

When are Timed Automata weakly timed bisimilar to Time Petri Nets ?

B. Bérard¹, F. Cassez², S. Haddad¹, Didier Lime³, O.H. Roux²

¹ LAMSADE, Paris, France

E-mail: {beatrice.berard | serge.haddad}@lamsade.dauphine.fr

² IRCCyN, Nantes, France

{Franck.Cassez | Olivier-h.Roux}@irccyn.ec-nantes.fr

³ CISS, Aalborg, Denmark

Didier@cs.aau.dk

Abstract. In this paper, we compare Timed Automata (TA) with Time Petri Nets (TPN) with respect to weak timed bisimilarity. It is already known that the class of bounded TPNs is included in the class of TA. It is thus natural to try and identify the (strict) subclass \mathcal{TA}^{wtb} of TA that is equivalent to TPN for the weak time bisimulation relation. We give a characterisation of this subclass and we show that the membership problem and the reachability problem for \mathcal{TA}^{wtb} are *PSPACE*-complete. Furthermore we show that for a TA in \mathcal{TA}^{wtb} with integer constants, an equivalent TPN can be built with integer bounds but with a size exponential w.r.t. the original model. Surprisingly, using rational bounds yields a TPN whose size is linear.

Keywords: Time Petri Nets, Timed Automata, Weak Timed Bisimilarity.

1 Introduction

Expressiveness of timed models. Adding explicit time to classical models was first done in the seventies for Petri nets [14,16]. Since then, timed models based on Petri nets and finite automata were extensively studied, and various tools were developed for their analysis. In this paper, we focus on two well known models: Timed Automata (TA) from [2] and Time Petri Nets (TPNs) from [14]. In [4], we studied the different semantics for TPNs w.r.t. weak timed bisimilarity. Here, we are interested in comparing the expressive power of TA and TPN for this equivalence. Recall that there are unbounded TPNs for which no bisimilar TA exists. This is a direct consequence of the following observation: the untimed language of a TA is regular which is not necessarily the case for TPNs. On the other hand, it was proved in [10] that bounded TPNs form a subclass of the class of timed automata, in the sense that for each bounded TPN \mathcal{N} , there exists a TA which is weakly timed bisimilar to \mathcal{N} . A similar result can be found in [13], where it is obtained by a completely different approach. In another line of work [12], Haar, Kaiser, Simonot and Toussaint compare Timed

State Machines (TSM) and Time Petri Nets, giving a translation from TSM to TPN that preserves timed languages. In [5], we propose an extended translation between TA and TPNs with better complexity.

Our Contribution. In this work, we consider TPNs and label-free TA, *i.e.* where two different edges have different labels (and no label is ε) and we give a characterisation of the subclass \mathcal{TA}^{wtb} of timed automata which admit a weakly timed bisimilar TPN. This non intuitive condition relates to the topological properties of the so-called region automaton associated with a TA. To prove that the condition is necessary, we introduce the notion of *uniform bisimilarity*, which is stronger than weak timed bisimilarity. Conversely, when the condition holds for a TA, we provide two effective constructions of bisimilar TPNs: the first one with rational constants has a size linear w.r.t. the TA, while the other one, which uses only integer constants has an exponential size. From this characterisation, we will deduce that given a TA, the problem of deciding whether there is a TPN bisimilar to it, is *PSPACE*-complete. Thus, we obtain that the membership problem is *PSPACE*-complete. Finally we also prove that the reachability problem is *PSPACE*-complete.

Outline of the paper. Section 2 recalls the semantics of TPNs and TA, and the notion of timed bisimilarity. Section 3 explains the characterisation while Section 4 is devoted to a sketch of its proof. We conclude in Section 5.

2 Time Petri Nets and Timed Automata

Notations. Let Σ be a finite alphabet, Σ^* (resp. Σ^ω) the set of finite (resp. infinite) words of Σ and $\Sigma^\infty = \Sigma^* \cup \Sigma^\omega$. We also use $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$ with ε (the empty word) not in Σ .

The sets \mathbb{N} , $\mathbb{Q}_{\geq 0}$ and $\mathbb{R}_{\geq 0}$ are respectively the sets of natural, non-negative rational and non-negative real numbers. We write $\mathbf{0}$ for the tuple $v \in \mathbb{N}^n$ such that $v(k) = 0$ for all $1 \leq k \leq n$. Let $g > 0$ in \mathbb{N} , we write $\mathbb{N}_g = \{\frac{i}{g} \mid i \in \mathbb{N}\}$. A tuple $v \in \mathbb{Q}^n$ belongs to the *g-grid* if $v(k) \in \mathbb{N}_g$ for all $1 \leq k \leq n$.

An interval I of $\mathbb{R}_{\geq 0}$ is a $\mathbb{Q}_{\geq 0}$ -interval iff its left endpoint belongs to $\mathbb{Q}_{\geq 0}$ and its right endpoint belongs to $\mathbb{Q}_{\geq 0} \cup \{\infty\}$. We set $I^\downarrow = \{x \mid x \leq y \text{ for some } y \in I\}$, the *downward closure* of I and $I^\uparrow = \{x \mid x \geq y \text{ for some } y \in I\}$, the *upward closure* of I . We denote by $\mathcal{I}(\mathbb{Q}_{\geq 0})$ the set of $\mathbb{Q}_{\geq 0}$ -intervals of $\mathbb{R}_{\geq 0}$.

Timed Transition Systems and Equivalence Relations. Timed transition systems describe systems which combine discrete and continuous evolutions. They are used to define and compare the semantics of TPNs and TA.

