Semilinearity of Vector Addition Systems with States

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Keywords: Vector Addition System with States (VASS), Presburger Arithmetic,

Semilinearity, Reachability Analysis

Summary

The decidability of the reachability problem for VASS has been open for a long time, and today the computational complexity of the problem is still open. Obviously, there is more hope of obtaining elementary complexity upper bounds by restricting the class of VASS, for example by considering those for which the reachability sets are effectively semilinear. Presburger arithmetic would then provide a means to represent symbolically the relevant sets of tuples of integers. Hence, an angle of attack for the general VASS reachability problem has been to determine subclasses of VASS that have effectively computable semilinear reachability sets.

There is, indeed, a rich literature on subclasses of VASS having an effectively computable semilinear reachability set: VASS of dimension 2 (Hopcroft and Pansiot, 1979), reversible and cyclic VASS (Taiclin, 1968; Araki and Kasami, 1977; Bouziane and Finkel, 1997), conflict-free and persistent VASS (Landweber and Robertson, 1978; Mayr, 1981), regular and context-free VASS (Valk and Vidal-Naquet, 1981; Schwer, 1992), BPP-nets (Esparza, 1997; Fribourg and Olsén, 1997). In addition to solving the reachability problem, a number of other interesting properties are immediately decidable when the reachability set is semilinear and effectively computable, such as boundedness, deadlock freedom, satisfaction of linear inequalities, etc. Moreover, the equivalence problem (two VASS are equivalent if they have the same reachability set), which is undecidable for general VASS, becomes decidable in that case.

The above-mentioned classes of VASS are contained in the class of semilinear VASS, i.e., the class of VASS having a semilinear reachability set. It was proved independently by Hauschildt and Lambert that the class of semilinear VASS is recursive: checking whether a given VASS has a semilinear reachability set is decidable (see the unpublished works (Hauschildt, 1990; Lambert, 1994)). Moreover, the reachability set is effectively computable when it is semilinear. Unfortunately, their proofs rely on the structures used by Kosaraju and Mayr for deciding the general VASS reachability problem, which means that they suffer from the same complexity and implementability issues.

The research program for this PhD thesis consists in revisiting the problem of deciding whether a given VASS has a semilinear reachability set: (a) find a simpler decidability proof, inspired by the recent methods developed in (Leroux, 2011) for the reachability problem, and (b) investigate the computational complexity of this problem, which is open. The new proof techniques should be amenable to an implementation.

Qualifications

Ideally, the candidate holds a Master degree in Computer Science (with courses in formal verification, theoretical computer science and mathematical structures for CS) or equiva-

lently is graduated from a Computer Science Engineering School with a strong background in theoretical computer science.

References

Araki, T. and Kasami, T., 1977. Decidable problems on the strong connectivity of Petri net reachability sets. *Theor. Comput. Sci.*, 4(1):99–119. doi:10.1016/0304-3975(77)90059-7.

Bouziane, Z. and Finkel, A., 1997. Cyclic petri net reachability sets are semi-linear effectively constructible. In *INFINITY'97, Elec. Notes Theor. Comput. Sci.* 9, pp. 15–24. Elsevier. doi:10.1016/S1571-0661(05)80423-2.

Esparza, J., 1997. Petri nets, commutative context-free grammars, and basic parallel processes. *Fund. Inform.*, 31(1):13–25.

Fribourg, L. and Olsén, H., 1997. A decompositional approach for computing least fixed-points of Datalog programs with Z-counters. *Constraints*, 2(3–4):305–335. doi:10.1023/A:1009747629591.

Hauschildt, D., 1990. *Semilinearity of the reachability set is decidable for Petri nets*. PhD thesis, University of Hamburg.

Hopcroft, J.E. and Pansiot, J.J., 1979. On the reachability problem for 5-dimensional vector addition systems. *Theor. Comput. Sci.*, 8:135–159. doi:10.1016/0304-3975(79)90041-0.

Lambert, J.L., 1994. Vector addition systems and semi-linearity. Accepted for publication in *SIAM J. Comp.*

Landweber, L. and Robertson, E., 1978. Properties of conflict-free and persistent Petri nets. *J. ACM*, 25(3):352–364. doi:10.1145/322077.322079.

Leroux, J., 2011. Vector addition system reachability problem: a short self-contained proof. In *POPL 2011*, pp. 307–316. ACM Press. doi:10.1145/1926385.1926421.

Mayr, E., 1981. Persistence of vector replacement systems is decidable. *Acta Inf.*, 15:309–318. doi:10.1007/BF00289268.

Schwer, S., 1992. The context-freeness of the languages associated with vector addition systems is decidable. *Theor. Comput. Sci.*, 98(2):199–247. doi:10.1016/0304-3975(92)90002-W.

Taiclin, M., 1968. Algorithmic problems for commutative semigroups. *Soviet Math. Doklady*, 9(1):201–204.

Valk, R. and Vidal-Naquet, G., 1981. Petri nets and regular languages. *J. Comput. Syst. Sci.*, 23(3):299–325. doi:10.1016/0022-0000(81)90067-2.