

# Imperfection is better!

1-clock priced TA with imprecisions on energy level

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# Outline of the presentation

- 1 Introduction
- 2 1-clock PTA with imperfect energy level
- 3 Where Presburger arithmetic helps us
- 4 Robust strategies
- 5 A case study
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# Time is necessary

we need a **quantitative notion of time** for modelization:

- the behaviour of most systems actually depends on time,
- modelling has to take it into account

→ Timed Automata, Timed Petri nets...

and also for specifications:

- to faithfully represent the specification,
- untimed specifications are not enough (e.g. bounded response time property)

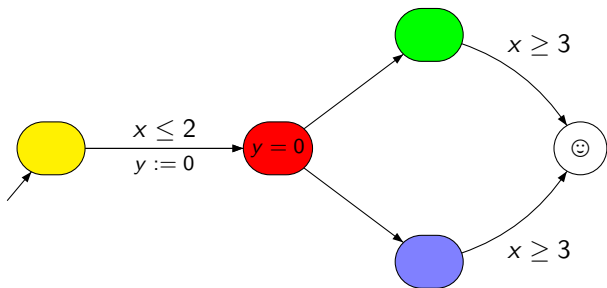
→ TCTL, MTL, TPTL, timed  $\mu$ -calculus...

# But time may not be sufficient

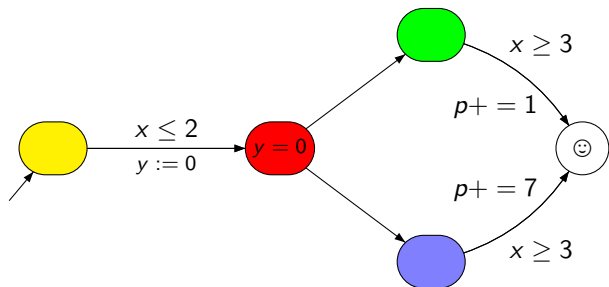
In some cases we don't want to measure time, but rather **energy consumption** to achieve some goal.

- **stopwatch automata**: allows to stop/start some clocks,  
→ useful for scheduling
- **hybrid automata**: allows variables whose derivative is not constant,  
→ leaking gas burner, water-level monitor...
- **priced (=weighted) timed automata**: similar to linear hybrid automata, but the **behaviour only depends on clock variables**

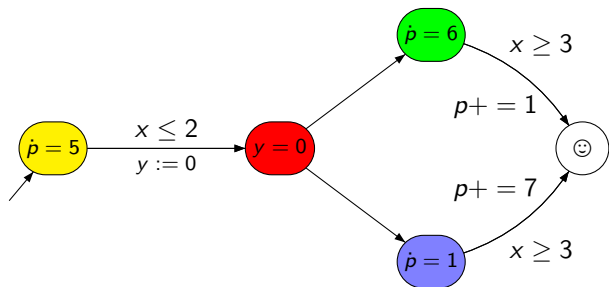
# Priced timed automata [ALP01, BFH+01]



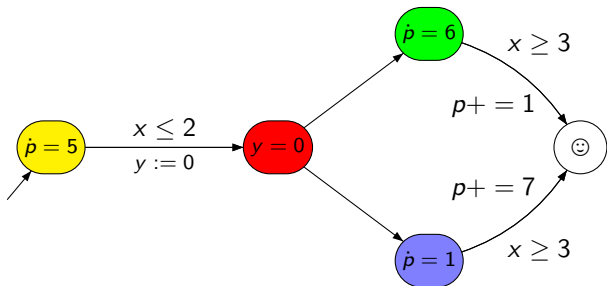
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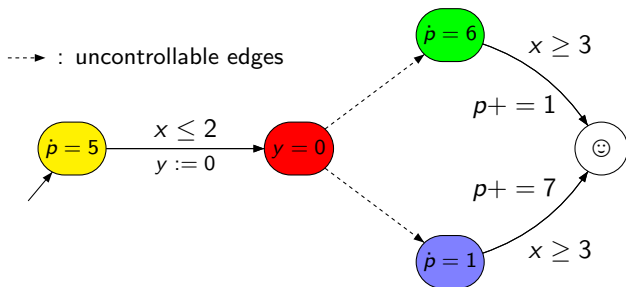


