How Much Memory is Needed to Win in Partial-Observation Games

Laurent Doyen
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&

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IST Austria

GAMES'11

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stochastic

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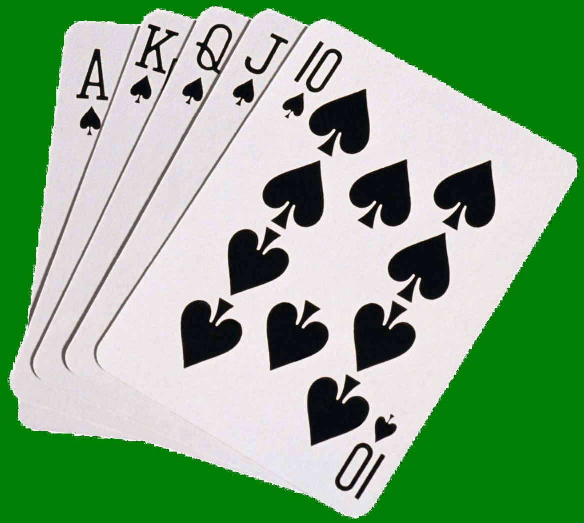
Examples

- Poker
  - partial-observation
  - stochastic



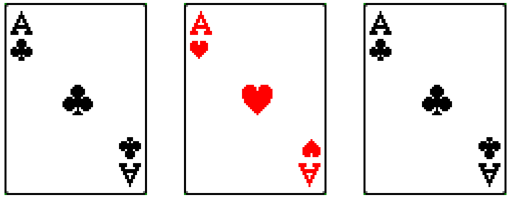
Examples

- Poker
  - partial-observation
  - stochastic

- Bonneteau

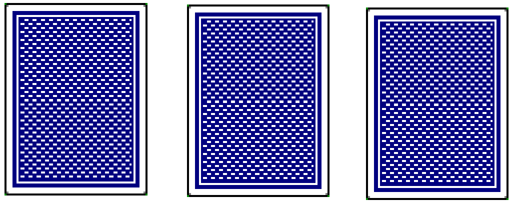
Bonneteau

2 black card, 1 red card



Initially, all are face down

Goal: find the red card



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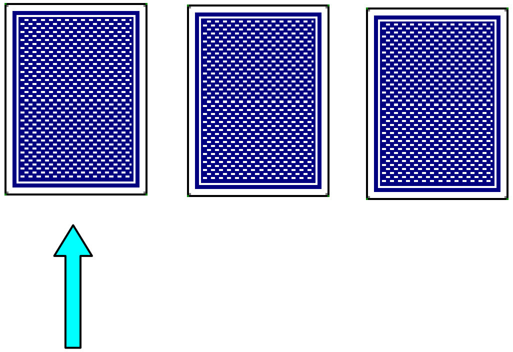

Rules:

1. Player 1 points a card
2. Player 2 flips one remaining black card
3. Player 1 may change his mind, wins if pointed card is red

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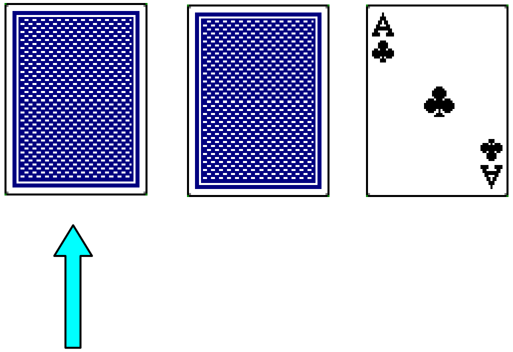

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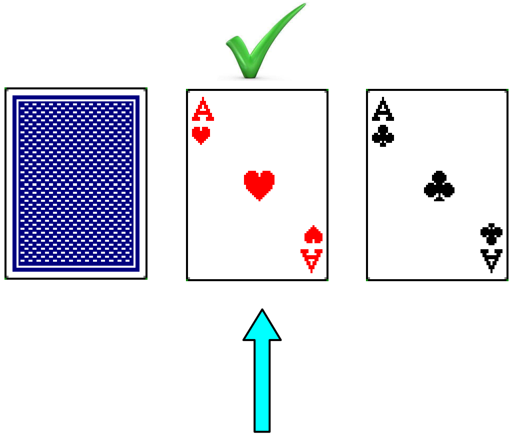

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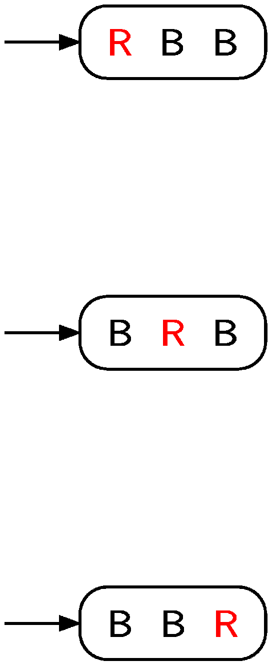
Bonneteau: Game Model



