

# A Symbolic Procedure for Control Reachability in the Asynchronous $\pi$ -calculus (Extended Abstract)

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## Abstract

We study the relationship between the asynchronous  $\pi$ -calculus and the specification language  $\text{MSR}_{\text{NC}}$  combining *multiset rewriting over first-order atomic formulas* (MSR) and *name constraints* (NC) proposed in [10]. We exploit this connection to define a sound and fully automatic procedure for attacking *control reachability* for infinite-state specifications given in asynchronous  $\pi$ -calculus, i.e., for specifications of *mobile processes* with unbounded control, name generation, and name mobility.

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## 1 Introduction

In [13] German and Sistla established a connection between Petri Nets and CCS by means of which automated verification methods like the covering graph construction could be transferred to CCS-like models (see e.g. [5]). In this setting individual processes are basically viewed as “communicating finite state machines”, whereas the entire system is composed of an arbitrary (but finite) number of processes. The connection between CCS and Petri Nets has been extended in several different ways. For instance, in [12] communication mechanisms like *broadcast* have been modelled via *transfer arcs*. Control reachability is still decidable for the extended Petri Nets models proposed in [12]. Formalisms used to specify *mobile processes*, often called *nominal calculi* [14], represent another important extension of *value passing CCS*. In this setting the use of channel names as values provides for a dynamic reconfiguration of the network (i.e. of the communication links between processes). A well-known example of nominal calculus is the  $\pi$ -calculus [16]. In the  $\pi$ -calculus *process mobility* is achieved by using names as communication ports. Automated verification of specifications in the  $\pi$ -calculus becomes particularly challenging due to the presence of *fresh name generation*, *name mobility*, and *unbounded control*, i.e., their state-space is infinite in *several dimensions*. The application of automatic verification techniques developed for Petri Nets to specifications

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given in the  $\pi$ -calculus has been explored in different works in the literature. For instance, in [4] control reachability has been shown to be decidable for different *fragments* of asynchronous  $\pi$ -calculus ( $\pi_a$ ) via a reduction to Petri Nets with *transfer arcs*. The use of Petri Nets indicates a restriction to models with a single infinite dimension (e.g. the number of processes or the number of names). Similar restrictions are taken in other verification methods for mobile systems like, e.g., [17,18,19], where processes are required to be *finitary* (there is a bound on the number of parallel components generated during execution).

The research direction that we are currently investigating concerns instead the applicability of *infinite-state verification methods* developed for concurrent systems for several sources of infiniteness [10] to mobile processes. Specifically, in this paper we will investigate the connection between specifications of *mobile processes* given in the asynchronous  $\pi$ -calculus and  $\text{MSR}_{NC}$  [10,11], a specification language based on *multiset rewriting over first order atomic formulas* (MSR) and *name constraints* (NC).  $\text{MSR}_{NC}$  is a conservative extension of Petri Nets, in which tokens are represented via *atomic formulas*, and in which constraints are used to specify the relationships defined over data attached to the tokens. To establish a formal connection, we embed the formulation of asynchronous  $\pi$ -calculus proposed in [4] based on the notion of *normalised equations* into  $\text{MSR}_{NC}$ . The proposed encoding preserves (control) reachability and opens a way for transferring the verification procedures for attacking control reachability for  $\text{MSR}_{NC}$  proposed in [10] to mobile processes. The verification method is based on a symbolic representation of upward closed sets of configurations of unbound  $\pi_a$  specifications. This data structure can be used then to attack the control reachability problem using *symbolic backward reachability*. In fact, the computation of the *pre-image* of a  $\pi_a$  specification can be made effective by using the encoding and by specializing the pre-image operator defined for  $\text{MSR}_{NC}$  specifications.

The resulting method gives us a *fully automatic* and *sound* procedure for the verification of safety properties (often reducible to control reachability) for mobile processes. Termination cannot be guaranteed for generic  $\pi_a$  specification. However, techniques like abstract interpretation or heuristics inspired to the Structural Theory of Petri Nets can be used here to enforce termination, to accelerate the speed of the analysis, or to simply compute approximated results. Furthermore, the study of fragments of  $\pi_a$  related to the monadic fragment of  $\text{MSR}_{NC}$  (for which backward reachability terminates) could represent a promising research line for finding new decidability results for mobile processes.