Main known results:

- Optimal infinite behaviours are computable (in PSPACE) minimal cost for reaching  $\text{☺}$
- Model Checking logic WCTL:  $\text{EF}_{p \leq 10} \text{☺}$ ,  $\text{AG}_{t=2}(\text{EF}_{p \leq 6} \text{☺})$ 
  - decidable over PTAs with  $\leq 1$  clocks (in PSPACE)
  - undecidable over PTAs with  $\geq 3$  clocks

Note: Prices and price rates are non-negative

# Priced timed games



Main known results:

- Computing (almost) optimal strategies for 1-clock turn-based PTGs is decidable
- It is undecidable for PTGs with  $\geq 3$  clocks

**Note:** Again, prices and price rates are non-negative

# Energy Constraints

Energy is not only consumed, but can be regained.

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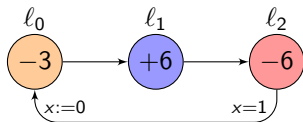
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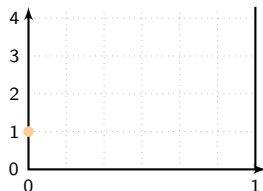
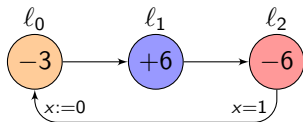


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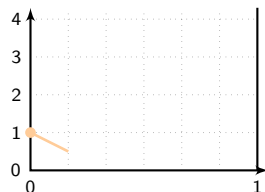
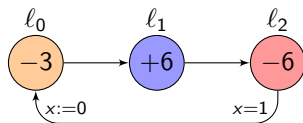
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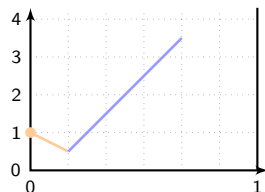
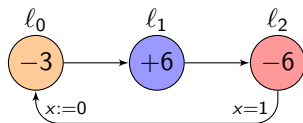
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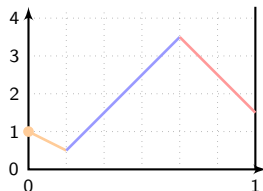
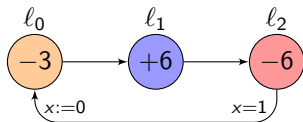
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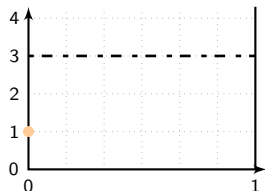
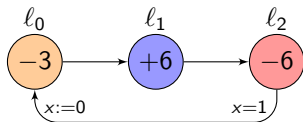
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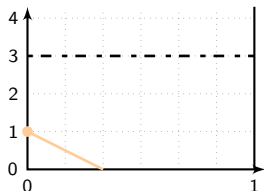
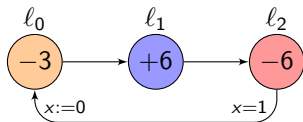
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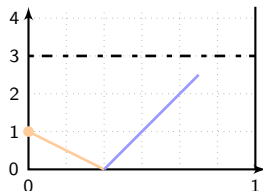
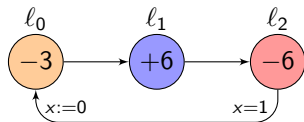
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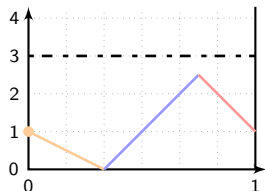
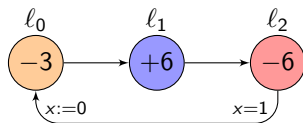
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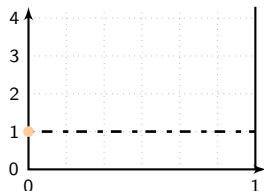
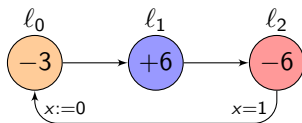
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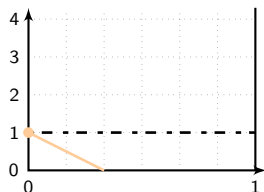
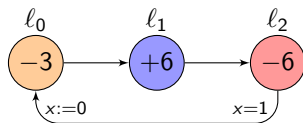
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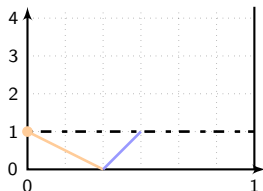
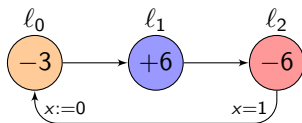
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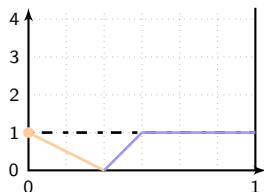
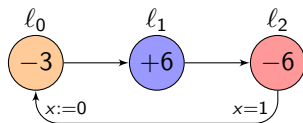
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## Existing results for the perfect 1-clock case

