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# ProNoBiS

## Activities in Verona

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# List of Activities

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- Comparative semantics
  - Alternating and non-alternating models
  - Simulation and bisimulation relations
- Logical characterizations
  - Extensions of HM logic
- Non-discrete measures
  - Stochastic Transition Systems
- Verification of crypto protocols
  - Task-based PIOAs
    - Oblivious transfer
  - Aproximate simulations
    - Authentication, matching conversations

# Probabilistic Automata (NA)

$$NA = (Q, q_0, E, H, D)$$

Transition relation

$$D \subseteq Q \times (E \cup H) \times \text{Disc}(Q)$$

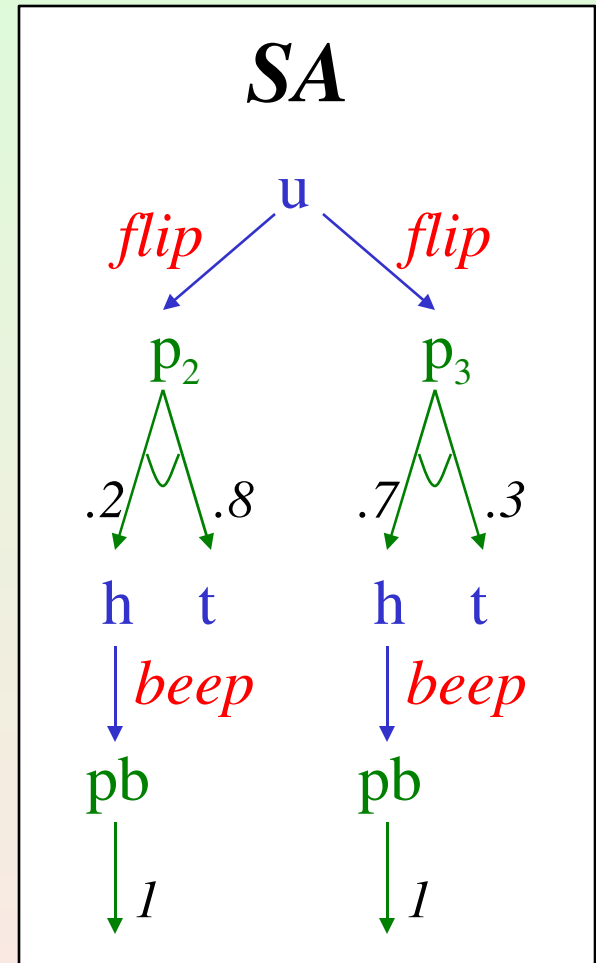
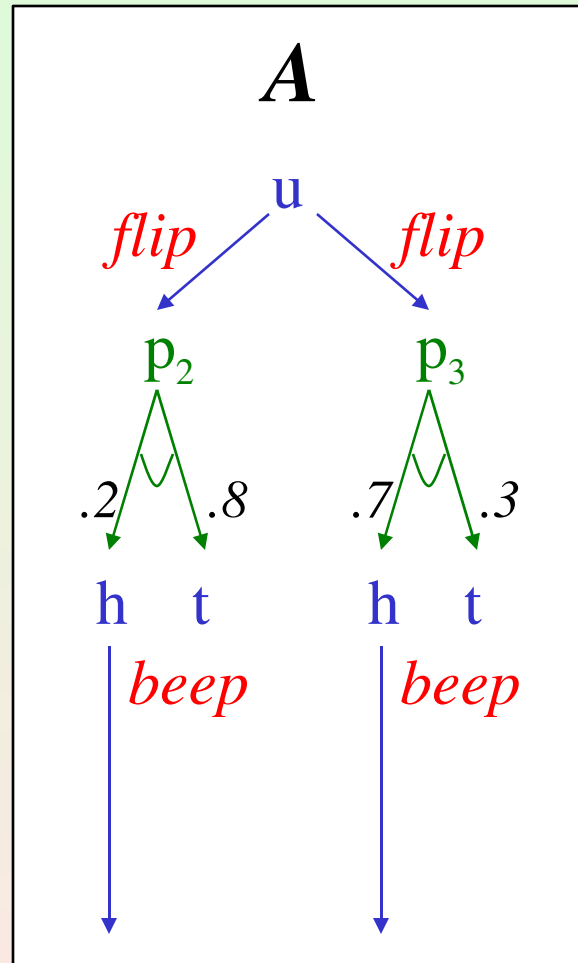
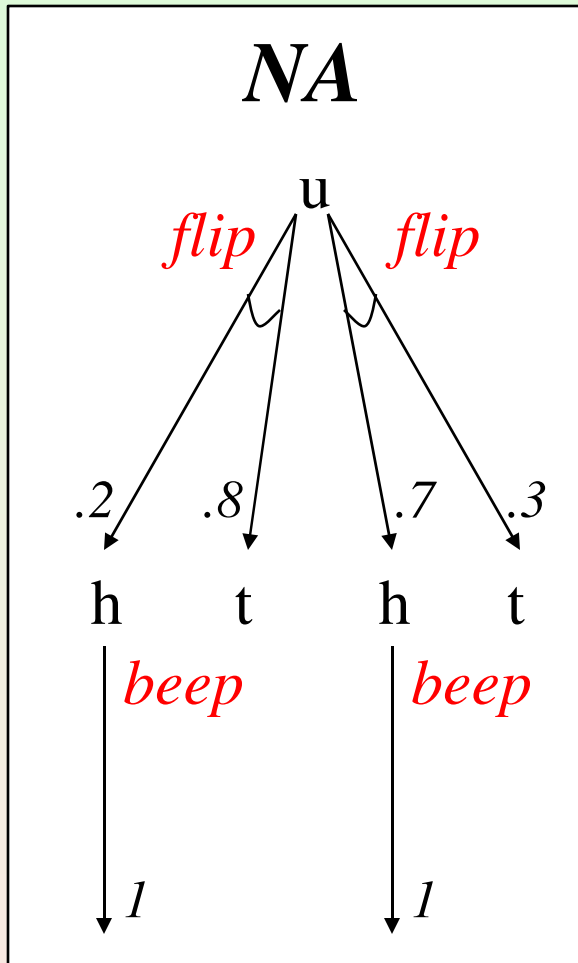
Internal (hidden) actions

