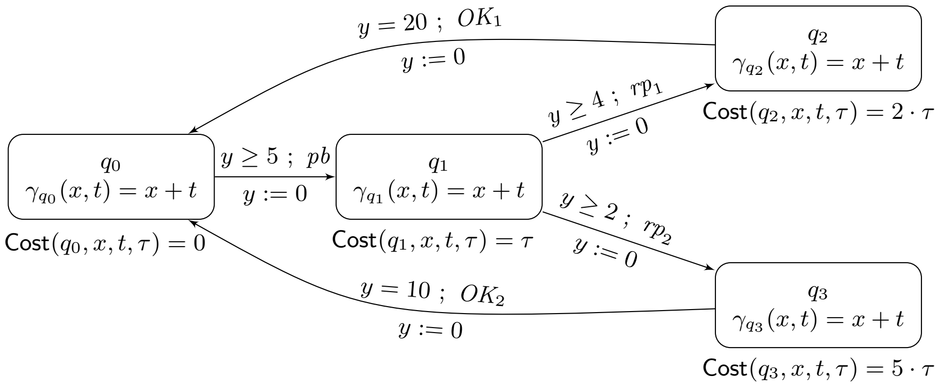

Fig. 1. A repair problem example

The cost-optimal control problem. The cost-optimal reachability control problem was first considered on weighted timed automata in [ABM04,BCFL04]. However it has been shown in [BBR05,BBM06] that the cost-optimal reachability control problem for weighted timed automata is undecidable.

In our context of weighted o-minimal games, the *cost-optimal control problem* asks what is the optimal cost for the controller to reach Goal whatever the environement does. In order to take the cost function into account, we now need to define the *cost of a strategy from a state* and the *optimal cost from a state.*

Definition 9 (Cost of a strategy from a state). *Let* $(\mathcal{H}, \mathsf{Cost})$ *be a weighted o-minimal game,* Goal *be a subset of winning locations,* s *be a state of the (output) space* V_2 *and* λ *be a strategy. The* cost $\mathsf{Cost}(s, \lambda)$ of λ from s is defined by:

$$\mathsf{Cost}(s,\lambda) = \sup\big\{\mathsf{Cost}(\varrho) \ \mid\ \varrho \in \mathsf{Outcome}(s,\lambda)\big\}.$$

Intuitively the presence of the supremum is explained by the fact that the environment tries to maximize the cost.

Definition 10 (Optimal cost from a state). *Let* $(\mathcal{H}, \mathsf{Cost})$ *be a weighted o-minimal game,* Goal *be a subset of winning locations and* s *be a state of the (output) space* V_2*. The* optimal cost $\mathsf{OptCost}(q)$ associated with s is defined by:

$$\mathsf{OptCost}(s) = \inf\big\{\mathsf{Cost}(s,\lambda) \ \mid\ \lambda \textit{ is a winning strategy}\big\}.$$

A winning strategy from s is called *optimal* whenever $\mathsf{Cost}(s, \lambda) = \mathsf{OptCost}(s)$.

Problem 1 (Cost-optimal control problem). Given a weighted o-minimal game $\mathcal{H}$, a definable state s and a definable constant c, decide if there exists a winning strategy λ from q such that $\mathsf{Cost}(s, \lambda) \leq c$.

Problem 2 (Computation of the optimal cost). Given a weighted o-minimal game $\mathcal{H}$ and a definable state s, compute the optimal cost $\mathsf{OptCost}(s)$.

Remark 1. There is an optimal winning strategy from state s iff the infimum can be replaced by a minimum in the definition of $\mathsf{OptCost}(s)$. If we solve problem 1 and 2, we can also determine if there is an optimal winning strategy by asking if there is a strategy λ with $\mathsf{Cost}(s,\lambda) \leq \mathsf{OptCost}(s)$. In [BBR05,BBM06], it has been shown that the variant of Problem 1 for weighted timed automata is undecidable.

The WCTL model-checking problem. The logic WCTL [4] has been proposed in the context of (weighted) timed systems as an extension of CTL with cost constraints on modalities [BBR04,BBM06,BLM07]. In our context, we define for every structure $\mathcal{M}$ the logic $\mathsf{WCTL}_{\mathcal{M}}$ over Σ inductively as follows:

$$\mathsf{WCTL}_{\mathcal{M}} \ni \varphi \ ::= \ a \ | \ \varphi \vee \varphi \ | \ \neg\varphi \ | \ \mathsf{E}\,\varphi \mathsf{U}_{\sim c}\varphi \ | \ \mathsf{A}\,\varphi \mathsf{U}_{\sim c}\varphi$$

where $a \in \Sigma$, $\sim \in \{<, \leq, =, \geq, >\}$ and c is an $\mathcal{M}$-definable constant.