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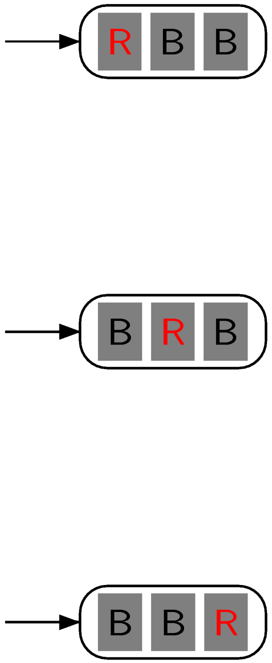
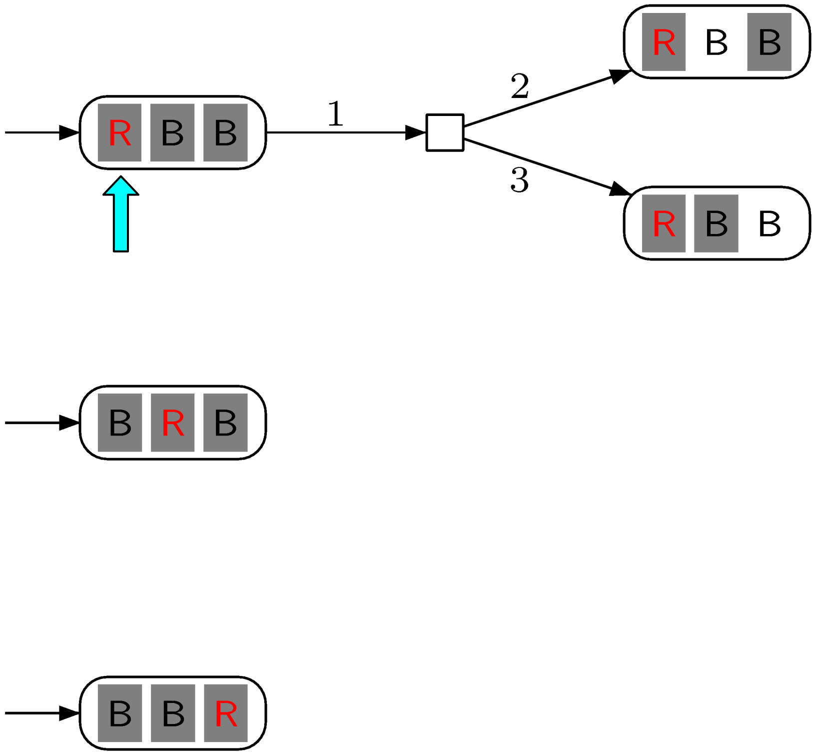
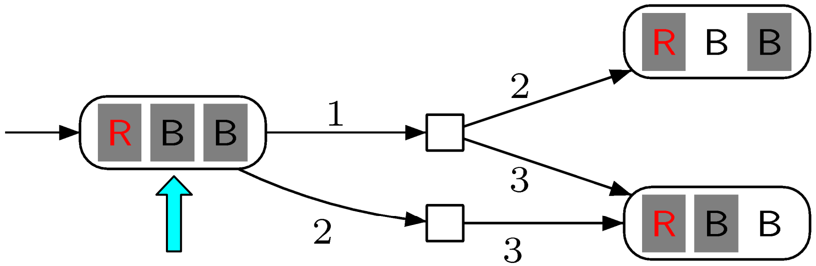
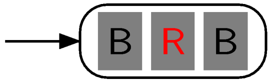