External actions:  $E \cap H = \emptyset$

Initial state:  $q_0 \in Q$

States

# Alternating vs. non-alternating



# Relations between models

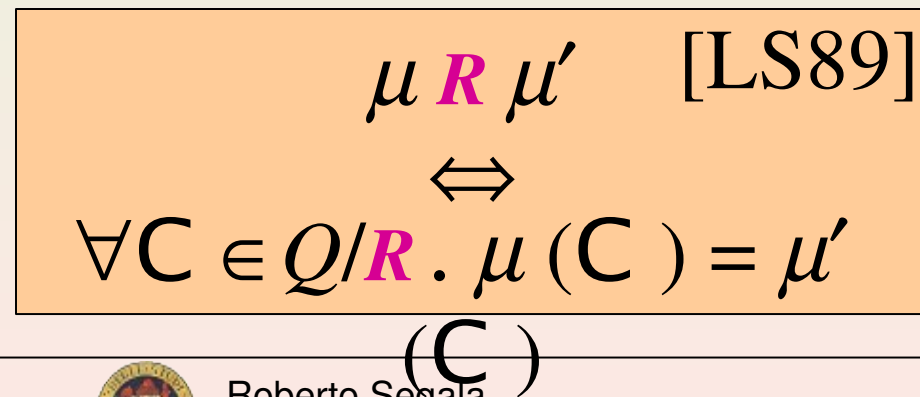
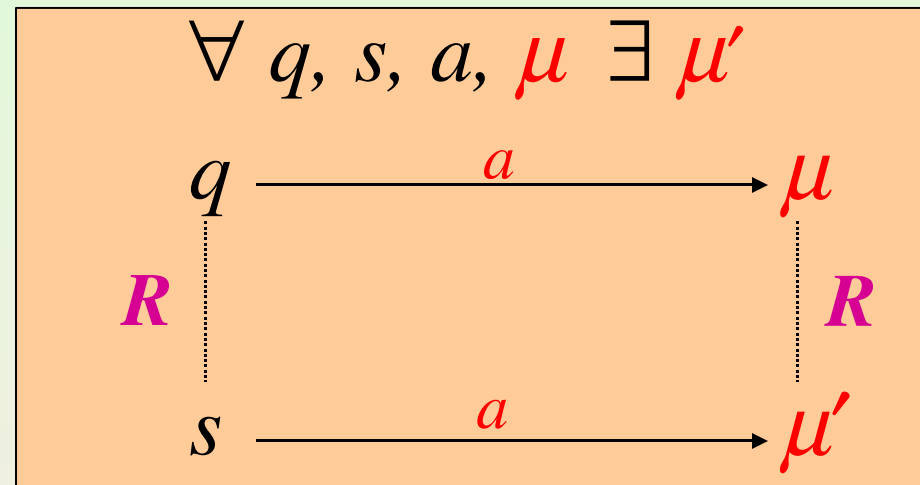
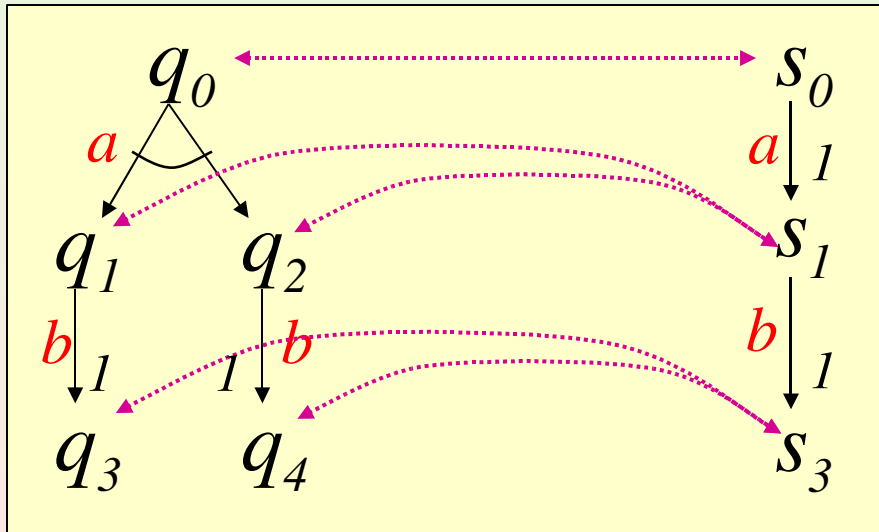
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- Embeddings ( $E$ )
  - $SA$  as an instance of  $A$  and of  $NA$
  - $A$  as an instance of  $NA$
  - Embeddings as structure restrictions
- Transformations ( $T$ )
  - Folkloristic ways to represent the same object within the three models