### 1.1 Asynchronous $\pi$ -calculus ( $\pi_a$ )

The *asynchronous  $\pi$ -calculus* ( $\pi_a$ ) is a subcalculus of the  $\pi$ -calculus without choice and match and in which message emission is non-blocking [15,6]. The

set of  $\pi_a$  processes is defined as follows

$$P ::= \mathbf{0} \mid \bar{x}y \mid x(y).P \mid P_1|P_2 \mid (\nu x)P \mid !P$$

The term  $\mathbf{0}$  denotes a null process. The output term  $\bar{x}y$  denotes an asynchronous message with target  $x$  and content  $y$ . With the input prefix  $x(y).P$  a process receives an arbitrary name  $z$  at channel  $x$  and then behaves like  $P[z \mapsto y]$ . The process  $P[z \mapsto y]$  is the result of substituting all free occurrences of  $y$  in  $P$  by  $z$ . The argument  $y$  of  $x(y)$  binds all free occurrences of  $y$  in  $P$ . The composition  $P|Q$  consists of  $P$  and  $Q$  running in parallel. The restriction  $(\nu x)P$  behaves like  $P$  except that it cannot exchange messages targeted to  $x$  with the environment; the argument  $x$  of  $(\nu x)$  binds all free occurrences of  $x$  in  $P$ . The replication  $!P$  provides an arbitrary number of copies of process  $P$  ( $!P \equiv P \mid !P$ ).

In [4], Amadio and Meyssonier proposed an equivalent reformulation based on the notion of *normalised* parametric equations in which repetition is replaced by recursion. In this paper we will take it as reference model. Let us use  $\vec{a}$  to denote a tuple  $a_1, \dots, a_n$  of names, and  $[\vec{a} \mapsto \vec{b}]$  to indicate a substitution mapping  $a_i$  to  $b_i$  for  $i : 1, \dots, n$ . Furthermore, let the term  $(\nu \vec{v})P$  denote the term  $(\nu v_1) \dots (\nu v_n)P$ . Following [4], a *normalised* parametric equation is defined as follows

$$A(\vec{x}) = \underbrace{a(\vec{u})}_{\text{input}} \cdot \underbrace{(\nu \vec{v})}_{\text{gen.}} \underbrace{(\bar{a}_1 \vec{y}_1 \mid \dots \mid \bar{a}_n \vec{y}_n)}_{\text{output}} \mid \underbrace{A_1(\vec{w}_1) \dots \mid A_m(\vec{w}_m)}_{\text{continuations}}$$

where  $A, A_1, \dots$  denote process identifier; the (bound) names in  $\vec{x}, \vec{u}, \vec{v}$  are all distinct each other;  $a_i, \vec{y}_i$  and  $\vec{w}_j$  are names taken from  $\vec{x}, \vec{u}, \vec{v}$  for  $i : 1, \dots, n$  and  $j : 1, \dots, m$ . We will use  $Fn(P)$  to denote the set of free names in the body of an equation  $A(\vec{x}) = P$ . A *process* is defined via a set  $\mathcal{E}$  of normalised parametric equations, and by an *initial configuration*. A *configuration* is formally defined as a normalised process of the shape

$$(\nu \vec{v}) \underbrace{(\bar{a}_1 \vec{y}_1 \mid \dots \mid \bar{a}_n \vec{y}_n)}_{\text{messages}} \mid \underbrace{A_1(\vec{w}_1) \dots \mid A_m(\vec{w}_m)}_{\text{processes}}$$

Two configurations  $P$  and  $Q$  are equivalent, written  $P \equiv Q$ , if  $P$  is syntactically equal to  $Q$  up to renaming of bound names, and associativity and commutativity of parallel composition.

The operational semantics of a process is defined as the reflexive-transitive closure of the reduction relation  $\cdot \Rightarrow_{\pi_a} \cdot$  defined over configurations as follows. Let  $P$  be the configuration  $(\nu \vec{b}')(A(\vec{b}) \mid \bar{c}(\vec{c}) \mid Q)$  where  $Q$  is a multiset of messages and continuations, and let  $D \in \mathcal{E}$  be the equation  $A(\vec{x}) = a(\vec{u}).(\nu \vec{v})R$  such that the set of names  $\vec{x}, \vec{u}, \vec{v}$  and  $\vec{b}'$  and  $\cup Fn(P)$  are all distinct each other. Given  $\sigma = [\vec{x} \mapsto \vec{b}, \vec{u} \mapsto \vec{c}]$  and its natural extensions  $\hat{\sigma}$  to expressions, if  $\hat{\sigma}(a) = c$ , then  $P$  reduces to  $P'$ , written  $P \Rightarrow_{\pi_a} P'$ , where  $P' = (\nu \vec{b}', \vec{v}) (\sigma(R) \mid Q)$ ,

and  $\sigma(R)$  is the natural extension of  $\sigma$  to a process expression.

The *control reachability problem* [4] is defined as follows. Given a process  $\mathcal{E}$  containing the process identifier  $A$ , and an initial configuration  $P$ , does  $P \Rightarrow_{\pi_a}^* Q$  hold with  $Q = \nu \vec{a}.(A(\vec{b}) \mid Q')$  for some  $\vec{b}$  and  $Q'$ ?