A *Timed Transition System (TTS)* is a transition system $S = (Q, q_0, \rightarrow)$, where Q is the set of configurations, $q_0 \in Q$ is the initial configuration and the relation \rightarrow consists of either delay moves $q \xrightarrow{d} q'$, with $d \in \mathbb{R}_{\geq 0}$, or discrete moves $q \xrightarrow{a} q'$, with $a \in \Sigma_\varepsilon$. Moreover, we require standard properties for the relation \rightarrow :

Time-Determinism: if $q \xrightarrow{d} q'$ and $q \xrightarrow{d} q''$ with $d \in \mathbb{R}_{\geq 0}$, then $q' = q''$

0-delay: $q \xrightarrow{0} q$

Additivity: if $q \xrightarrow{d} q'$ and $q' \xrightarrow{d'} q''$ with $d, d' \in \mathbb{R}_{\geq 0}$, then $q \xrightarrow{d+d'} q''$

Continuity: if $q \xrightarrow{d} q'$, then for every d' and d'' in $\mathbb{R}_{\geq 0}$ such that $d = d' + d''$, there exists q'' such that $q \xrightarrow{d'} q'' \xrightarrow{d''} q'$.

With these properties, a *run* of S can be defined as a finite or infinite sequence of moves $\rho = q_0 \xrightarrow{a_0} q'_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q'_1 \xrightarrow{a_3} \dots q_n \xrightarrow{a_n} q'_n \dots$ where discrete actions alternate with durations. We also write this run as $q \xrightarrow{d_0 a_0 \dots d_n \dots} q'$. The word $Untimed(\rho)$ in Σ^∞ is obtained by the concatenation $a_0 a_1 \dots$ of labels in Σ_ε (so empty labels disappear), and $Duration(\rho) = \sum_{i=0}^{|\rho|} d_i$.

From a TTS, we define the relation $\rightarrow \subseteq Q \times (\Sigma \cup \mathbb{R}_{\geq 0}) \times Q$ for $a \in \Sigma$ and $d \in \mathbb{R}_{\geq 0}$ by:

- $q \xrightarrow{d} q'$ iff $\exists \rho = q \xrightarrow{w} q'$ with $Untimed(\rho) = \varepsilon$ and $Duration(\rho) = d$,
- $q \xrightarrow{a} q'$ iff $\exists \rho = q \xrightarrow{w} q'$ with $Untimed(\rho) = a$ and $Duration(\rho) = 0$.

Definition 1 (Weak Timed Bisimilarity). Let $S_1 = (Q_1, q_0^1, \rightarrow_1)$ and $S_2 = (Q_2, q_0^2, \rightarrow_2)$ be two TTS and let \approx be a binary relation over $Q_1 \times Q_2$. We write $q \approx q'$ for $(q, q') \in \approx$. The relation \approx is a weak timed bisimulation between S_1 and S_2 iff $q_0^1 \approx q_0^2$ and for all $a \in \Sigma \cup \mathbb{R}_{\geq 0}$

- if $q_1 \xrightarrow{a} q'_1$ and $q_1 \approx q_2$ then $\exists q_2 \xrightarrow{a} q'_2$ such that $q'_1 \approx q'_2$;
- conversely, if $q_2 \xrightarrow{a} q'_2$ and $q_1 \approx q_2$ then $\exists q_1 \xrightarrow{a} q'_1$ such that $q'_1 \approx q'_2$.

Two TTS S_1 and S_2 are weakly timed bisimilar, written $S_1 \approx_{\mathcal{W}} S_2$, if there exists a weak timed bisimulation relation between them.

Strong timed bisimilarity would require similar properties for transitions labeled by $a \in \Sigma \cup \mathbb{R}_{\geq 0}$, but with \xrightarrow{a} instead of \xrightarrow{a} . Thus it does not take into account the possibility of splitting a run while keeping the global duration. On the other hand, weak timed bisimilarity is more precise than language equivalence and it is well-known to be central among equivalence relations between timed systems. In the rest of the paper, we abbreviate weak timed bisimilarity by bisimilarity and we explicitly name other equivalences when needed.

Time Petri Nets. Introduced in [14], and studied more recently in [15], Time Petri Nets (TPNs) associate a closed time interval with each transition.

Definition 2 (Labeled Time Petri Net). A Labeled Time Petri Net \mathcal{N} over Σ_ε is a tuple $(P, T, \Sigma_\varepsilon, \bullet(\cdot), (\cdot)^\bullet, M_0, \Lambda, I)$ where P is a finite set of places, T is a finite set of transitions with $P \cap T = \emptyset$, $\bullet(\cdot) \in (\mathbb{N}^P)^T$ is the backward incidence mapping, $(\cdot)^\bullet \in (\mathbb{N}^P)^T$ is the forward incidence mapping, $M_0 \in \mathbb{N}^P$ is the initial marking, $\Lambda : T \rightarrow \Sigma_\varepsilon$ is the labeling function and $I : T \mapsto \mathcal{I}(\mathbb{Q}_{\geq 0})$ associates with each transition a closed firing interval.

A TPN \mathcal{N} is a g -TPN if for all $t \in T$, the interval $I(t)$ has its bounds in \mathbb{N}_g . We also use $\bullet t$ (resp. t^\bullet) to denote the set of places $\bullet t = \{p \in P \mid \bullet t(p) > 0\}$ (resp. $t^\bullet = \{p \in P \mid t^\bullet(p) > 0\}$) as is common in the literature.