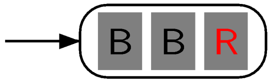
Game Model



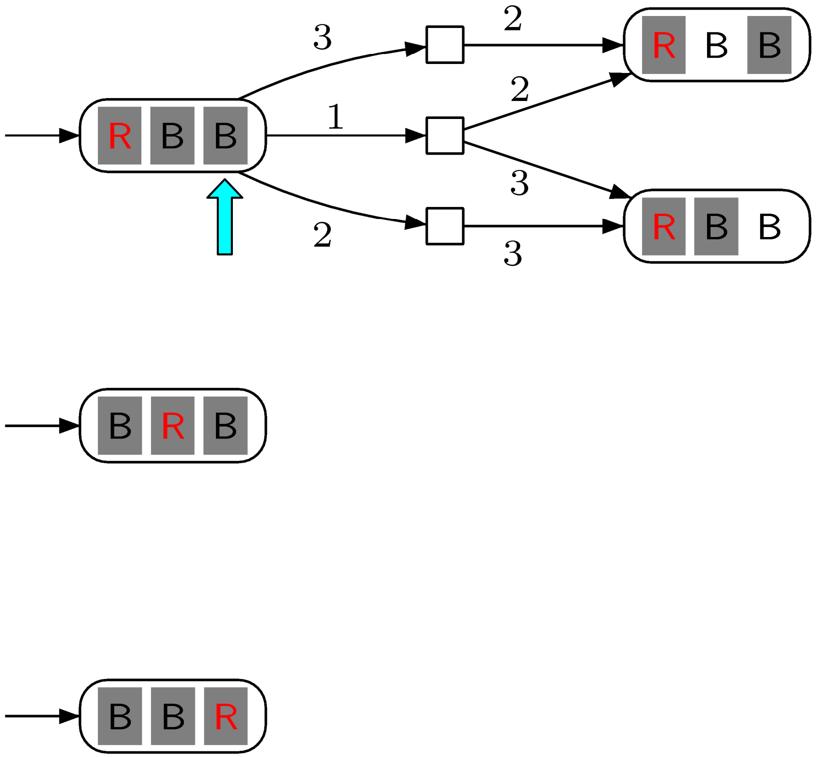
Game Model



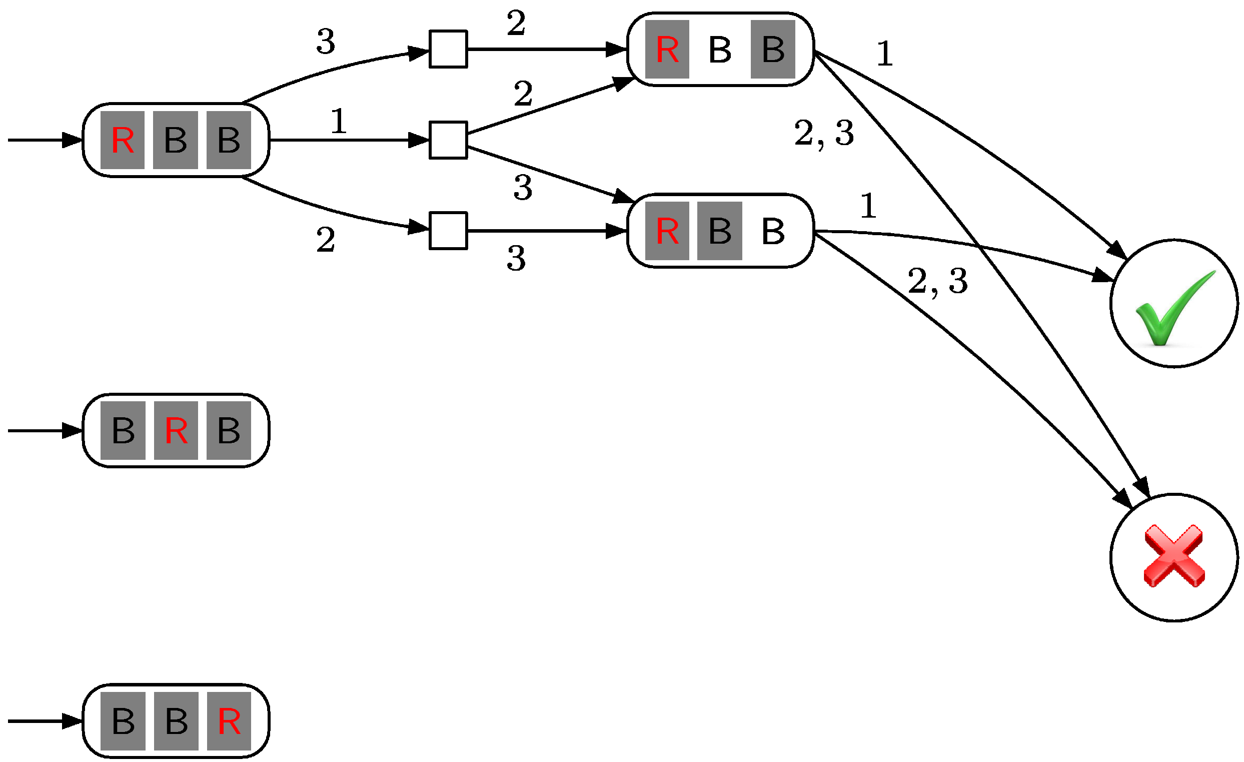