**Example 1.1** Let us consider the following equations:

$$Init(a) = (\nu p)(\bar{a}p \mid Wait(p)),$$

$$Wait(p) = p(x).EndI(p, x),$$

$$Resp(a) = a(y).(\nu ok)(\bar{y}ok \mid EndR(y, ok)).$$

Given  $P = (\nu c)(Init(c) \mid Resp(c))$ , a possible reduction is as follows

$$\begin{aligned} P &= (\nu c, p)(\bar{c}p \mid Wait(p) \mid Resp(c)) \\ &\Rightarrow_{\pi_a} (\nu c, ok, p)(Wait(p) \mid \bar{p} ok \mid EndR(p, ok)) \\ &\Rightarrow_{\pi_a} (\nu c, ok, p)(EndI(p, ok) \mid EndR(p, ok)) \end{aligned}$$

This reduction describes a run of the protocol in which *Init* and *Resp* exchange the private channel name  $p$  along which *Resp* sends an acknowledge to *Init*. If we add the equation

$$Start = (\nu c)(Init(c) \mid Resp(c) \mid Start)$$

then *Start* will generate an arbitrary number of *sessions* of the *Init* – *Resp* protocol.

## 1.2 The Specification Language $MSR_{NC}$

Let  $\mathcal{V}$  be a set of variables. We call *name constraint* a conjunction  $\varphi_1, \dots, \varphi_n$  of atomic formulas of the shape *true*,  $x > y$ ,  $x = y$ ,  $x \neq y$ , or  $x \geq y$  with  $x, y \in \mathcal{V}$ . The set of *solutions* *Sol* of a constraint  $\varphi$  consists of all evaluations from  $\mathcal{V}$  to  $\mathbb{Z}$  (*integer numbers*) that make  $\varphi$  true. A constraint  $\varphi$  is *satisfiable* whenever  $Sol(\varphi) \neq \emptyset$ .

Let  $\mathcal{P}$  be a set of predicate symbols. An *atomic formula*  $p(x_1, \dots, x_n)$  is such that  $p \in \mathcal{P}$ , and  $x_1, \dots, x_n \in \mathcal{V}$ . A *multiset* of atomic formulas is indicated as  $A_1, \dots, A_k$ , where the symbol “,” is an *associative-commutative* term constructor not occurring inside atomic formulas. We use “,” instead of the symbol “|” used in [10] to avoid confusion with parallel composition in  $\pi_a$ . In the rest of the paper will use  $\mathcal{M}, \mathcal{N}, \dots$  to denote *multisets* of atomic formulas,  $\epsilon$  to denote the *empty multiset*,  $\oplus$  to denote *multiset union* and  $\ominus$  to denote *multiset difference*.

A *configuration* is a multiset of *ground atomic formulas*, i.e., atomic formulas where all variables are instantiated with *integer values*. An  $MSR_{NC}$  rule has the form

$$A_1, \dots, A_n \longrightarrow B_1, \dots, B_m : \varphi$$

where  $\mathcal{M} = A_1, \dots, A_n$  and  $\mathcal{M}' = B_1, \dots, B_m$  are two (possibly empty) multisets of atomic formulas built on predicates in  $\mathcal{P}$ , and  $\varphi$  is a constraint such that  $Var(\varphi) \subseteq Var(\mathcal{M}) \cup Var(\mathcal{M}')$ , where  $Var(F)$  is the set of free variables in the formula  $F$ . Equality constraints between variables can be implicitly defined by multiple occurrences of the same variable in a rule. The ground instances of an  $MSR_{NC}$  rule are defined as

$$Inst(\mathcal{M} \longrightarrow \mathcal{M}' : \varphi) = \{\sigma(\mathcal{M}) \longrightarrow \sigma(\mathcal{M}') \mid \sigma \in Sol(\varphi)\}$$

where  $\sigma$  is extended in the natural way to multisets. The instances of a set of rules  $\mathcal{R} = \{R_1, \dots, R_n\}$  is defined as  $Inst(\mathcal{R}) = Inst(R_1) \cup \dots \cup Inst(R_n)$ .

An  $MSR_{NC}$  *specification*  $\mathcal{S}$  is a tuple  $\langle \mathcal{P}, \mathcal{I}, \mathcal{R} \rangle$ , where  $\mathcal{P}$  is a set of predicate symbols,  $\mathcal{I}$  is a set of (*initial*) configurations, and  $\mathcal{R}$  is a finite set of rules over  $\mathcal{P}$ . The *operational semantics* of  $\mathcal{S}$  is defined via the rewriting relation  $\Rightarrow_{\mathcal{R}}$  defined over configurations (i.e. ground multisets) as follows.