A *configuration* of a TPN is a pair (M, ν) , where M is a *marking* in the usual sense, *i.e.* a mapping in \mathbb{N}^P , with $M(p)$ the number of tokens in place p . A transition t is *enabled* in a marking M iff $M \geq \bullet t$. We denote by $En(M)$ the set of enabled transitions in M . The second component of the pair (M, ν) describes the values of clocks implicitly associated with transitions enabled in M : a *valuation* ν is a mapping in $(\mathbb{R}_{\geq 0})^{En(M)}$. For $d \in \mathbb{R}_{\geq 0}$, the valuation $\nu + d$ is defined by $(\nu + d)(t) = \nu(t) + d$ for each $t \in En(M)$. An enabled transition t can be fired if $\nu(t)$ belongs to the interval $I(t)$. The result of this firing is as usual the new marking $M' = M - \bullet t + t \bullet$. Moreover, some valuations are reset and we say that the corresponding transitions are newly enabled. Different semantics are possible for this operation. In this paper, we choose *persistent atomic semantics*, which is slightly different from the classical semantics [7,3], but equivalent when the net is bounded [4]. The predicate is defined by:

$$\uparrow enabled(t', M, t) = t' \in En(M - \bullet t + t \bullet) \wedge (t' \notin En(M)).$$

Thus, firing a transition is considered as an atomic step and the transition currently fired behaves like the other transitions ($\nu(t)$ need not be reset when t is fired). The set $ADM(\mathcal{N})$ of (*admissible*) *configurations* consists of the pairs (M, ν) such that $\nu(t) \in I(t)^\downarrow$ for each transition $t \in En(M)$. Thus time can progress in a marking only up to the minimal right endpoint of the intervals for all enabled transitions.

Definition 3 (Semantics of TPN). *The semantics of a TPN $\mathcal{N} = (P, T, \Sigma_\varepsilon, \bullet(\cdot), (\cdot)^\bullet, M_0, \Lambda, I)$ is a TTS $S_{\mathcal{N}} = (Q, q_0, \rightarrow)$ where $Q = ADM(\mathcal{N})$, $q_0 = (M_0, \mathbf{0})$ and \rightarrow is defined by:*

- either a delay move $(M, \nu) \xrightarrow{d} (M, \nu + d)$ iff $\forall t \in En(M), \nu(t) + d \in I(t)^\downarrow$,
- or a discrete move $(M, \nu) \xrightarrow{\Lambda(t)} (M - \bullet t + t \bullet, \nu')$ where $\forall t' \in En(M - \bullet t + t \bullet), \nu'(t') = 0$ if $\uparrow enabled(t', M, t)$ and $\nu'(t') = \nu(t)$ otherwise, iff $t \in En(M)$ is such that $\nu(t) \in I(t)$.

We simply write $(M, \nu) \xrightarrow{w}$ to emphasise that a sequence of transitions w can be fired. If $\text{Duration}(w) = 0$, we say that w is an *instantaneous firing sequence*. A net is said to be k -bounded if for each reachable configuration (M, ν) and for each place p , $M(p) \leq k$.

Note that taking into account the enabling degree of transitions would require to add components to ν , which leads to awkward notations, although our result holds in the bounded case.

Timed Automata. First defined in [2], the model of timed automata (TA) associates a set of non negative real-valued variables called *clocks* with a finite automaton. Let X be a finite set of *clocks*. We write $\mathcal{C}(X)$ for the set of *constraints* over X , which consist of conjunctions of atomic formulas of the form $x \bowtie h$ for $x \in X$, $h \in \mathbb{Q}_{\geq 0}$ and $\bowtie \in \{<, \leq, \geq, >\}$.

Definition 4 (Timed Automaton). *A Timed Automaton \mathcal{A} over Σ_ε is a tuple $(L, \ell_0, X, \Sigma_\varepsilon, E, Inv)$ where L is a finite set of locations, $\ell_0 \in L$ is the initial location, X is a finite set of clocks, $E \subseteq L \times \mathcal{C}(X) \times \Sigma_\varepsilon \times 2^X \times L$ is a finite set of edges and $Inv \in \mathcal{C}(X)^L$ assigns an invariant to each location. An*

edge $e = \langle \ell, \gamma, a, R, \ell' \rangle \in E$ represents a transition from location ℓ to location ℓ' with guard γ and reset set $R \subseteq X$. We restrict the invariants to conjunctions of terms of the form $x \bowtie h$ for $x \in X$, $h \in \mathbb{N}$ and $\bowtie \in \{<, \leq\}$.

When we need to consider label-free automata, we simply assume that each edge has a unique label, different from ε .

A valuation v is a mapping in $\mathbb{R}_{\geq 0}^X$. For $R \subseteq X$, the valuation $v[R \mapsto 0]$ maps each variable in R to the value 0 and agrees with v over $X \setminus R$. Constraints of $\mathcal{C}(X)$ are interpreted over valuations: we write $v \models \gamma$ when the constraint γ is satisfied by v .

Definition 5 (Semantics of TA). *The semantics of a TA $\mathcal{A} = (L, \ell_0, X, \Sigma_\varepsilon, E, Inv)$ is a TTS $S_{\mathcal{A}} = (Q, q_0, \rightarrow)$ where $Q = L \times (\mathbb{R}_{\leq 0})^X$, $q_0 = (\ell_0, \mathbf{0})$ and \rightarrow is defined by:*

- either a delay move $(\ell, v) \xrightarrow{d} (\ell, v + d)$ iff $v + d \models Inv(\ell)$,
- or a discrete move $(\ell, v) \xrightarrow{e} (\ell', v')$ iff there exists some $e = (\ell, \gamma, a, R, \ell') \in E$ s.t. $v \models \gamma$, $v' = v[R \mapsto 0]$ and $v' \models Inv(\ell')$.