Game Model



Game Model



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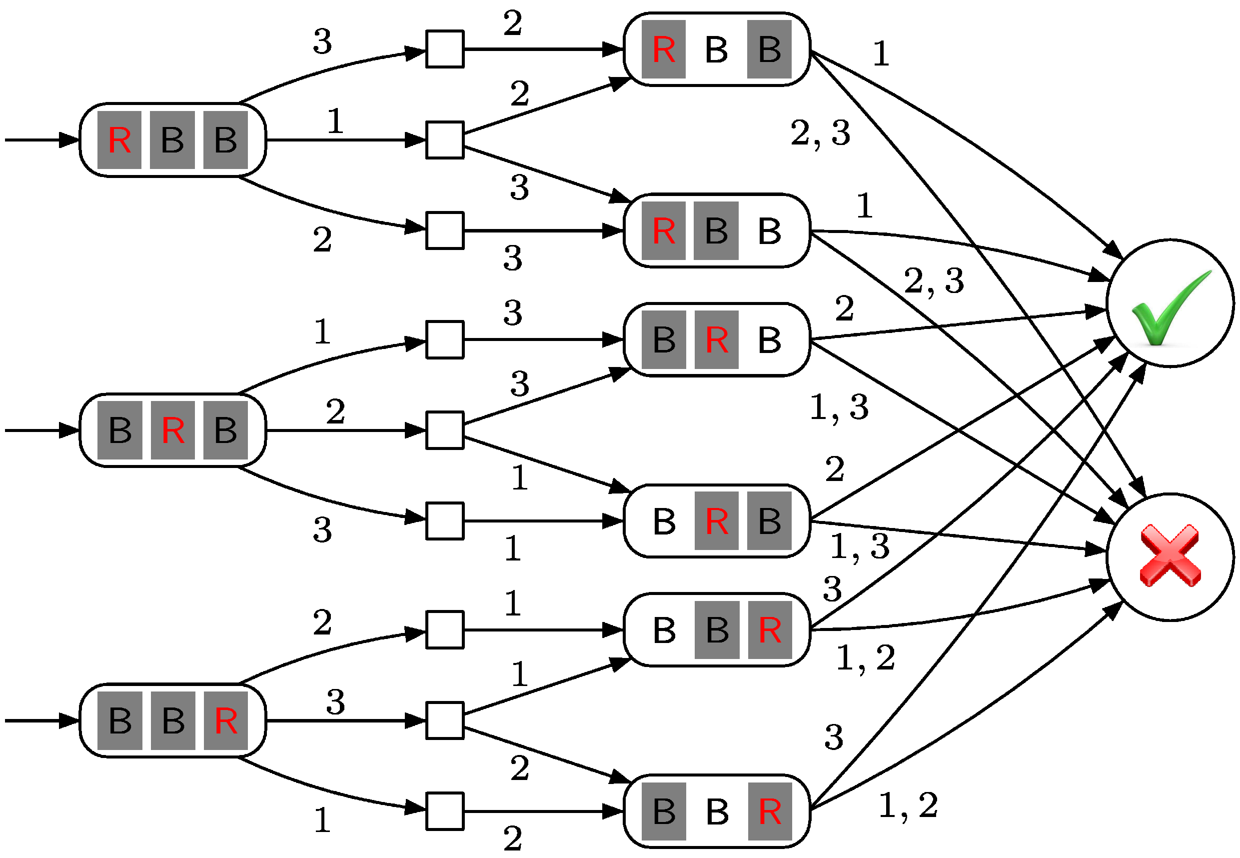
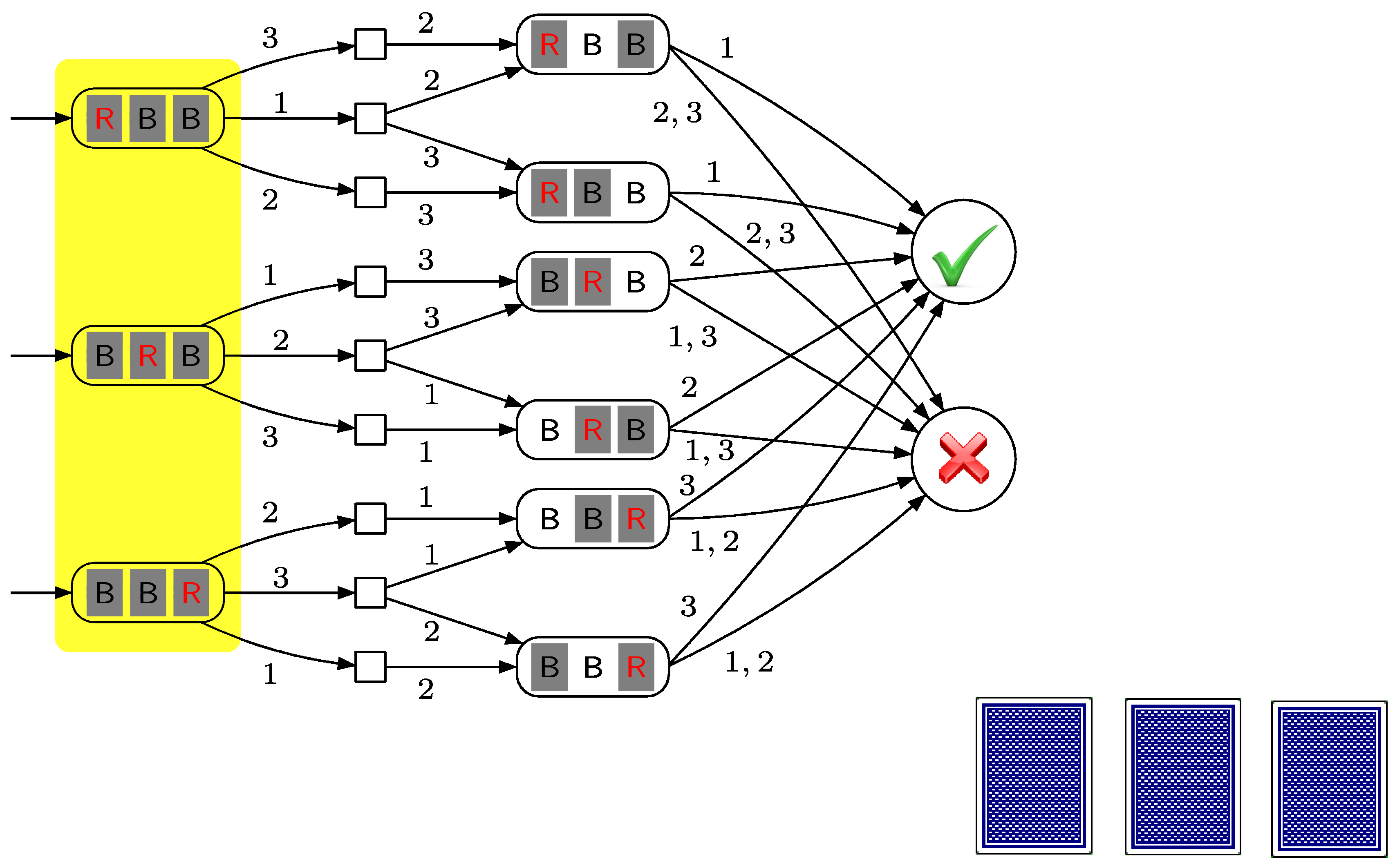