Let $(\mathcal{H}, \mathsf{Cost})$ be an $\mathcal{M}$-hybrid system. The semantics of $\mathsf{WCTL}_{\mathcal{M}}$ is defined for every state $(q,y) \in Q \times V_2$ of $(\mathcal{H}, \mathsf{Cost})$ as follows:

$$\begin{array}{rcl}
(q,y) \models a & \Leftrightarrow & (q,y) \xrightarrow{a} (q',y') \text{ for some } (q',y') \in Q \times V_2 \,^{5} \\
(q,y) \models \neg\varphi & \Leftrightarrow & (q,y) \not\models \varphi \\
(q,y) \models \varphi_1 \vee \varphi_2 & \Leftrightarrow & (q,y) \models \varphi_1 \text{ or } (q,y) \models \varphi_2 \\
(q,y) \models \mathsf{E}\,\varphi_1 \mathsf{U}_{\sim c}\varphi_2 & \Leftrightarrow & \text{there is an infinite run } \varrho \text{ in } \mathcal{H} \text{ from } (q,y) \\
 & & \text{s.t. } \varrho \models \varphi_1 \mathsf{U}_{\sim c}\varphi_2 \\
(q,y) \models \mathsf{A}\,\varphi_1 \mathsf{U}_{\sim c}\varphi_2 & \Leftrightarrow & \text{every infinite run } \varrho \text{ in } \mathcal{H} \text{ from } (q,y) \\
 & & \text{satisfies } \varrho \models \varphi_1 \mathsf{U}_{\sim c}\varphi_2 \\
\varrho \models \varphi_1 \mathsf{U}_{\sim c}\varphi_2 & \Leftrightarrow & \text{there exists } \pi \geq 0 \text{ position along } \varrho \text{ s.t.} \\
 & & \varrho[\pi] \models \varphi_2, \text{ for all positions } 0 \leq \pi' < \pi \text{ on } \varrho, \\
 & & \varrho[\pi'] \models \varphi_1 \vee \varphi_2, \text{ and } \mathsf{Cost}(\varrho_{\leq \pi}) \sim c^{6}
\end{array}$$

We use `true` for $a \vee \neg a$, and classical "eventually" and "always" operators: $\mathsf{E}\,\mathsf{F}_{\sim c}\phi$ (resp. $\mathsf{A}\,\mathsf{F}_{\sim c}\phi$) stands for $\mathsf{E}\,\mathtt{true}\mathsf{U}_{\sim c}\phi$ (resp. $\mathsf{A}\,\mathtt{true}\mathsf{U}_{\sim c}\phi$) and $\mathsf{A}\,\mathsf{G}_{\sim c}\phi$ (resp. $\mathsf{E}\,\mathsf{G}_{\sim c}\phi$) stands for $\neg\mathsf{E}\,\mathsf{F}_{\sim c}\neg\phi$ (resp. $\neg\mathsf{A}\,\mathsf{F}_{\sim c}\neg\phi$).

Let us give an example of WCTL formulae on the repair problem of Example 3.

Example 4. [BLM07] An example of a property that can be expressed with WCTL is "*Whenever a problem occurs it can be repaired within a total cost of* 55". It can be expressed with the following formula:

$$\mathsf{A}\,\mathsf{G}\big(pb \implies \mathsf{E}\,\mathsf{F}_{\leq 55}(OK_1 \vee OK_2)\big).$$

One can easily check that this formula holds for every state of the weighted o-minimal hybrid system of Figure 1.

[4] WCTL stands for "Weighted CTL."
[5] This can be viewed equivalently as atomic propositions.
[6] Following [Ras99] we use $\varphi_1 \vee \varphi_2$ to handle open intervals in timed models.

Problem 3 (Model-checking of WCTL*).* Given $(\mathcal{H}, \mathsf{Cost})$ an $\mathcal{M}$-weighted hybrid system, φ a $\mathsf{WCTL}_{\mathcal{M}}$-formula and $(q, y) \in Q \times V_2$ a definable state of $\mathcal{H}$, decide whether $(q, y) \models \varphi$.