Elementary zones of a TA. Recall [2] that, if m is the maximal constant appearing in atomic formulas $x \bowtie c$ of \mathcal{A} , an equivalence relation with finite index can be defined on clock valuations, leading to a partition \mathcal{P}_m of $(\mathbb{R}_{\geq 0})^X$, with the following property: two equivalent valuations have the same behaviour under progress of time and reset operations, with respect to the constraints. Note that the same property holds for any partition which refines \mathcal{P}_m . This is the case in particular if we replace m by any $K \geq m$ instead of m , even with $K = +\infty$ (as depicted in Figure 1 on the left). Of course, a finite constant is needed for decidability results. Finally, we can also consider a g -grid, where all constants are of the form $\frac{i}{g}$, $0 \leq i \leq K \cdot g$ instead of $\{0, 1, \dots, K\}$.

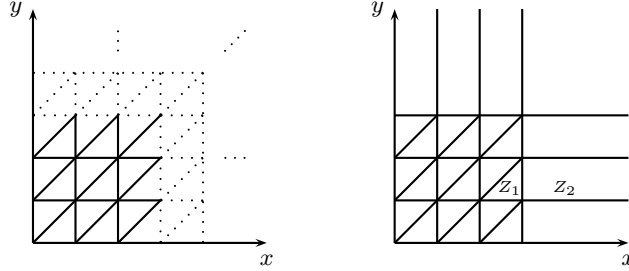


Fig. 1. Partitions of $(\mathbb{R}^+)^2$ with $K = +\infty$ and $K = 3$

In this paper, the elements of the partition are called *elementary zones* and we consider a slight variation for their definition: we take a constant $K \geq m + 1$ and with each clock $x \in X$, we associate an interval in the set $\{\{0\},]0, 1[, \{1\}, \dots, \{K-1\},]K-1, K[, [K, +\infty[\}$, instead of keeping $\{K\}$ separately. As usual, we also

specify the ordering on the fractional parts for all clocks x such that $x < K$. Such a partition is represented in Figure 1 (on the right) for the set of two clocks $X = \{x, y\}$ and $K = 3$. For this example, elementary zones Z_1 and Z_2 are described by the constraints: $Z_1 : (2 < x < 3) \wedge (1 < y < 2) \wedge (0 < \text{frac}(y) < \text{frac}(x))$ and $Z_2 : (x \geq 3) \wedge (1 < y < 2)$.

If Z and Z' are elementary zones, Z' is a time successor of Z , written $Z \leq Z'$, if for each valuation $v \in Z$, there is some $d \in \mathbb{R}_{\geq 0}$ such that $v + d \in Z'$. For each elementary zone Z , there is at most one elementary zone such that (i) Z' is a time successor of Z , (ii) $Z \neq Z'$ and (iii) there is no time successor Z'' different from Z and Z' such that $Z \leq Z'' \leq Z'$. When it exists, this elementary zone is called the immediate successor of Z and denoted by $\text{succ}(Z)$.

Standard topological notions on $(\mathbb{R}_{\geq 0})^X$ apply to elementary zones. Moreover, due to the particular form of the constraints, the topological closure of any elementary zone has a minimal element.

3 A characterisation of TA bisimilar to TPNs

Regions of a timed automaton. Since our results are mainly based on the *region automaton*, we recall its definition [2]. For a TA \mathcal{A} , a constant K and a granularity g , the region automaton $R(\mathcal{A})_{g,K}$ is a finite automaton with states of the form (ℓ, Z) , where ℓ is a location of \mathcal{A} and Z an elementary zone of $(\mathbb{R}_{\geq 0})^X$.

We call *region* a pair (ℓ, Z) . The regions of $R(\mathcal{A})_{g,K}$ are built inductively from the initial one $(\ell_0, \mathbf{0})$ by the following transitions over the set of labels $\{\text{succ}\} \cup \Sigma_\varepsilon$: $(\ell, Z) \xrightarrow{\text{succ}} (\ell, \text{succ}(Z))$ if $\text{succ}(Z) \models \text{Inv}(\ell)$ and $(\ell, Z) \xrightarrow{a} (\ell', Z')$ if there is a transition $(\ell, \gamma, a, R, \ell') \in E$ such that $Z \models \gamma$ and $Z' = Z[R \mapsto 0]$, with $Z' \models \text{Inv}(\ell')$. Thus, only *reachable* regions appear in $R(\mathcal{A})$. A region $r = (\ell, Z)$ is said to be *maximal* in $R(\mathcal{A})$ with respect to ℓ if no *succ*-transition is possible from r . In the sequel, the topological properties of r are implicitly derived from those of Z . We write \bar{r} for the topological closure of r (it is a finite union of regions), and we denote by min_r the minimal vector of \bar{r} .

We now give a definition which distinguishes time-closed and time-open descriptions for regions. It is equivalent to the original one but more convenient for our proofs and it fits both cases, whether K is finite or infinite.

Definition 6 (Region description for automaton $R(\mathcal{A})_{g,K}$).

A time-closed description of a region r is given by:

- ℓ_r the location of r ,
- $\text{min}_r \in \mathbb{N}_g^X$ with $\forall x, \text{min}_r(x) \leq K$, the minimal vector of the topological closure of r ,
- $\text{Act}X_r = \{x \in X \mid \text{min}_r(x) < K\}$ the subset of relevant clocks,
- the number size_r of different fractional parts for the values of relevant clocks in the $\mathbb{N}_g^{\text{Act}X_r}$ grid, with $1 \leq \text{size}_r \leq \text{Max}(|\text{Act}X_r|, 1)$ and the onto mapping $\text{ord}_r : X \mapsto \{1, \dots, \text{size}_r\}$ giving the ordering of the fractional parts.