Given two configurations  $\mathcal{M}_1$  and  $\mathcal{M}_2$ ,  $\mathcal{M}_1 \Rightarrow_{\mathcal{R}} \mathcal{M}_2$  if and only if there exists a multiset of ground atomic formulas  $\mathcal{Q}$  s.t.  $\mathcal{M}_1 = \mathcal{N}_1 \oplus \mathcal{Q}$ ,  $\mathcal{M}_2 = \mathcal{N}_2 \oplus \mathcal{Q}$ , and  $\mathcal{N}_1 \longrightarrow \mathcal{N}_2$  is in  $Inst(\mathcal{R})$ . A configuration  $\mathcal{M}$  is *reachable* if there exists  $\mathcal{M}_0 \in \mathcal{I}$  such that  $\mathcal{M}_0 \Rightarrow_{\mathcal{R}}^* \mathcal{M}$ , where  $\Rightarrow_{\mathcal{R}}^*$  is the transitive closure of  $\Rightarrow_{\mathcal{R}}$ .

## 2 From $\pi_a$ to $MSR_{NC}$

In this section we define an encoding of infinite-state asynchronous  $\pi$ -calculus specifications into  $MSR_{NC}$ . In this preliminary work we will restrict ourselves to *closed*  $\pi_a$  specifications and configurations. Specifically, we will consider normalised equations of the form  $A(\vec{x}) = a(\vec{u}).(\nu \vec{v})R$  such that  $Fn(R) \subseteq \{\vec{x}, \vec{u}, \vec{v}\}$ , and configurations  $(\vec{v}).Q$  such that  $Fn(Q) \subseteq \{\vec{v}\}$ , i.e., we assume that all names with scope over different equations already occur in the quantifier in the initial configuration. Closed specifications and configurations present all the features of  $\pi_a$  we are interested in (fresh name generation, unbound parallelism, name and process mobility).

The encoding of closed specifications is defined as follows. Names are encoded as integer values. Relations over names are symbolically represented as constraints. We first encode an input action  $\bar{a}\vec{x}$  and an output action  $a(\vec{x})$  as the atomic formula  $m(a, \vec{x})$ , where  $a, x_1, \dots, x_n$  are free variables. Input messages will occur in the left-hand side of a  $MSR_{NC}$  rule encoding a process definition, whereas output messages will occur in the right-hand side. Then, we encode a *process identifier*  $A$  using a predicate symbol  $p_A$  taking as arguments as many variables as the parameters in its defining equation. Finally, we use an atomic formula  $new(f)$  to keep track of fresh values (i.e. to separate used and unused names). We will explain its meaning in few lines.

Let us consider the initial configuration  $P$  defined as

$$(\nu \vec{v})(\bar{a}_1 \vec{y}_1 \mid \dots \mid \bar{a}_n \vec{y}_n \mid A_1(\vec{w}_1) \dots \mid A_m(\vec{w}_m))$$

The encoding of  $P$  is defined via the  $\text{MSR}_{NC}$  rule  $P^\bullet$  defined as

$$\begin{aligned} & \text{init}, \text{new}(f) \longrightarrow \\ & m(a_1, \vec{y}_1), \dots, m(a_n, \vec{y}_n), p_{A_1}(\vec{w}_1), \dots, p_{A_m}(\vec{w}_m), \text{new}(f') : \\ & f' > v_1, v_1 > v_2, \dots, v_{r-1} > v_r, v_r > f. \end{aligned}$$

The constraint over  $f, f', \vec{v}$  ensures that the names in  $\vec{v}$  are distinct each other, and that the global memory  $\text{new}(f)$  contains a name strictly greater than all used names;  $\text{new}(\cdot)$  will be used then to *separate* the set of used names from the set of unused ones.

Let us consider now a *normalised* parametric equation  $D$  defined as

$$A(\vec{x}) = a(\vec{u}).(\nu\vec{v})(\bar{a}_1\vec{y}_1 \mid \dots \mid \bar{a}_n\vec{y}_n \mid A_1(\vec{w}_1) \mid \dots \mid A_m(\vec{w}_m))$$

The encoding of  $D$  is defined via the  $\text{MSR}_{NC}$  rule  $D^\bullet$  defined as

$$\begin{aligned} & p_A(\vec{x}), m(a, \vec{u}), \text{new}(f) \longrightarrow \\ & m(a_1, \vec{y}_1), \dots, m(a_n, \vec{y}_n), p_{A_1}(\vec{w}_1), \dots, p_{A_m}(\vec{w}_m), \text{new}(f') : \\ & f' > v_1, v_1 > v_2, \dots, v_{r-1} > v_r, v_r > f \end{aligned}$$

The constraint on  $\vec{v}, f, f'$  ensures the freshness of the names in  $\vec{v}$ . Note that, if  $D$  has no generation of fresh values (i.e.  $r = 0$ ), then we can simplify the rule by removing  $\text{new}(f)$  and  $\text{new}(f')$  from the left- and from the right-hand side, respectively.