Remark 2. Note that classical results on o-minimal hybrid systems cannot be used to solve the two problems presented above (for instance, weighted o-minimal hybrid systems do not have a finite bisimulation), and hence, *ad-hoc* proofs have to be developed.

4 Solving the Cost-Optimal Control Problem

In this section we prove the decidability of Problem 1.

Definition 11. *Let* $(\mathcal{H}, \mathsf{Cost})$ *be a weighted o-minimal game. We say that a run* $(q_0, x_0, t_0, y_0) \xrightarrow{\tau_1, a_1} \ldots$ *crosses the transition* $e = (q, g, a, R, q')$ *at step* i *if* $q = q_{i-1}$*,* $q' = q_i$*,* $a = a_i$*,* $\gamma_{q_{i-1}}(x_{i-1}, t_{i-1} + \tau_i) \in g$ *and* $y_i \in R$*.*

We are now going to prove a key proposition: we can restrict to winning strategies that cross each edge at most once. In order to prove this proposition we use the fact that the cost functions have non-negative values.

Proposition 12. *Let* $\mathcal{H}$ *be a weighted o-minimal game and* (q, y) *a state of* $\mathcal{H}$*. If there exists a winning strategy* λ *from* (q, y) *then there exists a winning strategy* λ_{new} *with* $\mathsf{Cost}((q, y), \lambda_{new}) \leq \mathsf{Cost}((q, y), \lambda)$ *such that every run starting in* (q, y) *compatible with* λ_{new} *crosses each transition at most once.*

Proof (Sketch). The idea of the proof is the following: for each transition e, we iteratively ensure that there is a winning strategy (of cost $c' \leq c$) crossing e at most once. See $\mathsf{Outcome}((q, y), \lambda)$ as a(n infinitely branching) tree: every path in this tree is finite and reaches Goal, but it may cross e several times. We construct a new strategy λ_{new} which short-cuts λ in the following sense: it simulates λ until e is crossed for the first time and then switches to a descendant in the tree from which all paths do not cross e anymore (this is possible as λ is winning). Such a strategy crosses e at most once, its cost is smaller than that of λ as its compatible runs are "shorter" than ones of λ and the cost function is non-negative. □

We now give a backward algorithm which computes the optimal cost, based on the formulation of the problem given in [LMM02,ABM04]. The termination of the algorithm relies on Proposition 12. For this, given $(q, y) \in Q \times V_2$ and $n \in \mathbb{N}$ we define $c_n(q, y)$, the optimal cost of reaching Goal from (q, y) in at most n steps. Formally we have that:

- $c_0(q, y) = 0$ if $q \in \mathsf{Goal}$, $+\infty$ if $q \notin \mathsf{Goal}$.
- $c_{n+1}(q, y) =$
$$\sup_{\gamma_q(x,t)=y} \inf_{(\tau,a) \in \mathsf{Enb}(q,x,t,y)} \max \begin{cases} \mathsf{Cost}^{\tau,a}_{c_n}(q, x, t, y) \\ \sup\{\mathsf{Cost}^{\tau',u}_{c_n}(q, x, t, y) \mid {\scriptstyle (\tau',u) \in \mathsf{Enb}(q,x,t,y), \tau' \leq \tau}\} \end{cases}$$
where $(\tau, e) \in \mathsf{Enb}(q, x, t, y)$ iff $e \in \mathsf{Enb}(q, x, t + \tau, \gamma_q(x, t + \tau))$, and
$\mathsf{Cost}^{\tau,e}_{c}(q, x, t, y) = \mathsf{Cost}(q, x, t, y, \tau) + \sup\{c(q', y') \mid (q, x, t, y) \xrightarrow{\tau, a} (q', x', t', y')\}$.