By convention, $\forall x \in X \setminus \text{Act}X_r, \text{ord}_r(x) = 1$.

Then $r = \{(\ell_r, \text{min}_r + \delta) \mid \delta \in \mathbb{R}_{\geq 0}^X \wedge \forall x, y \in \text{Act}X_r [\text{ord}_r(x) = 1 \Leftrightarrow \delta(x) = 0] \wedge \delta(x) < 1/g \wedge [\text{ord}_r(x) < \text{ord}_r(y) \Leftrightarrow \delta(x) < \delta(y)]\}$

A time-open description of a region r is defined with the same attributes (and conditions) as the time-closed one with:

$$r = \{(\ell_r, \min_r + \delta + d) \mid d \in \mathbb{R}_{>0} \wedge \forall x \in \text{Act}X_r, \delta(x) + d < 1/g\}.$$

The set $[X]_r$ is the set of equivalence classes of clocks w.r.t. their fractional parts, i.e. x and y are equivalent iff $\text{ord}_r(x) = \text{ord}_r(y)$.

Remark that $\min_r \notin r$ except if there is a single class of clocks relative to r (for instance if the corresponding zone is a singleton). Of course, when $K = +\infty$, the part about *relevant* clocks, for which the value is less than K , can be omitted (since $\text{Act}X_r = X$). This hypothesis makes some proofs simpler, because the extremal case where a clock value is greater than K is avoided, and it can be lifted afterward. Furthermore when K is finite, some regions admit both time-open and time-closed descriptions (for instance a region associated with zone Z_2 in fig. 1), whereas when $K = +\infty$, a region admits a single description, so that time elapsing leads to an alternation of time-open regions (where time can elapse) and time-closed ones (where no time can elapse).

Reachability. For a reachable region r of $R(\mathcal{A})_{g,K}$, not all configurations of r are reachable. Nevertheless, by induction on the reachability relation, the following property can be shown: For any reachable region r , there is a region $\text{reach}(r)$ w.r.t. the 1-grid and constant $K = \infty$ such that (i) $\text{reach}(r) \subset r$, (ii) each configuration of $\text{reach}(r)$ is reachable and (iii) if $\text{reach}(r)$ is a time-open region then r admits a time-open description else r admits a time-closed description. As a consequence, we have: $\forall x \in \text{Act}X_r, \min_{\text{reach}(r)}(x) = \min_r(x)$ and $\forall x \in X \setminus \text{Act}X_r, \min_{\text{reach}(r)}(x) \geq K$ and ord_r restricted to $\text{Act}X_r$ is identical to $\text{ord}_{\text{reach}(r)}$.

Consider now the relation \mathcal{R} defined by $(l, v) \mathcal{R} (l, v')$ iff $\forall x \in X, v'(x) = v(x) \vee (v(x) \geq K \wedge v'(x) \geq K)$. It is a strong timed bisimulation relation. From the previous observations, we note that each configuration of a reachable region is strongly timed bisimilar to a reachable configuration of this region. Thus speaking about reachability of regions is a slight abuse of notations.

We can now state our main results.

Theorem 1 (Characterisation of TA bisimilar to some TPN). *Let \mathcal{A} be a (label-free) timed automaton and $R(\mathcal{A})$ its region automaton w.r.t. the 1-grid and a constant K strictly greater than any constant occurring in the automaton, then \mathcal{A} is weakly timed bisimilar to a time Petri net iff for each region r of $R(\mathcal{A})$ and for each edge e from \mathcal{A} ,*

- (a) *Every region r' such that $r' \cap \bar{r} \neq \emptyset$ is reachable*
- (b) *$\forall (\ell_r, v) \in r$, if $(\ell_r, v) \xrightarrow{e}$ then $(\ell_r, \min_r) \xrightarrow{e}$*
- (c) *$\forall (\ell_r, v) \in \bar{r}$, if $(\ell_r, \min_r) \xrightarrow{e}$ then $(\ell_r, v) \xrightarrow{e}$.*

Furthermore, if these conditions are satisfied then we can build a 1-bounded 2-TPN bisimilar to \mathcal{A} whose size is linear w.r.t. the size of \mathcal{A} and a 1-bounded 1-TPN bisimilar to \mathcal{A} whose size is exponential w.r.t. the size of \mathcal{A} .

We denote by \mathcal{TA}^{wtb} the corresponding subclass of timed automata.

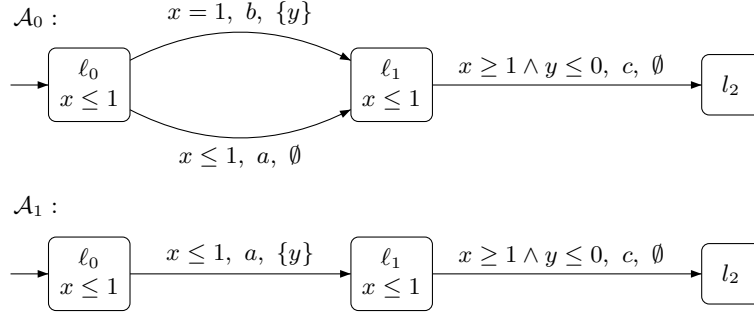


Fig. 2. Two automata with different behaviours w.r.t bisimulation with a TPN

Theorem 2 (Complexity results). *Given a (label-free) timed automaton \mathcal{A} , deciding whether there is a TPN weakly timed bisimilar to \mathcal{A} is PSPACE-complete. The reachability problem for the class \mathcal{TA}^{wtb} is PSPACE-complete.*

The characterisation of Theorem 1 is closely related to the topological closure of reachable regions: it states that any region intersecting the topological closure of a reachable region is also reachable and that a discrete step either from a region or from the minimal vector of its topological closure is possible in the whole topological closure. Consider the two TA \mathcal{A}_0 and \mathcal{A}_1 in Figure 2. The automaton \mathcal{A}_0 admits a bisimilar TPN whereas \mathcal{A}_1 does not. Indeed, the region $r = \{(\ell_1, x = 1 \wedge 0 < y < 1)\}$ is reachable. The guard of edge c is true in $\min_r = (\ell_1, (1, 0))$ whereas it is false in r .