R B B
B R B
B B R
3
1
2
1
2
3
2
3
1
2
2
3
3
3
3
1
1
1
1
2
2
R B B
R B B
B R B
B R B
B B R
B B R
1
2, 3
1
2, 3
2
1, 3
2
1, 3
3
1, 2
3
1, 2

Observations (for player 1)



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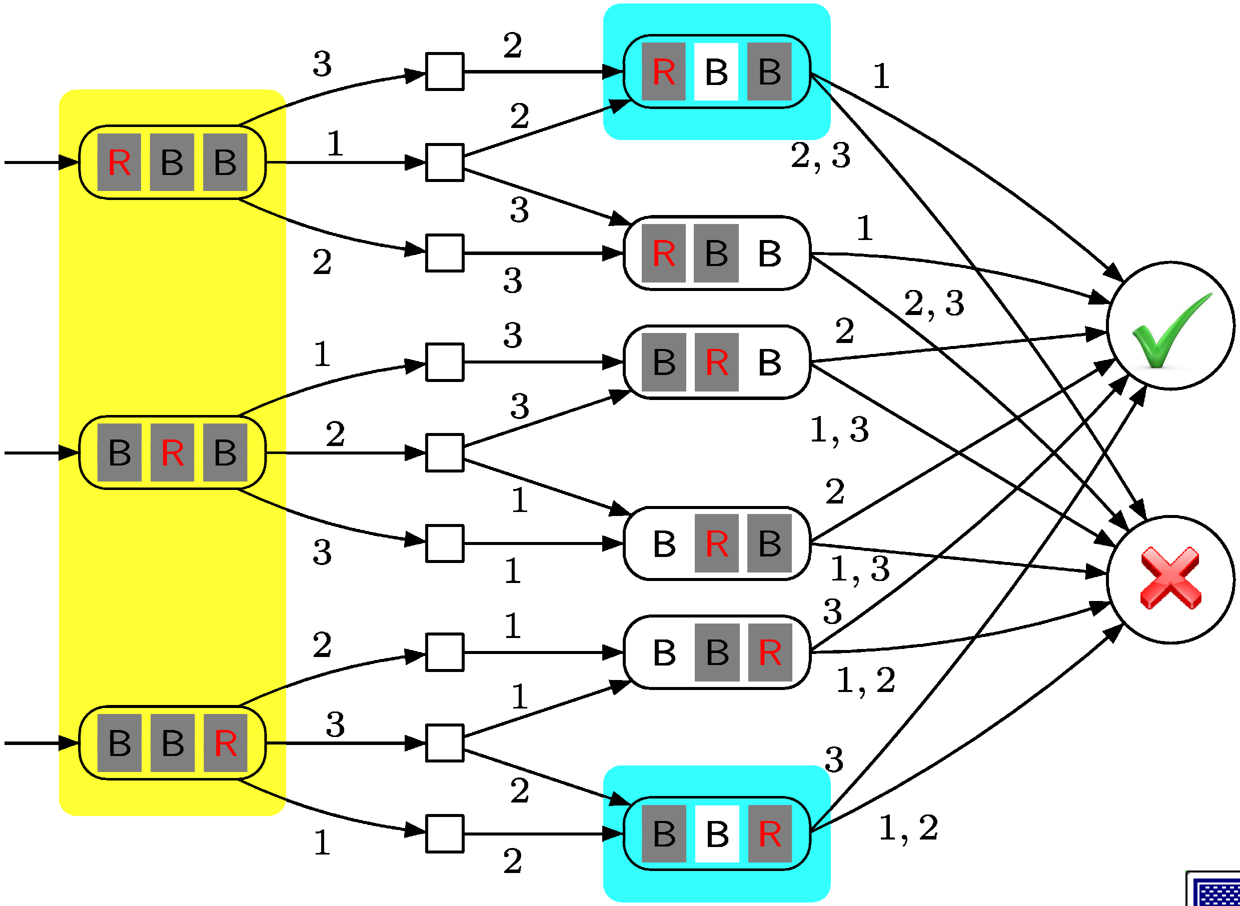

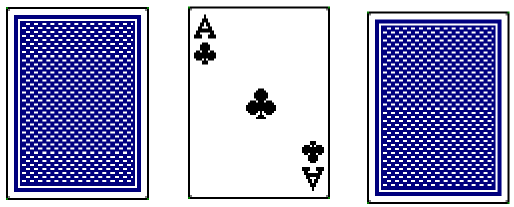

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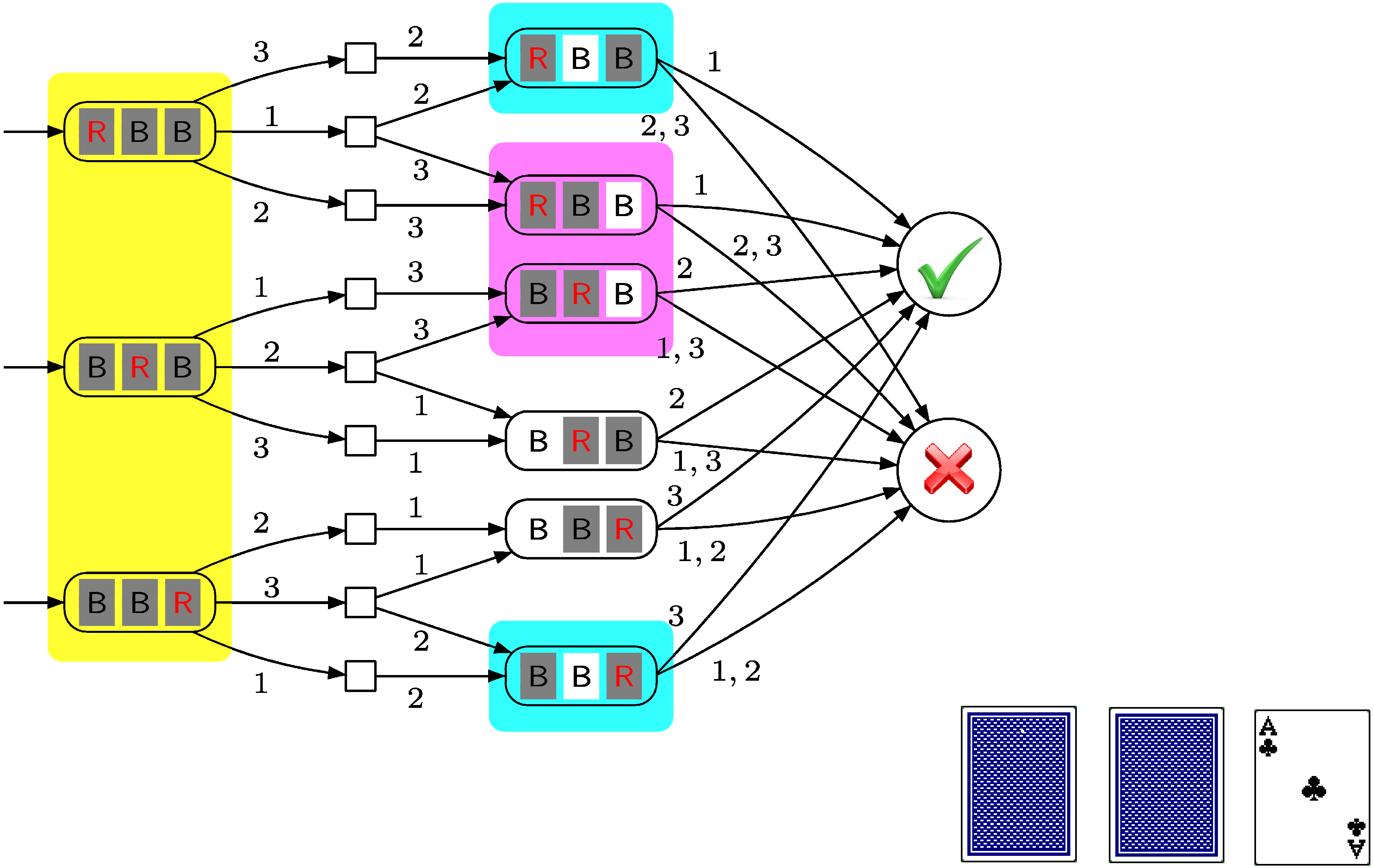