Definition 13. *A strategy λ is said n-bounded from (q, y) if every run compatible with λ starting from (q, y) has length at most n.*

Lemma 14. *For every $\varepsilon > 0$, for every $(q, y) \in Q \times V_2$ s.t. $c_n(q, y) < +\infty$, there exists a definable n-bounded strategy λ from (q, y) such that $\mathsf{Cost}((q, y), \lambda) \leq c_n(q, y) + \varepsilon$.*

Lemma 15. *If λ is an n-bounded strategy from (q, y) then $\mathsf{Cost}((q, y), \lambda) \geq c_n(q, y)$.*

Theorem 16. *Let $\mathcal{M} = \langle M, +, 0, 1, \ldots \rangle$ be an o-minimal structure such that $\mathsf{Th}(\mathcal{M})$ is decidable. The cost-optimal control problem over $\mathcal{M}$-weighted hybrid systems is decidable.*

Proof. Let $\mathcal{H}$ be a weighted o-minimal game and s a state of $\mathcal{H}$. Lemmas 14 and 15 imply that $c_k(s) = \inf \big\{ \mathsf{Cost}(s, \lambda) \ \mid\ \lambda \text{ is a } k\text{-bounded strategy strategy} \big\}$.

Let n be the number of transitions of $\mathcal{H}$. Proposition 12 shows that for every winning strategy from s there is an $(n + 1)$-bounded winning strategy from s with smallest cost. Thus $\mathsf{OptCost}(s) = \inf \big\{ \mathsf{Cost}(s, \lambda) \ \mid\ \lambda \text{ is an } (n + 1)\text{-bounded strategy} \big\} = c_{n+1}(s)$. Note that $c_{n+1}(s)$ is computable since $\mathsf{Th}(\mathcal{M})$ is decidable.

We can moreover decide if the optimal cost can be achieved by a strategy: by Proposition 12 it is sufficient to enumerate all $(n + 1)$-bounded strategies using τ_i's as parameters and check if the cost of one of them is equal to $c_{n+1}(s)$. □

Remark 3. Let us notice that Theorem 16 encompasses the decidability of the time-bounded reachability problem considered in [Gen05]. Moreover, a consequence of Theorem 16 is the decidability of the (cost-)optimal reachability problem formulated in [ALP01,BFH$^+$01] for weighted timed automata.

5 Solving the WCTL Model-Checking Problem

The aim of this section is to prove the following decidability theorem. The techniques that we will develop for proving this result are partly inspired by the recent decidability proof for WCTL over one-clock weighted timed automata [BLM07].

Theorem 17. *Let $\mathcal{M} = \langle M, +, 0, 1, <, \ldots \rangle$ be an o-minimal structure such that $\mathcal{M}$ is Archimedian*[7] *and $\mathsf{Th}(\mathcal{M})$ is decidable. The model-checking of $\mathsf{WCTL}_{\mathcal{M}}$ over $\mathcal{M}$-weighted hybrid systems is decidable.*

If $\mathcal{P}$ and $\mathcal{P}'$ are two partitions, we write $\mathcal{P} \sqcap \mathcal{P}'$ for the joint partition,*i.e.* the smallest partition that refines both $\mathcal{P}$ and $\mathcal{P}'$. Let us notice that if $\mathcal{P}$ and $\mathcal{P}'$ are both finite and definable the joint partition $\mathcal{P} \sqcap \mathcal{P}'$ is also finite and definable.

Let $\mathcal{H} = (Q, \mathsf{Goal}, \Sigma, \delta, \gamma)$ be an $\mathcal{M}$-weighted hybrid system. On each location $q \in Q$, let us denote by $\mathcal{P}_q$ the partition induced by the guards and the resets

[7] A structure is *Archimedian* whenever for every pair $(a, b) \in M^2$ such that $a \neq 0$, there exists some integer n such that $n \cdot a > b$.

associated with location q. We denote by $\mathcal{P}_{\mathcal{H}}$ the partition of state space $S = Q \times V_2$ induced by the $\mathcal{P}_q$'s. $\mathcal{P}_{\mathcal{H}}$ is a finite definable partition of S.

Two states of the same piece P of $\mathcal{P}_{\mathcal{H}}$ agree on all atomic formulae $a \in \Sigma$. We will now inductively construct for every $\mathsf{WCTL}_{\mathcal{M}}$-formula φ a refined (finite and definable) partition $\mathcal{P}_\varphi$ of $\mathcal{P}_{\mathcal{H}}$ such that two states of a piece of $\mathcal{P}_\varphi$ agree on formula φ. We will proceed in three steps: from partitions for ϕ and ψ we will successively construct a partition for $\mathsf{E}\,\phi \mathsf{U} \psi$ and $\mathsf{A}\,\phi \mathsf{U} \psi$, then for $\mathsf{E}\,\phi \mathsf{U}_{\sim c} \psi$ and finally for $\mathsf{A}\,\phi \mathsf{U}_{\sim c} \psi$.