[BFLMS08], [BFLM10]

	exist. problem	univ. problem	games
L	$\in PTIME$	$\in PTIME$	?
L+W	$\in PTIME$	$\in PTIME$	?
L+U	?	?	undecidable

**Note:** PTIME membership results hold if there are no discrete costs (EXPTIME otherwise).

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→ We will introduce imperfect information on the energy level to solve a L+U problem

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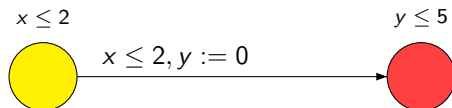
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# 1-clock PTA with imperfect energy level

- The **exact value of the energy level (=price)** is not known in general
- The energy level is only observed when firing so-called **update transitions**
- Derivative of the price is defined by a **non-punctual interval**

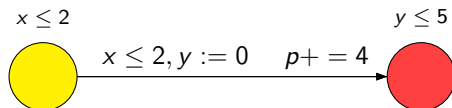
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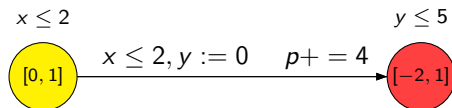
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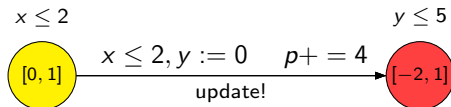
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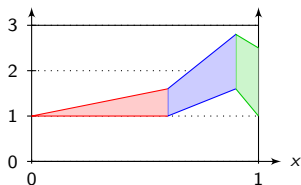
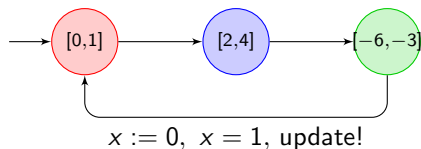


- We allow discrete updates of the price
- The dynamics of the price is defined as in rectangular hybrid automata:  $\dot{p} \in [0, 1]$
- Transitions with “update!” mean that after having fired this transition, the exact value of the energy level is observed.

## 1-clock PTA with imperfect energy level (2)

We see this model as a two-player game:

- the **controller** chooses the transitions and the **delays**.
- the **environment** chooses the **rate** of the energy



The controller only observes the energy level after having fired “update!” transitions.

## More formally

A *strategy* for the controller represents here a mapping:  
Finite prefixes of runs  $\longrightarrow$  Discrete transitions  $\cup$  Delays

A strategy is *admissible* if it respects the imperfect information:  
if two runs give the same observation, then the strategy  
must give the same decision.

A strategy is *winning* if all its outcomes are safe runs.