**Example 2.1** The  $\text{MSR}_{NC}$  encoding of the equations of Example 1.1 is defined as follows (for brevity, we write  $p_{\text{Init}}, \dots$  as  $\text{init}, \dots$ )

$$\begin{aligned} & \text{init}, \text{new}(f) \rightarrow \text{init}(c), \text{resp}(c), \text{new}(f') : f' > c, c > f. \\ & \text{start}, \text{new}(f) \rightarrow \text{start}, \text{init}(c), \text{resp}(c), \text{new}(f') : f' > c, c > f. \\ & \text{init}(a), \text{new}(f) \rightarrow m(a, p), \text{wait}(p), \text{new}(f') : f' > p, p > f. \\ & \text{wait}(p), m(p, x) \rightarrow \text{endI}(p, x) : \text{true}. \\ & \text{resp}(a), m(a, y), \text{new}(f) \rightarrow m(y, \text{ok}), \text{endR}(y, \text{ok}), \text{new}(f') : f' > \text{ok}, \text{ok} > f. \end{aligned}$$

Now, let  $P$  be the  $\pi_a$  configuration

$$(\nu\vec{v})(\bar{a}_1\vec{y}_1 \mid \dots \mid \bar{a}_n\vec{y}_n \mid A_1(\vec{w}_1) \mid \dots \mid A_m(\vec{w}_m))$$

Let  $\eta$  be an *injective mapping* from  $\vec{v}$  to  $\mathbb{Z}$  and let  $\eta(\vec{v})$  denote  $\eta(v_1), \dots, \eta(v_n)$ . We define  $P^\bullet(\eta, N)$  as the  $\text{MSR}_{NC}$  configuration

$$m(\eta(a_1), \eta(\vec{y}_1)), \dots, m(\eta(a_n), \eta(\vec{y}_n)), p_{A_1}(\eta(\vec{w}_1)), \dots, p_{A_m}(\eta(\vec{w}_m)), \text{new}(N)$$

where  $N$  is an integer strictly greater than  $\eta(v_1), \dots, \eta(v_r)$ . Let  $\eta : \vec{v} \rightsquigarrow \mathbb{Z}$ ,  $\eta' : \vec{v}' \rightsquigarrow \mathbb{Z}$ , and  $\{\vec{v}\} \subseteq \{\vec{v}'\}$ , then we define  $\eta \leq \eta'$  if  $\eta(\vec{v}) = \eta'(\vec{v})$ . The adequacy of the encoding is established then by the following propositions.

**Proposition 2.2** *Let  $\mathcal{E}$  be a set of closed normalised equations, and  $P_i$  be a closed configuration for  $i : 1, \dots, k$  such that  $P_1 \Rightarrow_{\pi_a} \dots \Rightarrow_{\pi_a} P_k$ . Then, there exists  $\eta_1 \leq \dots \leq \eta_k$ , and  $N_1 \leq \dots \leq N_k$  such that  $P_1^\bullet(\eta_1, N_1) \Rightarrow_{\mathcal{E}^\bullet} \dots \Rightarrow_{\mathcal{E}^\bullet} P_k^\bullet(\eta_k, N_k)$ .*

**Proof.** The proof is by induction on the length  $k$  of the derivation, the interesting case being the inductive step in which  $k \geq 1$ . Suppose that  $P_1 \Rightarrow_{\pi_a} \dots \Rightarrow_{\pi_a} P_k$  and that there exist  $\eta_1 \leq \eta_2 \leq \dots \leq \eta_k$ , and  $N_1 \leq N_2 \leq \dots \leq N_k$  such that  $P_1^\bullet(\eta_1, N_1) \Rightarrow_{\mathcal{E}^\bullet} \dots \Rightarrow_{\mathcal{E}^\bullet} P_k^\bullet(\eta_k, N_k)$ . Let  $P_k$  be the configuration

$$(\nu \vec{v}) \bar{c} \bar{z} \mid A(\vec{b}) \mid Q$$

Suppose there exists a normalised equation

$$A(\vec{x}) = a(\vec{u}).(\nu \vec{v})R$$

and a substitution  $\sigma = [\vec{x} \mapsto \vec{b}, \vec{u} \mapsto \vec{z}]$  such that  $\hat{\sigma}(a) = c$ . Then,  $P_k \Rightarrow_{\pi_a} P_{k+1}$  where  $P_{k+1}$  is the configuration