The first step is achieved using the following lemma, which applies the construction of a finite and definable partition (the so-called suffix-partition) correct w.r.t. bisimulation (see Theorem 5), and thus for CTL-formulae [AHLP00,HMR05].

Lemma 18. *Let $\mathcal{P}_\phi$ (resp. $\mathcal{P}_\psi$) be a partition for ϕ (resp. ψ). The partition $\mathsf{Suf}(\mathcal{P}_\phi \sqcap \mathcal{P}_\psi)$ is a time-abstract bisimulation and is a partition for formula $\mathsf{E}\,\phi \mathsf{U} \psi$. That is, if P is a piece of $\mathsf{Suf}(\mathcal{P}_\phi \sqcap \mathcal{P}_\psi)$, then all states (q, y) with $y \in P$ either satisfy $\mathsf{E}\,\phi \mathsf{U} \psi$ or do not satisfy this formula. The same holds for $\mathsf{A}\,\phi \mathsf{U} \psi$.*

The second step, the construction of a (finite and definable) partition respecting $\mathsf{E}\,\phi \mathsf{U}_{\sim c} \psi$, is more involved.

Let us denote by $\mathcal{P}$ the partition $\mathsf{Suf}(\mathcal{P}_\phi \sqcap \mathcal{P}_\psi)$. Let q and q' be two locations of $\mathcal{H}$. Let P and P' be two pieces of $\mathcal{P}$. We give formulae which define the set of possible costs of paths from a state (or a piece of $\mathcal{P}$) to another one:

- $\theta^{\phi \vee \psi}_{(q,P) \to (q',P')}(c, y)$ expresses that it is possible to go from some (q, y) with $y \in P$ to some (q', y') with $y' \in P'$ by a continuous step followed by a discrete action, with cost c, and always satisfying $\phi \vee \psi$.
- $\theta^{\phi \vee \psi}_{(q,P \rightsquigarrow P')}(c, y)$ expresses that it is possible to go from (q, y) with $y \in P$ to some (q, y') with $y' \in P'$ by a continuous step (and no discrete action), with cost c, and always satisfying $\phi \vee \psi$.

From the two previous formulae, we define the following definable sets:

$$\kappa^{\phi \vee \psi}_{(q,P) \to (q',P')} = \{c \in M^+ \mid \exists y \in P \text{ s.t. } \theta^{\phi \vee \psi}_{(q,P) \to (q',P')}(c, y)\} \ ;$$

$$\kappa^{\phi \vee \psi}_{(q,P \rightsquigarrow P')} = \{c \in M^+ \mid \exists y \in P \text{ s.t. } \theta^{\phi \vee \psi}_{(q,P \rightsquigarrow P')}(c, y)\} \ ;$$

$$\lambda^{\phi \vee \psi}_{(q,P) \to (q',P')}(y) = \{c \in M^+ \mid \theta^{\phi \vee \psi}_{(q,P) \to (q',P')}(c, y)\} \ ;$$

$$\lambda^{\phi \vee \psi}_{(q,P \rightsquigarrow P')}(y) = \{c \in M^+ \mid \theta^{\phi \vee \psi}_{(q,P \rightsquigarrow P')}(c, y)\}.$$

We then construct a finite graph which abstracts away all dynamical parts of $\mathcal{H}$ and which will be restricted to parts of $\mathcal{H}$ which satisfy formula $\mathsf{E}\,\phi \mathsf{U} \psi$. Each edge of this graph will be labeled with a weight (indeed a definable set) which will represent the set of costs of all paths in $\mathcal{H}$ witnessing formula $\mathsf{E}\,\phi \mathsf{U} \psi$. More formally, we construct a *(definable) weighted finite graph* $\mathcal{G}_{\phi,\psi} = (V, E)$ as follows:

– its set of vertices V is

$$\{(q,P),(q,P,\mathsf{init}) \mid (q,P) \models \phi \vee \psi\}^8 \quad \cup \quad \{(q,P,\mathsf{final}) \mid (q,P) \models \psi\}$$