**Problem:** Given a 1cPTA with imp. energy level, an initial state  $s_0$  and a safe interval  $S$ , does there exist a winning admissible alternating strategy?

### Theorem

*Under some assumptions on 1c-PTAs, this problem is decidable.*

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# Outline of the proof

We build a Presburger formula with real variables which characterizes the existence of a winning admissible strategy.

If one has a winning predicate, one can express that in a given configuration, there exists a move which only leads to winning states.

**Key ingredient:** we use imperfect information to show that winning configurations can be gathered into a finite number of intervals.

→ The Presburger formula expresses the existence of these intervals.

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→ The Presburger formula expresses the existence of these intervals.

**Our Hypothesis:**

- each cycle carries at least one “update!” transition,
- when the energy is observed, the clock is reset, and it is compared to a positive lower bound.

## How imperfect information helps

In the following we consider “meta-transitions” that consist in sequence of non-update transitions ending by an update transition.

Using our hypothesis on the model, we prove:

### Lemma

*There exists a positive value  $\Delta > 0$  such that after having fired a meta-transition, the imprecision on the value of the energy is at least  $\Delta$ .*

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## Corollary

*Let  $S$  be a safe interval. If a configuration is winning and has an entering meta-transition, then there exists a winning interval of size at least  $\Delta$  containing it.*

*As a consequence, in each location, winning configurations are included in at most  $N := \lceil \frac{|S|}{\Delta} \rceil$  intervals.*

# Construction of the formula

Let  $e$  be an energy level,  $t$  be a meta-transition and  $\vec{\tau}$  be a vector of timestamps for  $t$ . We define the following operators:

- the predicate  $Safe(e, t, \vec{\tau})$
- the interval  $Outcomes(e, t, \vec{\tau})$

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We consider variables  $a_{i,j}, b_{i,j}$ , with  $1 \leq i \leq n$  and  $1 \leq j \leq N$ .

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Let  $q \in Q$ . We define the **winning predicate**:

$$Win(q, e) \iff \bigvee_{q \xrightarrow{t} q_k} \exists \vec{\tau} \text{ s.t. } \left\{ \begin{array}{l} Safe(e, t, \vec{\tau}) \\ \bigwedge_{\ell=1}^N Outcomes(e, t, \vec{\tau}) \subseteq [a_{k,\ell}, b_{k,\ell}] \end{array} \right.$$

## Construction of the formula (2)

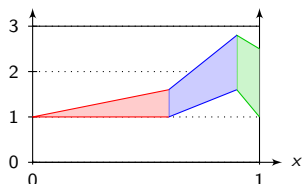
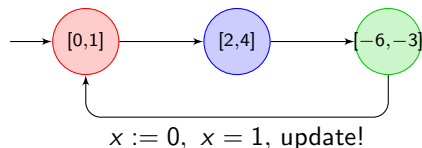
Let  $q_0$  (resp.  $e_0$ ) denote the initial location (resp. energy).

We define the Presburger formula as follows:

$$\Phi(q_0, e_0) = \exists_{i=1}^n \exists_{j=1}^N a_{i,j}, b_{i,j} \text{ s.t. } \left\{ \begin{array}{l} \bigwedge_{i,j} [a_{i,j}, b_{i,j}] \subseteq S \\ \wedge \text{Win}(q_0, e_0) \\ \bigwedge_{i,j} \forall e, a_{i,j} \leq e \leq b_{i,j} \Rightarrow \text{Win}(q_i, e) \end{array} \right.$$

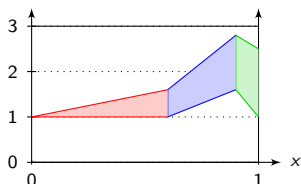
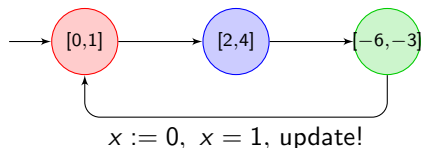
**Bonus:** Using Fourier-Motzkin elimination, we end up with the set of initial values of the energy for which there exists a winning strategy.