# Strong Bisimulation of NA

Strong bisimulation between  $A_1$  and  $A_2$

Relation  $R \subseteq Q \times Q$ ,  
 $Q = Q_1 \uplus Q_2$ , such that



# Bisimulation Literature

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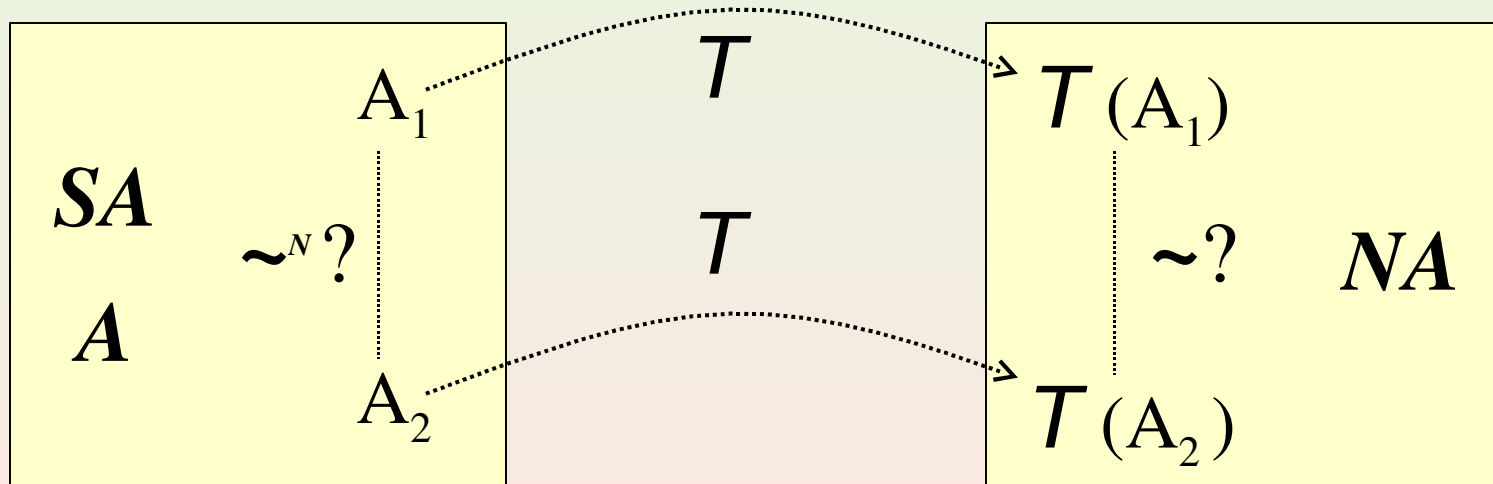
In literature there are also

- Strong bisimulation of Hansson on  $SA$ 
  - Relates only nondeterministic states
- Strong bisimulation of Philippou on  $A$ 
  - Relates all states
  - Probabilistic states are a technicality
- Weak bisimulation of Philippou on  $A$ 
  - Relates all states
  - Probabilistic states are meaningful
  - Uses conditional probabilities on self loop

# Taxonomy

Nondeterministic typology  $N$

- Based on  $T$  transformations
- Check bisimilarity of images in  $NA$

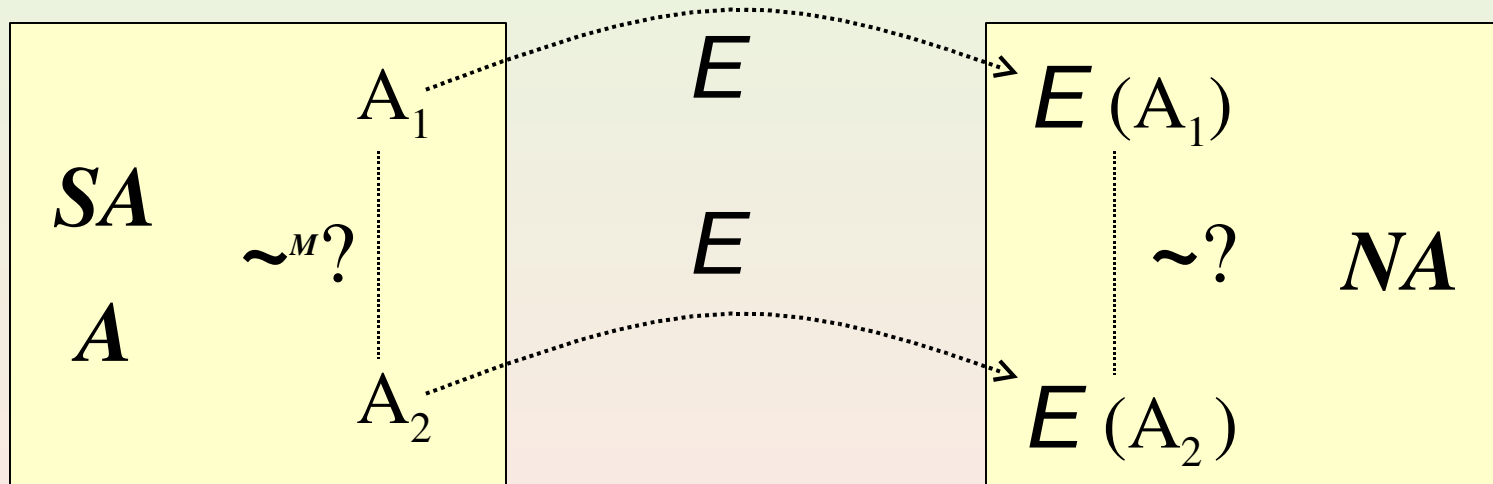




# Taxonomy

## Mixed typology $M$

- Based on E mbeddings
- Check bisimilarity of images in  $NA$



# Taxonomy and Literature

## [Segala, Turrini]

Equivalences	$SA$	$A$
Strong $\sim$	$\sim^N \sim^M$	$\sim^N$
Weak $\approx$		$\approx^{pM}$