– its set of edges E is

$$\begin{aligned}
&\{(q,P,\mathsf{init}) \xrightarrow{\lambda^{\phi\vee\psi}_{(q,P)\to(q',P')}(y)} (q',P')\} \cup \{(q,P,\mathsf{init}) \xrightarrow{\lambda^{\phi\vee\psi}_{(q,P\rightsquigarrow P'')}(y)} (q,P'',\mathsf{final})\}\\
\cup\ &\{(q,P,\mathsf{init}) \xrightarrow{[0]} (q,P,\mathsf{final})\} \cup \{(q,P) \xrightarrow{\kappa^{\phi\vee\psi}_{(q,P)\to(q',P')}} (q',P')\}\\
\cup\ &\{(q,P) \xrightarrow{\kappa^{\phi\vee\psi}_{(q,P)\to(q',P')}} (q',P',\mathsf{final})\} \cup \{(q,P) \xrightarrow{\kappa^{\phi\vee\psi}_{(q,P\rightsquigarrow P'')}} (q,P'',\mathsf{final})\}
\end{aligned}$$

Let $\varrho = (q_0,P_0,\mathsf{init}) \xrightarrow{\lambda(y)} (q_1,P_1) \xrightarrow{\kappa_1} \dots \xrightarrow{\kappa_n} (q_n,P_n,\mathsf{final})$ be a finite path of $\mathcal{G}_{\phi,\psi}$. Then we define $K_\varrho(y)$ to be the set of all possible costs of the path ϱ: $K_\varrho(y) = \{\lambda(y) + c_2 + \cdots + c_n \mid c_i \in \kappa_i\}$.

The following proposition shows that the graph $\mathcal{G}_{\phi,\psi}$ can be used to model-check the formula $\mathsf{E}\,\phi\mathsf{U}_{\sim c}\psi$.

Proposition 19. *Let $q_0 \in Q$, $P_0 \in \mathcal{P}$ and $y_0 \in P_0$. Then, $(q_0,y_0) \models \mathsf{E}\,\phi\mathsf{U}_{\sim c}\psi$ iff there exists a path $\varrho = (q_0,P_0,\mathit{init}) \xrightarrow{\lambda(y)} (q_1,P_1) \xrightarrow{\kappa_1} \dots \xrightarrow{\kappa_n} (q_n,P_n,\mathit{final})$ in $\mathcal{G}_{\phi,\psi}$, there exists $\zeta \in M$ such that $\zeta \in K_\varrho(y_0)$ and $\zeta \sim c$.*

We prove that, under the Archimedian hypothesis, we can bound the length of witnessing paths in the graph $\mathcal{G}_{\phi,\psi}$. This hypothesis has not been used yet, as everything holds without it, but is required by the next proposition. For every $(m,c) \in M^2$ such that $m \neq 0$, we note $\left[\frac{c}{m}\right]$ the smallest integer k such that $k \cdot m > c$.

Proposition 20. *We assume that $(q_0,y_0) \models \mathsf{E}\,\phi\mathsf{U}_{\sim c}\psi$, and we define $N = 2 + (|Q|\cdot|\mathcal{P}|+1)\cdot\left(\left[\frac{c}{m}\right]+2\right)$ where m is the smallest positive constant defining an interval of some weight labelling the transitions of $\mathcal{G}_{\phi,\psi}$. Then there exists a path $\varrho = (q_0,P_0,\mathit{init}) \xrightarrow{\lambda(y)} (q_1,P_1) \xrightarrow{\kappa_1} \dots \xrightarrow{\kappa_n} (q_n,P_n,\mathit{final})$ in $\mathcal{G}_{\phi,\psi}$ such that $y_0 \in P_0$, $n \leq N$, and there exists $\zeta \in M$ such that $\zeta \in K_\varrho(y_0)$ and $\zeta \sim c$.*

Proof (Sketch). By Proposition 19, there exists a path $\varrho = (q_0,P_0,\mathsf{init}) \xrightarrow{\lambda(y)} (q_1,P_1) \xrightarrow{\kappa_1} \dots \xrightarrow{\kappa_n} (q_n,P_n,\mathsf{final})$ in $\mathcal{G}_{\phi,\psi}$ s.t. $\exists\zeta \in K_\varrho(y_0)$, $\zeta \sim c$. From ϱ, we will construct a run of length smaller than N which witnesses the formula.