The next section is devoted to a sketch of the proof of Theorem 1. The complete proofs can be found in [6].

4 Proof of Theorem 1

4.1 Necessary condition

From bisimulation to uniform bisimulation. As a first step, we prove that when a TPN and a TA are bisimilar, this relation can in fact be strengthened in what we call *uniform bisimulation*. We first need a lemma which points out the effect of time granularity on the behaviour of TPN.

Lemma 1. *Let (M, ν) and $(M, \nu + \delta)$ be two admissible configurations of a g -TPN with $\nu, \delta \in \mathbb{R}_{\geq 0}^{En(M)}$. Let w be an instantaneous firing sequence, then:*

- (i) $(M, \nu) \xrightarrow{w}$ implies $(M, \nu + \delta) \xrightarrow{w}$
- (ii) If $\nu \in \mathbb{N}_g^{En(M)}$ and $\delta \in [0, 1/g]^{En(M)}$ then $(M, \nu + \delta) \xrightarrow{w}$ implies $(M, \nu) \xrightarrow{w}$

Lemma 2 is the central point for the proof of necessity. It shows that bisimulation implies uniform bisimulation for the g -grid with $K = \infty$. Roughly speaking,

uniform bisimulation means that a unique mechanism is used for every configuration of the topological closure of the region to obtain a bisimilar configuration of the net.

Lemma 2 (From bisimulation to uniform bisimulation). *Let \mathcal{A} be a timed automaton bisimilar to some g -TPN \mathcal{N} via some relation \mathcal{R} and let $R(\mathcal{A})_{g,\infty}$ be a region automaton of \mathcal{A} . Then:*

- if a region r belongs to $R(\mathcal{A})_{g,\infty}$ then \bar{r} also belongs to $R(\mathcal{A})_{g,\infty}$;
- for each reachable region r , there exist a configuration of the net (M_r, ν_r) with $\nu_r \in \mathbb{N}_g^{En(M_r)}$ and a mapping $\phi_r : En(M_r) \rightarrow [X]_r$ such that:
 - If r is time-closed, then for each $\delta \in \mathbb{R}_{\geq 0}^X$ such that $(\ell_r, \min_r + \delta) \in \bar{r}$, $(\ell_r, \min_r + \delta) \mathcal{R} (M_r, \nu_r + \text{proj}_r(\delta))$,
 - If r is time-open, then for each $\delta \in \mathbb{R}_{\geq 0}^X$, $d \in \mathbb{R}_{\geq 0}$ such that $(\ell_r, \min_r + \delta + d) \in \bar{r}$, $(\ell_r, \min_r + \delta + d) \mathcal{R} (M_r, \nu_r + \text{proj}_r(\delta) + d)$,
 where $\text{proj}_r(\delta)(t) = \delta(\phi_r(t))$.

Proof. First note that the choice of a particular clock x in the class $\phi_r(t)$ is irrelevant when considering the value $\delta(x)$. Thus the definition of proj_r is sound. The proof is an induction on the transition relation in the region automaton. The basis case is straightforward with $\{(l_0, \mathbf{0})\}$ and $\{(M_0, \mathbf{0})\}$. The induction part relies on lemma 1, with 4 cases, according to the incoming or target region and to the nature of the step: 1. a time step from a time-closed region, 2. a time step from a time-open region, 3. a discrete step into a time-closed region, and 4. a discrete step into a time-open region. \square

Proof of Necessity. The fact that conditions (a), (b) and (c) of Theorem 1 hold for $R(\mathcal{A})_{g,\infty}$ is straightforward:

- (a) This assertion is included in the inductive assertions.
- (b) Let r be a reachable region, let $(\ell_r, \min_r + \delta) \in r$ be a configuration with $\delta \in [0, 1/g]^X$, then $\exists(M, \nu) \nu \in \mathbb{N}_g^{En(M)}$ bisimilar to (ℓ_r, \min_r) and $(M, \nu + \delta')$ with $\delta' \in [0, 1/g]^{En(M)}$ bisimilar to $(\ell_r, \nu + \delta)$. Suppose that $(\ell_r, \min_r + \delta) \xrightarrow{e}$, then $(M, \nu + \delta') \xrightarrow{w}$ with w an instantaneous firing sequence and $\text{label}(w) = e$. Now by lemma 1-(ii), $(M, \nu) \xrightarrow{w}$, thus $(\ell_r, \min_r) \xrightarrow{e}$.
- (c) Let r be a region, and $(\ell_r, \min_r + \delta) \in \bar{r}$ with $\delta \in [0, 1/g]^X$ thus $\exists(M, \nu)$ bisimilar to (ℓ_r, \min_r) and $(M, \nu + \delta')$ with $\delta' \in [0, 1/g]^{En(M)}$ bisimilar to $(\ell_r, \min_r + \delta)$. Suppose that $(\ell_r, \min_r) \xrightarrow{e}$, then $(M, \nu) \xrightarrow{w}$ with w an instantaneous firing sequence and $\text{label}(w) = e$. By lemma 1-(i), we have $(M, \nu + \delta') \xrightarrow{w}$, thus $(\ell_r, \min_r + \delta) \xrightarrow{e}$.