# Logical Characterizations

## [Parma, Segala]

- Logic:  $\text{true} \mid \neg\phi \mid \phi \wedge \psi \mid \diamond a\phi \mid [\phi]_p$
- Semantics:  $\mu$  satisfies a formula
  - $\diamond a\phi$  : for each  $q$  in support of  $\mu$  there is a transition  $(q, a, \mu')$  such that  $\mu' \models \phi$
  - $[\phi]_p$  :  $\mu(\{q \mid q \models \phi\}) \geq p$
- Observation:  $\diamond_p a\phi$  corresponds to  $\diamond a[\phi]_p$

# Stochastic Transition Systems

[Cattani, Segala, Kwiatkowska, Norman]

$$ST = (Q, q_0, E, H, F_Q, F_A, D)$$

Transition relation

$$D \subseteq Q \times (E \cup H) \times \mathbf{P}(Q, F_Q)$$

$\sigma$ -field on actions

$\sigma$ -field on states

Internal (hidden) actions

External actions:  $E \cap H = \emptyset$

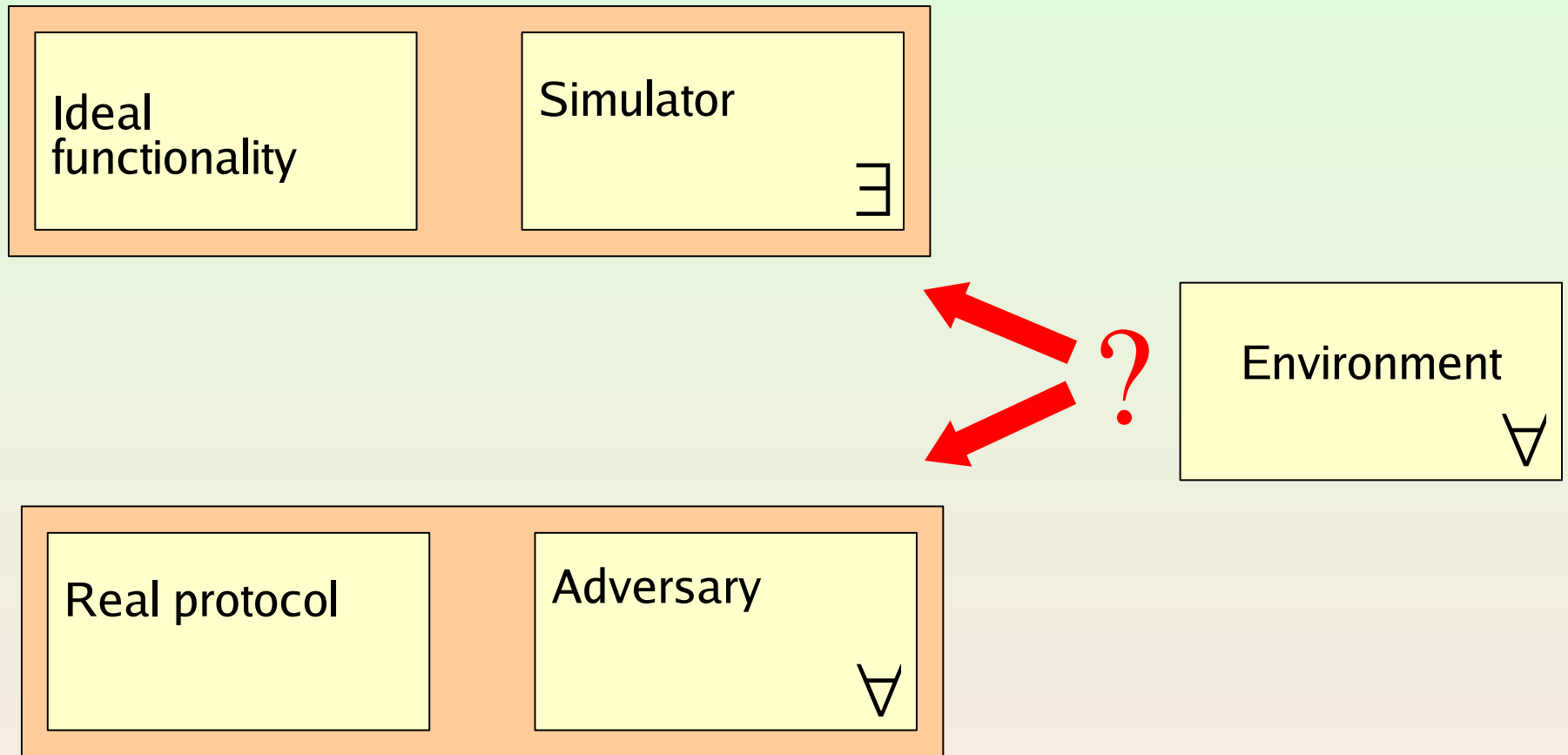
Initial state:  $q_0 \in Q$

States

# STS: Problems

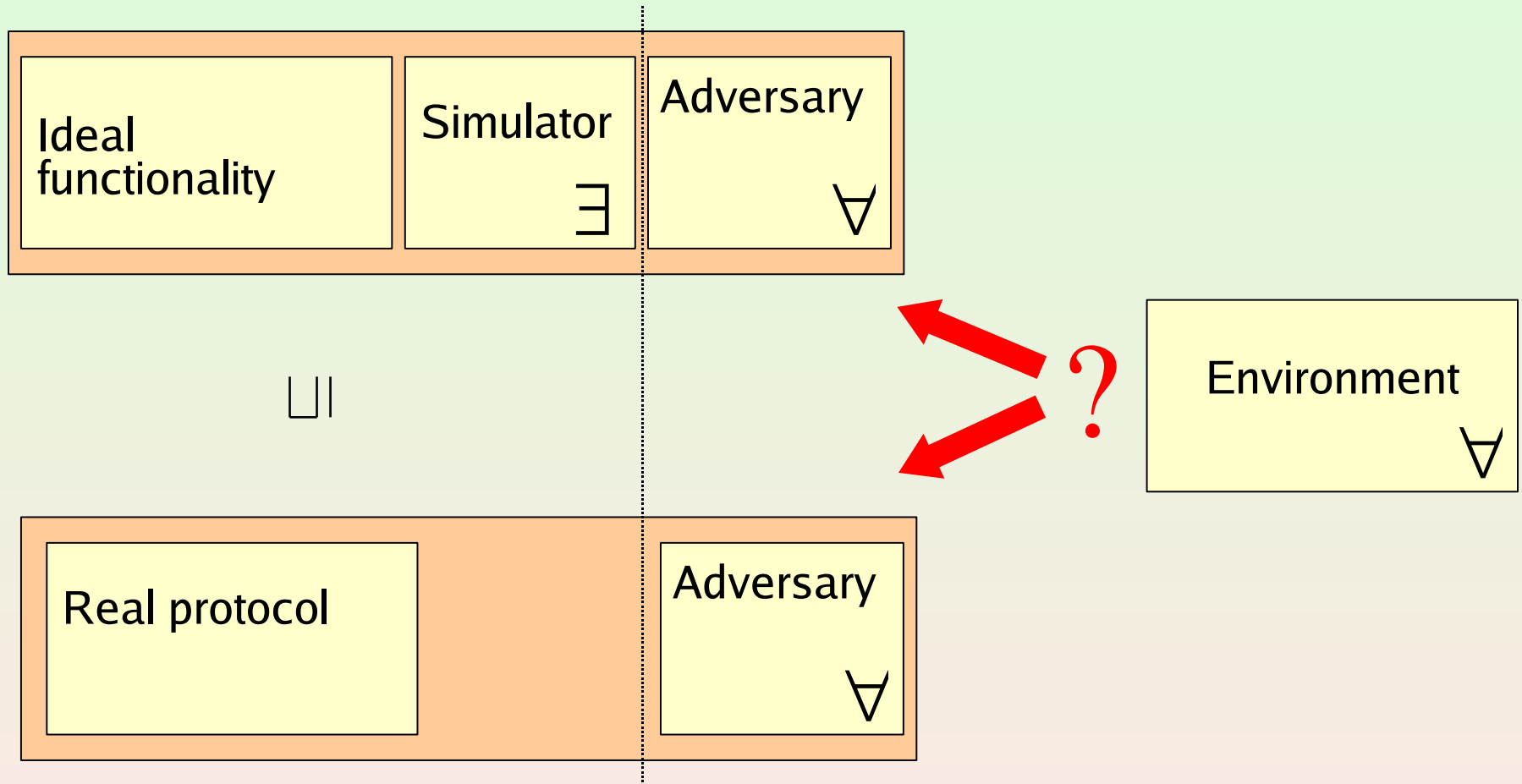
- Not all schedulers lead to measurability
  - Let  $X \subseteq [0,1]$  be non measurable
  - Choose  $x$  uniformly in  $[0,1]$
  - Schedule  $a$  only if  $x \in X$
  - What is the probability of  $\diamond a$ ?
- Define measurable schedulers
  - From  $F_{\text{EXEC}}$  to  $F_{A \times Q}$
  - Then we obtain Markov Kernels
- Markov kernels preserved by projection
  - Important for modular reasoning
- How about bisimulation?