$$(\nu \vec{w}, \vec{u})(\sigma(R) \mid Q)$$

By definition of the encoding,  $P_k^\bullet(\eta_k, N_k)$  is the  $\text{MSR}_{NC}$  configuration

$$m(\eta_k(c), \eta_k(\vec{z})), p_A(\eta_k(\vec{b})), \text{new}(N_k), Q'$$

where  $\eta_k(c) < N_k$ , and  $d < N_k$  for any  $d \in \{\eta_k(\vec{z}), \eta_k(\vec{b})\}$ , and  $Q'$  is the encoding of the remaining part of the configuration  $Q$ . Furthermore,  $D^\bullet$  is the rule

$$p_A(\vec{x}), m(a, \vec{u}), \text{new}(f) \longrightarrow R', \text{new}(f') : f' > v_1, \dots, v_r > f.$$

where  $R'$  is the multiset corresponding to the encoding of the body of the equation. Let  $\gamma$  be a solution for  $f' > v_1, \dots, v_r > f$  such that  $\gamma(a) = \eta_k(c)$ ,  $\gamma(\vec{x}) = \eta_k(\vec{b})$ , and  $\gamma(\vec{u}) = \eta_k(\vec{z})$ , and  $\gamma(f) = N_k$ . Furthermore, let  $\eta_{k+1}$  be defined in such a way that  $\eta_k \leq \eta_{k+1}$  and  $\eta_{k+1}(v_i) = \gamma(v_i)$  for  $i : 1, \dots, r$  and let  $N_{k+1} = \gamma(f')$ . Then,

$$p_A(\eta_k(\vec{b})), m(\hat{\eta}_k(c), \eta_k(\vec{z})), \text{new}(N_k) \longrightarrow \gamma(R'), \text{new}(N_{k+1}) \in \text{Inst}(D^\bullet).$$

where  $\hat{\eta}_k, \hat{\gamma}$  represent the natural extensions of  $\eta_k$  and  $\gamma$  to (multiset of) terms. Furthermore,  $P_{k+1}^\bullet(\eta_{k+1}, N_{k+1})$  is the  $\text{MSR}_{NC}$  configuration

$$\text{new}(N_{k+1}), \eta_{k+1}(Q'), \eta_{k+1}(R)$$

where  $\eta_{k+1}^{\hat{\cdot}}$  is the natural extension of  $\eta_{k+1}$  to multiset of terms. By definition of  $\Rightarrow_{\mathcal{E}^{\bullet}}$ , it follows then that  $P_k^{\bullet}(\eta_k, N_k) \Rightarrow_{\mathcal{E}^{\bullet}} P_{k+1}^{\bullet}(\eta_{k+1}, N_{k+1})$ .  $\square$

**Proposition 2.3** *Let  $\mathcal{E}$  be a set of closed normalised equations and  $\mathcal{E}^{\bullet}$  its  $MSR_{NC}$  encoding,  $P_1$  an initial closed configuration, and  $\mathcal{M}_1 = P_1^{\bullet}(\eta_1, N_1)$  for some  $\eta_1$  and  $N_1$ . If  $\mathcal{M}_1 \Rightarrow_{\mathcal{E}^{\bullet}} \dots \Rightarrow_{\mathcal{E}^{\bullet}} \mathcal{M}_k$ , then there exists  $P_2, \dots, P_k$ ,  $\eta_2 \dots \leq \eta_k$ , and  $N_2 \leq N_3 \leq \dots \leq N_k$  such that  $\eta_1 \leq \eta_2$ ,  $N_1 \leq N_2$  such that  $P_1 \Rightarrow_{\pi_a} \dots \Rightarrow_{\pi_a} P_k$  and  $\mathcal{M}_i = P_i^{\bullet}(\eta_i, N_i)$  for  $i : 2, \dots, k$ .*

**Proof.** The proof is by induction on the length  $k$  of the derivation and follows a schema similar to the proof of the previous proposition.  $\square$

### 3 A General Procedure for Control Reachability

Control reachability is undecidable for generic specifications in asynchronous  $\pi$ -calculus, while its decidable for restricted fragments that can be embedded into Petri Nets (with transfer) [4]. However, the encoding described in the previous sections allows us to tackle this problem in its more general form via the *symbolic model checking* procedure we defined for  $MSR_{NC}$  in [10]. For studying the *control reachability problem* for specifications in asynchronous  $\pi$ -calculus we are interested in finitely representing configurations with an *arbitrary* number of names and processes. We will achieve this goal by resorting to the  $MSR_{NC}$  encoding of  $\pi$ -calculus configurations. Specifically, we introduce a symbolic representation of *upward closed* sets of configurations, called *constrained configuration*. A  $\pi_a$  *constrained configuration* is a formula