By o-minimality, each κ_i is a finite union of intervals. We first choose accurately one such interval per κ_i and obtain a *realisation* of ϱ noted $\varrho[\mathcal{I}]$, so that the accumulated union of all intervals along $\varrho[\mathcal{I}]$ contains ζ such that $\zeta \sim c$. We call [0]-cycle a cycle labeled only by the interval [0]. We can suppose that there is no [0]-cycle in $\varrho[\mathcal{I}]$ otherwise we can remove this cycle and get a shorter path also witnessing the formula.

[8] This is a notation misuse, but $(q,P) \models \phi \vee \psi$ means that for all $y \in P$, $(q,y) \models \phi \vee \psi$, which is equivalent to the property that $(q,y) \models \phi \vee \psi$ for some $y \in P$.

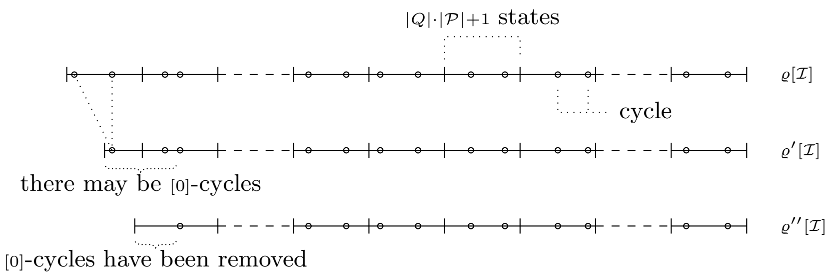

Fig. 2. The construction in the proof of Proposition 20

If the length of $\varrho[\mathcal{I}]$ is shorter than N then we are done. Otherwise ϱ contains at least $\left(\left[\frac{c}{m}\right]+2\right)$ simple cycles (see Fig. 2), each of which is labeled by an (accumulated) interval whose right bound is greater than m. Thus, $K_{\varrho[\mathcal{I}]}(y_0) = \langle a, b\rangle$ [9] with $b \geq c + 2m$. By hypothesis, there exists $\zeta \in \langle a, b\rangle$ such that $\zeta \sim c$.

We then remove the first cycle of the run and remove [0]-cycles that can have been potentially created (see Fig. 2). We obtain $\varrho''[\mathcal{I}]$ a strictly smaller run. Note that $K_{\varrho''[\mathcal{I}]}(y_0)$ might be different from $K_{\varrho[\mathcal{I}]}(y_0)$; nevertheless $\varrho''[\mathcal{I}]$ still has at least $\left[\frac{c}{m}\right]$ simple cycles so $K_{\varrho''[\mathcal{I}]}(y_0) = \langle a', b'\rangle$ with $b' > c$ (as previously). It proves that there exists $\zeta \in K_{\varrho''[\mathcal{I}]}(y_0)$ with $\zeta \sim c$ (since $c < b' \leq b$ and $a' \leq a$). We iterate this process until we find a run smaller than N with this property. $\square$

Applying the previous proposition, we can build a first-order formula which checks if a given state (q, y) satisfies the WCTL formula $\mathsf{E}\,\phi \mathsf{U}_{\sim c}\psi$, and we thus get the following corollary.

Corollary 21. *Let $\mathcal{M} = \langle M, +, 0, 1, <, \ldots\rangle$ be an o-minimal structure such that $\mathcal{M}$ is Archimedean and $\mathsf{Th}(\mathcal{M})$ is decidable. Let $\mathcal{H}$ be an $\mathcal{M}$-weighted hybrid system and ϕ and ψ be two* WCTL *formulae. Assume we have built two finite and definable partitions $\mathcal{P}_\phi$ and $\mathcal{P}_\psi$ for ϕ and ψ, then we can compute a definable finite partition correct for the formula $\mathsf{E}\,\phi\mathsf{U}_{\sim c}\psi$.*

We do not enter into the details of the third step. However, the construction of the partition respecting $\mathsf{A}\,\phi\mathsf{U}_{\sim c}\psi$ (from the partition $\mathcal{P}_\phi$ and $\mathcal{P}_\psi$) shares ideas with the construction of the partition respecting $\mathsf{E}\,\phi\mathsf{U}_{\sim c}\psi$ that we have described. In particular, the idea exploited in the proof of Proposition 20 plays an important role.

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[9] $\langle a, b\rangle$ stands for either $[a, b]$ or $[a, b)$ or $(a, b]$ or (a, b).

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