# UC-Security [Canetti]



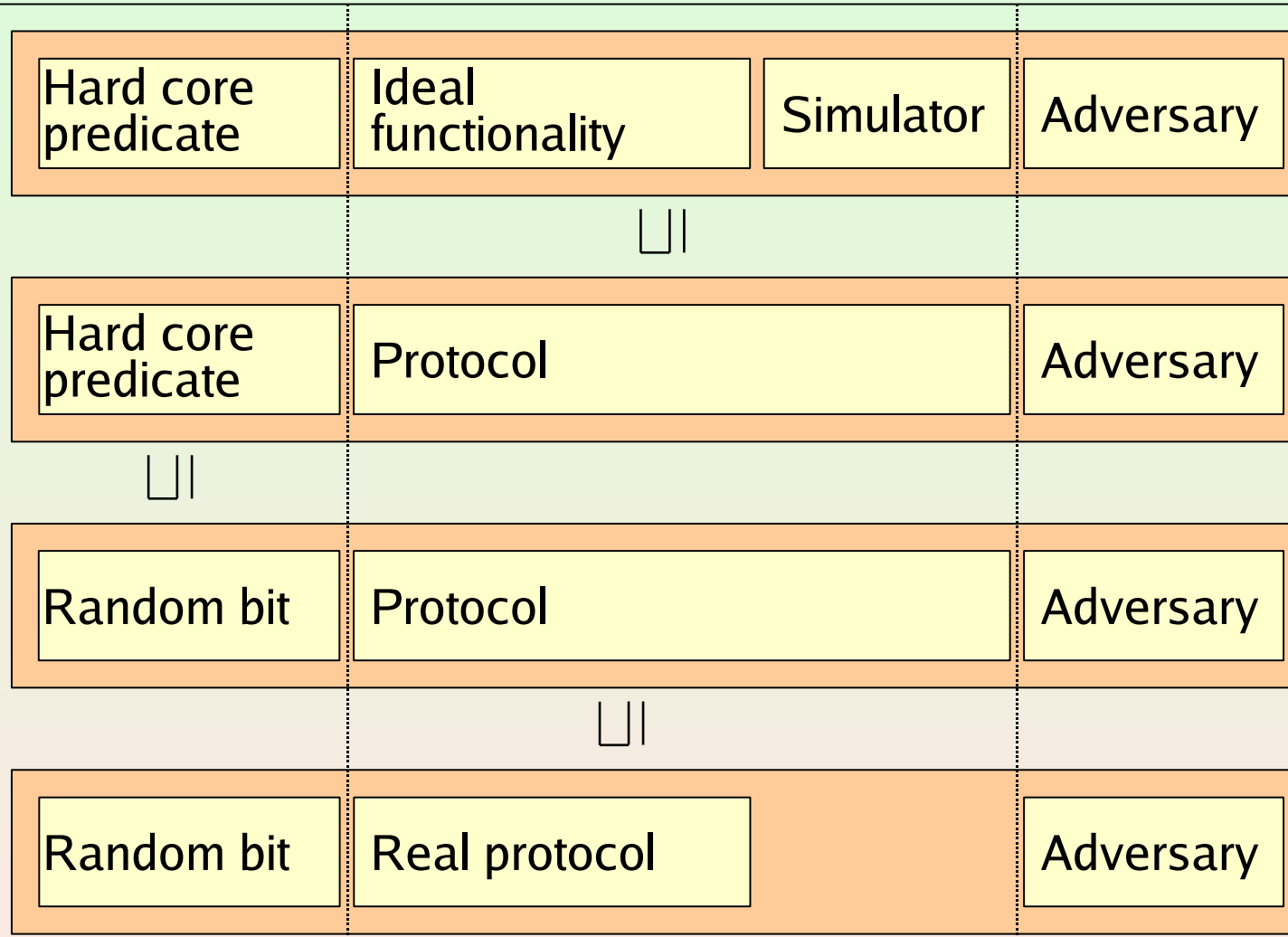
# UC-Security with PIOAs

[Canetti, Cheung, Kaynar, Liskov, Lynch, Pereira, Segala]



# Oblivious Transfer

[Canetti, Cheung, Kaynar, Liskov, Lynch, Pereira, Segala]





# Aproximate Simulations

## [Segala, Turrini]

Given  $\{A_k\}$  and  $\{B_k\}$  consider  $\{R_k\}$ .  $R \subseteq Q_{A_k} \times Q_{A_k}$

For each  $c \in \mathbb{N}$ ,  $p \in \text{Poly}$ , exists  $k \in \mathbb{N}$ , for each  $k > k$ ,  $\varepsilon > 0$ ,  $\mu_1, \mu_2$

If  $\mu_1 \xrightarrow{+}$

$\forall \mu_1$  reached in at most  $p(k)$  steps

$\forall \mu_1 L(R_k, \varepsilon) \mu_2$

$\forall \mu_1 \xrightarrow{\text{}} \mu_1'$

Then

$\forall \mu_2 \xrightarrow{\text{}} \mu_2'$

$\forall \mu_1' L(R_k, \varepsilon + k^{-c}) \mu_2'$

$\mu_1 L(R, \varepsilon) \mu_2$

$\forall \mu_1 = (1 - \varepsilon)\mu_1' + \varepsilon\mu_1''$

$\forall \mu_2 = (1 - \varepsilon)\mu_2' + \varepsilon\mu_2''$

$\forall \mu_1' L(R) \mu_2'$

# Implications on executions

Let  $\{R_k\}$  be an aprox sim from  $\{A_k\}$  to  $\{B_k\}$

For each  $c \in \mathbb{N}$ ,  $p \in \text{Poly}$ , exists  $k \in \mathbb{N}$ , for each  $k > k$ ,  $\mu_1$

If

$\forall \mu_1$  is reachable in  $A_k$  in  $p(k)$  steps

Then exists  $\mu_2$

$\forall \mu_2$  reachable in  $B_k$  in  $p(k)$  steps

$\forall \mu_1 \ L(R, p(k)k^{-c}) \ \mu_2$

# Application to Authentication

## Matching Conversation

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- **Specification:**
  - Actual protocol
  - States keep history
  - Adversary does almost everything
  - All invalid transitions removed
- **Implementation**
  - Actual protocol
  - States keep history
  - Adversary is a PPT algorithm
- **Simulation**
  - Identity on states
- **Properties**
  - All executions of specification satisfy matching conversations
  - Failure of simulation imply breaking a signature protocol

# Open problems

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- Logics
  - Complete the picture with simulations
- Stochastic Transition Systems
  - Understand bisimulation
  - Get soundness results
  - Understand restrictions to the model
- Verification
  - Refine the methods
  - Test on more complex case studies
  - Compare with soundness proofs for symbolic methods