$$m(a_1, \vec{y}_1), \dots, m(a_n, \vec{y}_n), p_{A_1}(\vec{w}_1), \dots, p_{A_m}(\vec{w}_m), new(f) : \varphi \quad (1)$$

defined over the set of variables  $V = \{f, a_1, \dots, a_n, \vec{y}_1, \dots, \vec{y}_n, \vec{w}_1, \dots, \vec{w}_m\}$  such that  $\varphi$  is an NC constraints over  $V$ , and  $\varphi, f > x$  is satisfiable for *any*  $x \in V$   $x \neq f$  (in every reachable configuration  $new(f)$  separates used from unused names). The *denotation* of a set  $\mathbf{S}$  of  $\pi_a$  constrained configurations is the *upward closure* of the *ground instances* of its elements, namely

$$\llbracket \mathbf{S} \rrbracket = \{ \mathcal{N} \mid \mathcal{M} \preceq \mathcal{N}, \mathcal{M} \in Inst(M), M \in \mathbf{S} \}$$

where  $\preceq$  is *multiset inclusion*, and the operator *Inst* is defined as

$$Inst(\mathcal{M} : \varphi) = \{ \sigma(\mathcal{M}) \mid \sigma \in Sol(\varphi) \}.$$

Thus, a  $\pi_a$  constrained configuration like (1) represents the set of  $\pi_a$ -calculus configurations of the shape

$$(\nu \vec{v})(\xi(\vec{a}_1)\xi(\vec{y}_1) \mid \dots \mid \xi(\vec{a}_n)\xi(\vec{y}_n) \mid A_1(\xi(\vec{w}_1)) \mid \dots \mid A_m(\xi(\vec{w}_m)) \mid Q)$$

where  $\xi$  is obtained by composing a solution  $\sigma$  for  $\varphi$  with an injective (possibly not surjective) mapping from  $\mathbb{Z}$  to the set of names  $\vec{v}$ , and  $Q$  is any pool of

messages and processes defined over a set of names containing  $\vec{v}$ .

This symbolic representation allows us to reason on *infinite sets* of configurations, thus forgetting about the actual number or processes/messages of a given run. Furthermore, the use of first order terms allows us to symbolically represent an infinite number of different instances of the same collection of processes/messages. Constraints define the *relationship* between the data of different processes/messages.

### 3.1 Symbolic State Exploration for $\pi_a$

We can now define a *symbolic backward reachability* procedure that computes all predecessor states of a given set of  $\pi_a$  constrained configurations with respect to  $\mathcal{E}^\bullet$ . The procedure is based on a breadth-first visit of the infinite state space of the  $\text{MSR}_{NC}$  specification resulting from the encoding presented in the previous sections. The search is defined on the basis of a *symbolic predecessor operator* and on an entailment relation (over constrained configurations) both formally defined in [10].

To briefly explain the idea underlying the procedure given in [10], in the rest of this section we will present a specialization of the symbolic predecessor operator to the class of  $\text{MSR}_{NC}$  specification resulting from the encoding of  $\pi_a$  processes.

Let us first recall some definitions. Given two (multisets of) atomic formulas with distinct free variables  $t$  and  $t'$ , a *unifier* for  $t$  and  $t'$  is a substitution  $\sigma$  such that  $\sigma(t) = \sigma(t')$ . The *most general unifier*  $\text{mgu}(t, t')$  is the *idempotent* substitution  $\sigma$  such that any other unifier  $\gamma$  can be obtained from  $\sigma$  as  $\gamma = \sigma \circ \eta$  for some substitution  $\eta$ ; the most general unifier always exists and it is unique. In our settings unification might give rise to new bindings for integer variables. An  $\text{mgu}$   $\sigma$  can also be viewed (and used) as an NC constraint of the shape of a conjunction of equalities  $x = y$  for some variables  $x, y$ .

Let  $\mathbf{S}$  be a set of  $\pi_a$  constrained configurations with distinct variables each other. The symbolic predecessor operator for an encoding in  $\text{MSR}_{NC}$   $\mathcal{R} = \mathcal{E}^\bullet$  of a specification in the asynchronous  $\pi$ -calculus  $\mathcal{E}^\bullet$  is defined in Fig. 1. In the definition of Fig. 1 we combine *unification* (via the calculation of the most general unifier  $\sigma$ ) and *constraint solving* (via satisfiability test and variable elimination);  $\sigma$  is needed to remember constraints on integer variables introduced via term unification. Condition 3 – 4 in Fig. 1 ensures the existence of a common pool of messages and processes shared between the *right-hand side* of a rule and a  $\pi_a$  constrained configurations in  $\mathbf{S}$ . Condition 5 in Fig. 1 allows us to prune all configurations that violate the *freshness* of generated names (this is specific to the  $\pi_a$  encoding). Condition 6 ensures that the selected common multisets agree on the data part (i.e. the conjunction of their constraints is satisfiable). Existential quantification is used to project away all variables (Cond. (7)) not needed in the symbolic pre-image. The symbolic