## Example



There is only one meta-transition: the cycle. Its minimal imprecision is  $\Delta = 1$ . We thus obtain  $N = \frac{3}{1} = 3$ .

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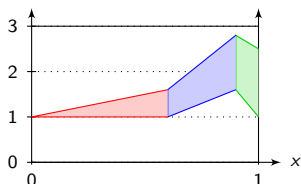
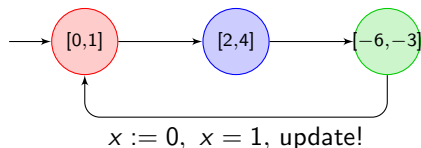


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We consider three timestamps  $\tau_1, \tau_2$  and  $\tau_3$  (which verify  $\sum_i \tau_i = 1$ ).

$$\text{Safe}(e, t, \vec{\tau}) \iff \begin{cases} 0 \leq e & \wedge e \leq 3 \\ \wedge 0 \leq e & \wedge e + \tau_1 \leq 3 \\ \wedge 0 \leq e + 2\tau_2 & \wedge e + \tau_1 + 4\tau_2 \leq 3 \\ \wedge 0 \leq e + 2\tau_2 - 6\tau_3 & \wedge e + \tau_1 + 4\tau_2 - 3\tau_3 \leq 3 \end{cases}$$

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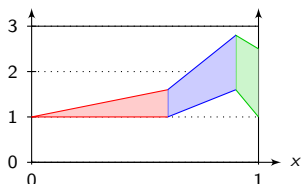
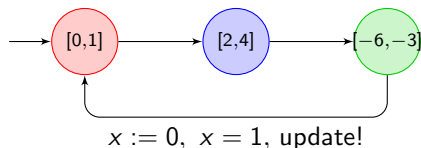
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We consider three timestamps  $\tau_1, \tau_2$  and  $\tau_3$  (which verify  $\sum_i \tau_i = 1$ ).

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Here we end up with the constraint  $0 \leq e_0 \leq \frac{12}{5}$

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- 1 Introduction
- 2 1-clock PTA with imperfect energy level
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- 4 Robust strategies**
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→ definition of  $Win_\delta$

→ definition of  $\Phi_\delta(q_0, e_0)$

## New Presburger formula

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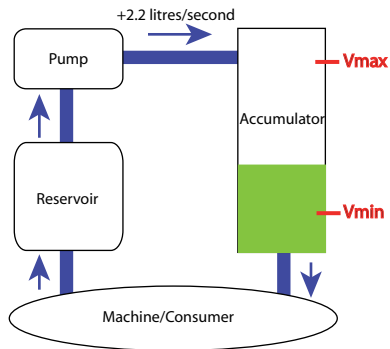
→ Consider the formula  $\Phi_{robust}(q_0, e_0) \stackrel{def}{=} \exists \delta > 0 \mid \Phi_\delta(q_0, e_0)$

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# A case study

The study of L+U energy problems has been motivated by a collaboration within the [european research project Quasimodo](#).



Synthesis of a pump controller for an oil-accumulator system.

**L+U:** The oil volume must stay within a **safety interval**.

**Infinite run:** The machine repeats a cycle of production of plastic cells forever.

## A case study (2)

The oil level can be modelled as a price:

- it is a linear hybrid variable,
- it can both increase (pump on) and decrease (machine consumes oil)
- price rates are not singletons: there are imprecisions in the machine rates.

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→ We have to **develop a tool** to perform this analysis!

We plan to rely on the SMT solver Z3 of Nikolaj Bjorner.

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# Conclusion

## Contributions:

- decidability for a **L+U** problem
- **Presburger arithmetic** as a new tool for energy constraints problems
- extends for free to **robustness** issues

## Challenges/Perspectives:

- extraction of **strategies**
- develop a **tool** (case study)
- works also for **1-clock prices timed games**
- other extensions? (**multiple costs, optimization...**)
- exact **complexity** of the procedure?