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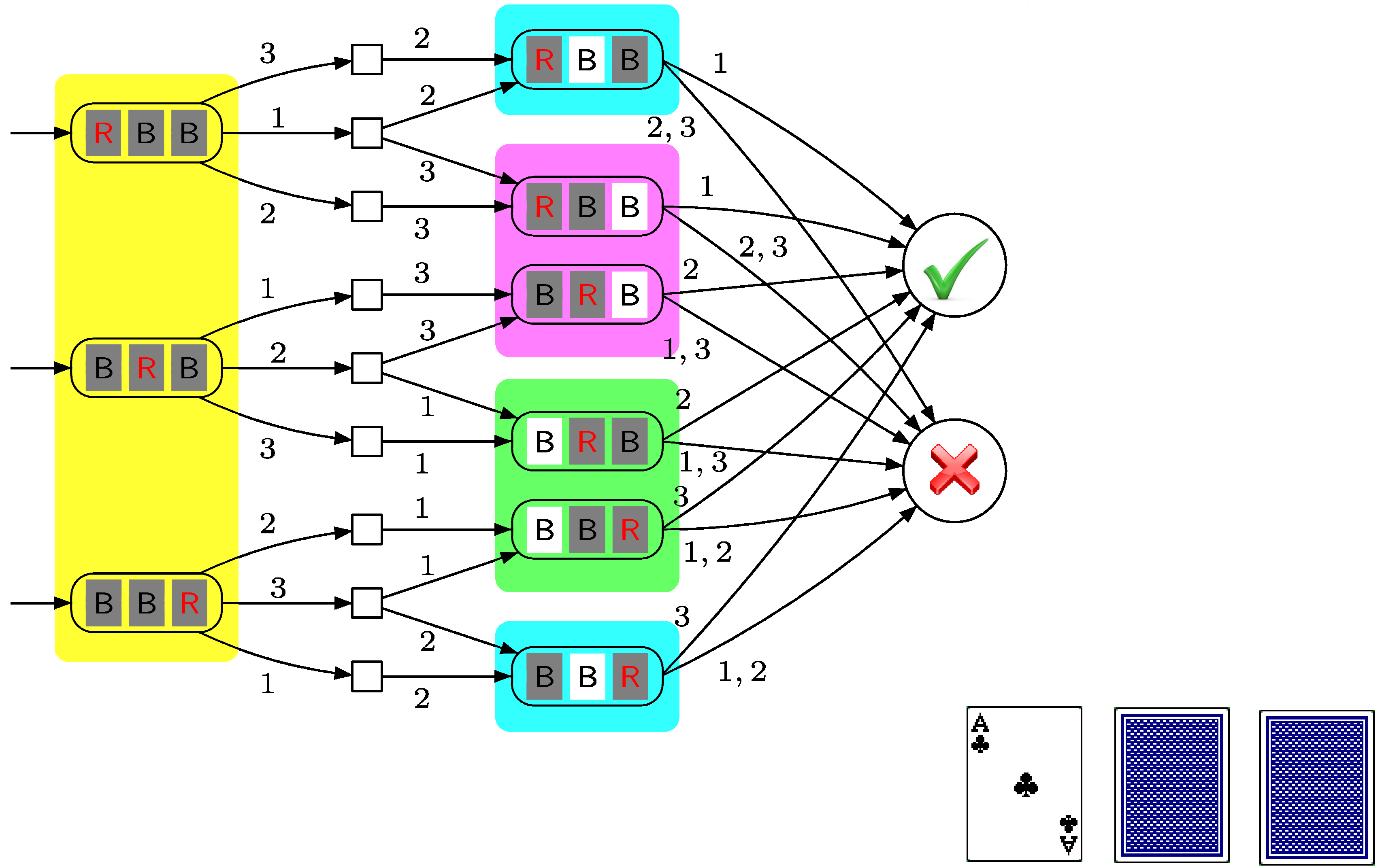