$\mathbf{Pre}_{\mathcal{R}}(\mathbf{S}) = \{ \mathcal{A} \oplus \mathcal{N} \oplus \{new(x)\} : \xi \mid \text{cond (1-7) listed below holds} \}$

- (1)  $\mathcal{A} \oplus \{new(x)\} \longrightarrow \mathcal{B} \oplus \{new(x')\} : \psi \in \mathcal{R},$
- (2)  $\mathcal{M} \oplus \{new(y)\} : \varphi \in \mathbf{S},$
- (3)  $\mathcal{B}' \preceq \mathcal{B}, \mathcal{M}' \preceq \mathcal{M}, \mathcal{N} = \mathcal{M} \ominus \mathcal{M}',$
- (4)  $\sigma = mgu(\mathcal{M}', \mathcal{B}'),$
- (5)  $\gamma = \bigwedge_{w \in Var(\mathcal{A} \oplus \mathcal{N})} x > w,$
- (6)  $\xi = \exists x', y, \vec{z}. (\sigma \wedge \varphi \wedge \psi \wedge \gamma)$  is satisfiable
- (7)  $\vec{z} = Var(\sigma \wedge \varphi \wedge \psi) \setminus (Var(\mathcal{A} \oplus \mathcal{N}) \cup \{x\}).$

Fig. 1. Symbolic Predecessor Operator for Logical Encoding of  $\pi_a$  operator  $\mathbf{Pre}_{\mathcal{R}}$  returns a set of  $\pi_a$  constrained configurations such that

$$\llbracket \mathbf{Pre}_{\mathcal{R}}(\mathbf{S}) \rrbracket = Pre_{\mathcal{R}}(\llbracket \mathbf{S} \rrbracket)$$

for any set of  $\pi_a$  constrained configurations  $\mathbf{S}$ . This result follows from the result proved in [11] for the symbolic predecessor operator associated to an  $MSR_{NC}$  specification. The specialized operator of Fig. 1 is presented here only for giving an intuition on how the general search technique for  $MSR_{NC}$  works.

The symbolic model checking procedure resulting from iterating the application of  $\mathbf{Pre}_{\mathcal{R}}$  can be used then to attack *control reachability* for unrestricted  $\pi_a$  specifications. Let  $P$  be the initial configuration and  $A$  be the process identifier we would like to reach. Then, we can run the symbolic backward search starting from the symbolic configuration  $A(\vec{x}), new(f) : \varphi$ . If the search terminates we have to check then if *init* belongs to the resulting fixpoint. Clearly, in the  $MSR_{NC}$  we can use search procedure to check generalization of this problems in which the target set of configurations is defined via  $\pi_a$  constrained configurations like  $A_1(\vec{x}_k), \dots, A_k(\vec{x}_k), new(f) : \varphi$  and  $\varphi$  expresses the relation over the names of the different processes.

**Example 3.1** As an example, suppose we want to check that in Example 1.1, the initial configuration *start* | *new(f)* always generates sessions that do not interfere with each other, i.e., we never reach configurations like

$$(\nu c, d, d')(EndI(c, d) \mid EndR(c, d') \mid \dots)$$

where  $d' \neq d$ , i.e., the processes involved in a session always exchange both names. To check this property we can apply the symbolic backward analysis described above starting from the  $\pi_a$  configuration

$$endI(x, y), endR(x, z), new(f) : z \neq y, f > x, f > y, f > z$$

Using our CLP-based implementation, this computation terminates in 6s, af-

ter a 4 steps, computing 22  $\pi_a$  constrained configurations. The resulting fix-point does not contain the configuration  $start, new(f) : true$ . This proves our original specification correct for an arbitrary number of Init-Resp sessions.

## 4 Related and Future Work

To our knowledge the present paper is the first attempt of establishing a connection between the infinite-state verification techniques based on constraints [1,3,2,10,11] and calculi for expressing mobility of processes as the asynchronous  $\pi_a$  calculus. Our verification method generalizes the ideas proposed for Time Petri Nets in [1,3] to more general classes of concurrent systems that can be specified via multiset rewriting and constraints. In previous work (see e.g. the technical report [11]) we applied our framework to mutual-exclusion and consistency protocols. Multiset rewriting over first order atomic formulas has been proposed for specifying security protocols by Cervesato et al. in [8].

As future work we plan to extend the encoding to specifications in full  $\pi$  calculus, and to study the possible impact of the presented relationship for finding new decidable verification problems for  $\pi_a$  and  $\pi$  specifications.

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