In order to complete the proof, we successively show that if the conditions are satisfied in $R(\mathcal{A})_{g,\infty}$ for some g , they also hold for $R(\mathcal{A})_{1,\infty}$, and finally that they are satisfied in $R(\mathcal{A})_{1,K}$, with a finite constant K .

4.2 Sufficient condition

Starting from a TA \mathcal{A} satisfying the conditions of Theorem 1, we build a 2-TPN bisimilar to \mathcal{A} . We describe the construction, the proof of correctness as well as the construction of a 1-TPN can be found in [6].

For this construction, all edges are weighted by 1. Omitted labels for transitions stand for ε . A firing interval $[0, 0]$ is indicated by a blackened transition and intervals $[0, \infty[$ are omitted. A double arrow between a place p and a transition t indicates that p is both an input and an output place for t .

W.l.o.g. we assume that an invariant never forbids to enter a state (by adding constraints to the input transitions). We then remark that $x < c$ occurring in an invariant of \mathcal{A} may be safely omitted. If it would forbid the progress of time in some configuration, then the associated region would be a maximal time-open region r . Due to condition (a), \bar{r} is reachable but since r is time-open, $\bar{r} \cap \text{succ}(r) \neq \emptyset$, so that $\text{succ}(r)$ is reachable which contradicts the maximality of r .

Clock constraints. The atomic constraints associated with a clock x are arbitrarily numbered from 1 to $n(x)$ where $n(x)$ is the number of such conditions. When $x \leq h$ occurs in at least one transition and in at least one invariant, we consider it as two different conditions. Then we add places $(Rtodo_i^x)_{i \leq n(x)+1}$ for the reset operations. We build a subnet for each atomic constraint $x \bowtie h$ occurring in a transition of the TA, and one for each condition $x \leq h$ occurring in an invariant. Figure 3 below shows the subnets corresponding to $x < h$ (with $h > 0$) on the left and $x \leq h$ on the right. Since constant $\frac{1}{2}$ appears in interval bounds, the resulting TPN is a 2-TPN.

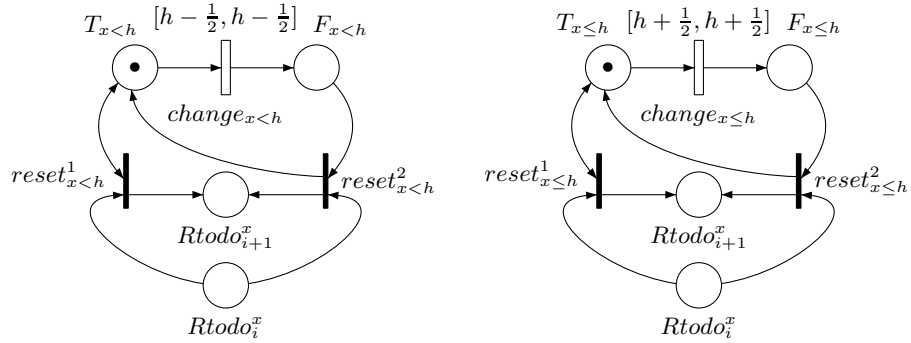


Fig. 3. The subnets for $x < h$ (with $h > 0$) and $x \leq h$

Locations and edges. With each location ℓ of the automaton, we associate an eponymous place ℓ . The place ℓ is initially marked iff the location ℓ is the initial one. The invariant $Inv(\ell)$ is tested with the subnets corresponding to its atomic constraints. To simulate an edge $(\ell, \gamma, a, R, \ell')$, we must test the atomic constraints from $\gamma = \gamma_1 \wedge \dots \wedge \gamma_{m(e)}$, using the places corresponding to true in the associated subnets, and reset successively all the clocks in $R = \{x_1, \dots, x_{n(e)}\}$ by instantaneous transitions. This is done by the subnet in Figure 4, which must be connected to some subsets like those of Figure 3.

This construction is illustrated in Figure 4.2 for the timed automaton \mathcal{A}_0 from Figure 2 with some simplifications related to this particular TA. Note that the subnet associated to the constraint $y \leq 0$ switches the condition to false (firing

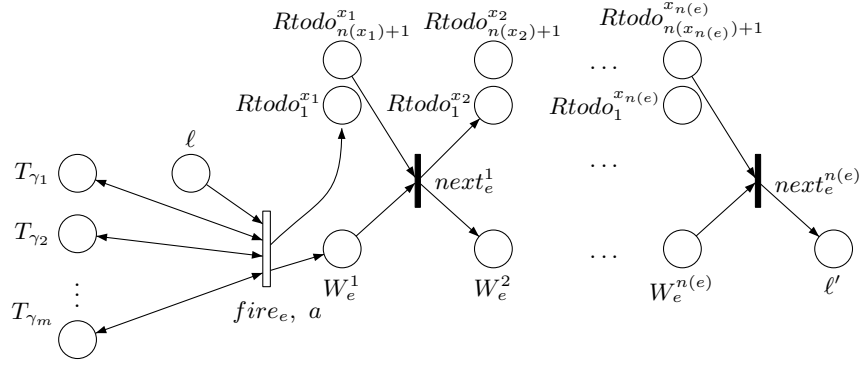


Fig. 4. The subnet for edge $e = (\ell, \gamma = \gamma_1 \wedge \dots \wedge \gamma_{m(e)}, a, R = \{x_1, \dots, x_{n(e)}\}, \ell')$

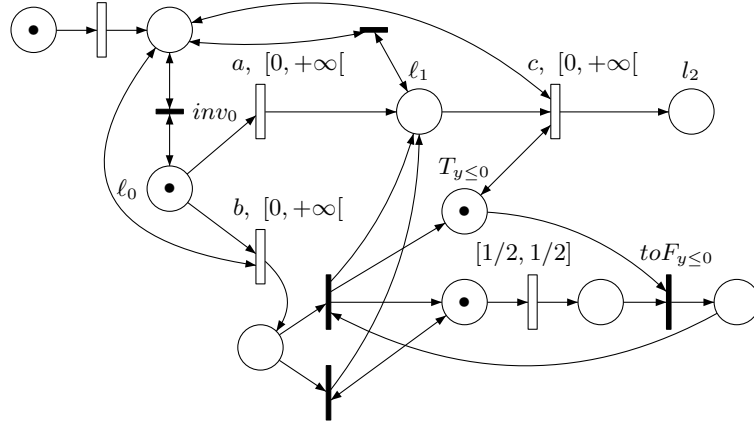


Fig. 5. The 2-TPN bisimilar to \mathcal{A}_0

of $toF_{y \leq 0}$) when the implicit value of y maintained in the net reaches $1/2$. This translation thus seems less constrained than the original condition. However, conditions (b) and (c) ensure that the configurations where both constraints do not simultaneously hold are not reachable.

5 Conclusion

In this paper, we considered the (semantic) subclass \mathcal{TA}^{wtb} of labeled-free TA such that a timed automaton \mathcal{A} is in \mathcal{TA}^{wtb} if and only if there is a TPN \mathcal{N} weakly timed bisimilar to \mathcal{A} . We obtained a characterisation of this class, based on the region automaton associated with \mathcal{A} . To prove that our condition is necessary, we introduced the notion of uniform bisimulation between TA and TPNs. For the sufficiency, we proposed two constructions. From this characterisation, we have proved that for the class \mathcal{TA}^{wtb} , the membership problem and the reachability problem are $PSPACE$ -complete. The techniques introduced here also lead to a similar characterisation for TA with diagonal constraints and to

a simpler one for TA without strict or diagonal constraints (see [6]), the complexity results being unchanged. These techniques also give some insight for use of the region automaton in order to obtain expressivity results. Further work would consist in finding a characterization for a larger class of intervals.

References

1. L. Aceto and F. Laroussinie. Is Your Model Checker on Time? On the Complexity of Model Checking for Timed Modal Logics. *Journal of Logic and Algebraic Programming*, volume 52-53, pages 7-51. Elsevier Science Publishers, august 2002.
2. R. Alur and D. Dill. A theory of timed automata. *Theoretical Computer Science B*, 126:183–235, 1994.
3. T. Aura and J. Lilius. A causal semantics for time Petri nets. *Theoretical Computer Science*, 243(1–2):409–447, 2000.
4. B. Bérard, F. Cassez, S. Haddad, D. Lime and O.H. Roux. Comparison of Different Semantics for Time Petri Nets. *ATVA '05, Taipei, Taiwan*, volume 3707 of *LNCS*, 2005, to appear.
5. B. Bérard, F. Cassez, S. Haddad, D. Lime and O.H. Roux. Comparison of the Expressiveness of Timed Automata and Time Petri Nets . *FORMATS'05, Uppsala, Sweden, LNCS*, 2005, to appear.
6. B. Bérard, F. Cassez, S. Haddad, D. Lime and O.H. Roux. Comparison of the Expressiveness of Timed Automata and Time Petri Nets. Research Report IRCCyN R2005-2 available at <http://www.lamsade.dauphine.fr/haddad/publis.html> 2005.
7. B. Berthomieu and M. Diaz. Modeling and verification of time dependent systems using time Petri nets. *IEEE Transactions on Software Engineering*, 17(3):259–273, March 1991.
8. B. Berthomieu and M. Menasche. An enumerative approach for analyzing time Petri nets. In *Information Processing*, volume 9 of *IFIP congress series*, pages 41–46. Elsevier Science Publishers, 1983.
9. B. Berthomieu and F. Vernadat. State class constructions for branching analysis of time Petri nets. In *TACAS'2003*, volume 2619 of *LNCS*, pages 442–457, 2003.
10. F. Cassez and O. H. Roux. Structural Translation of Time Petri Nets into Timed Automata. In Michael Huth, editor, *Workshop on Automated Verification of Critical Systems (AVoCS'04)*, Electronic Notes in Computer Science. Elsevier, August 2004.
11. D. L. Dill. Timing assumptions and verification of finite-state concurrent systems. In Proc. Workshop on Automatic Verification Methods for Finite State Systems, Grenoble, volume 407 of *LNCS*, 1989.
12. S. Haar, F. Simonot-Lion, L. Kaiser, and J. Toussaint. Equivalence of Timed State Machines and safe Time Petri Nets. In *Proceedings of WODES 2002*, Zaragoza, Spain, pages 119–126.
13. D. Lime and O. H. Roux. State class timed automaton of a time Petri net. In *PNPM'03*. IEEE Computer Society, September 2003.
14. P. M. Merlin. *A study of the recoverability of computing systems*. PhD thesis, University of California, Irvine, CA, 1974.
15. M. Pezzé and M. Young. Time Petri Nets: A Primer Introduction. Tutorial presented at the Multi-Workshop on Formal Methods in Performance Evaluation and Applications, Zaragoza, Spain, september 1999.
16. C. Ramchandani. *Analysis of asynchronous concurrent systems by timed Petri nets*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, 1974.