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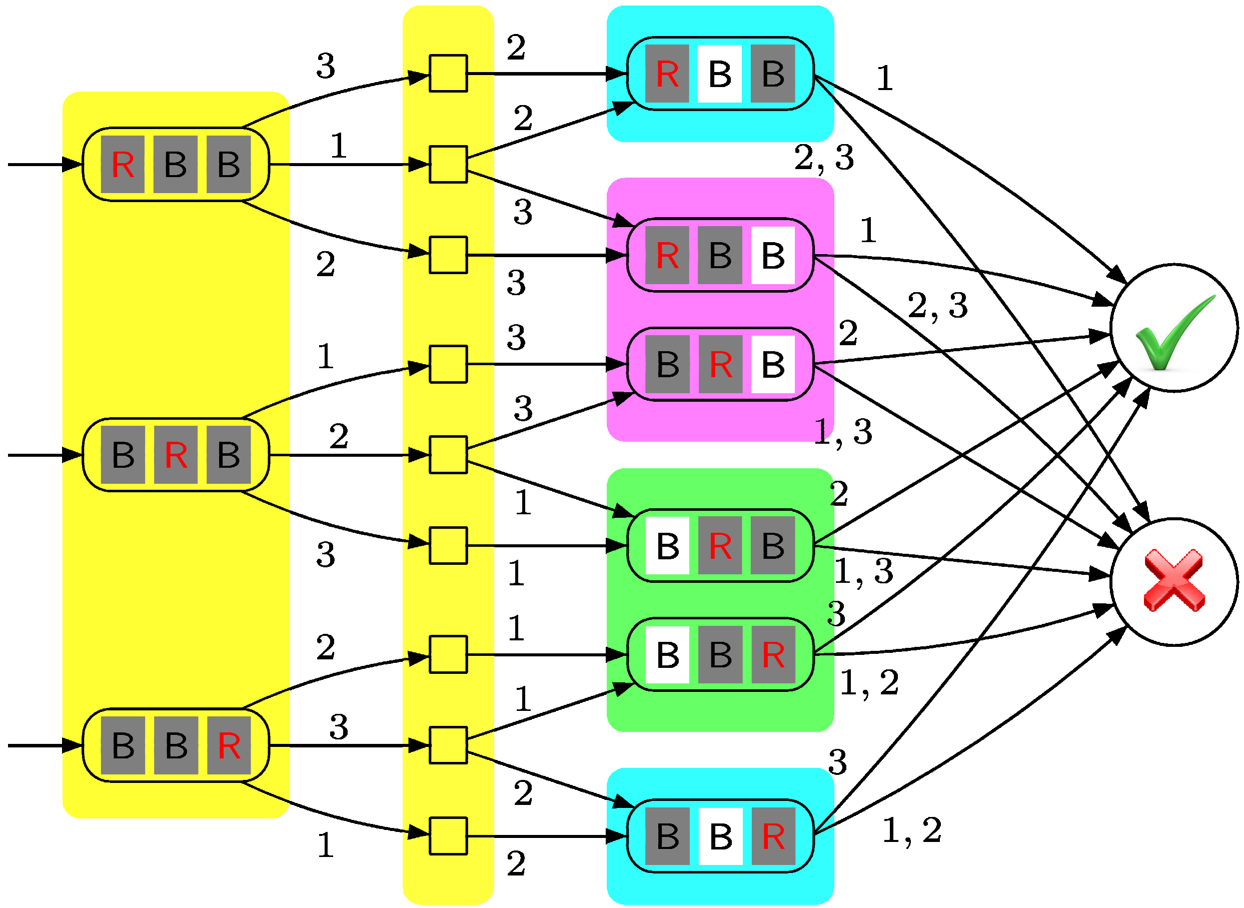
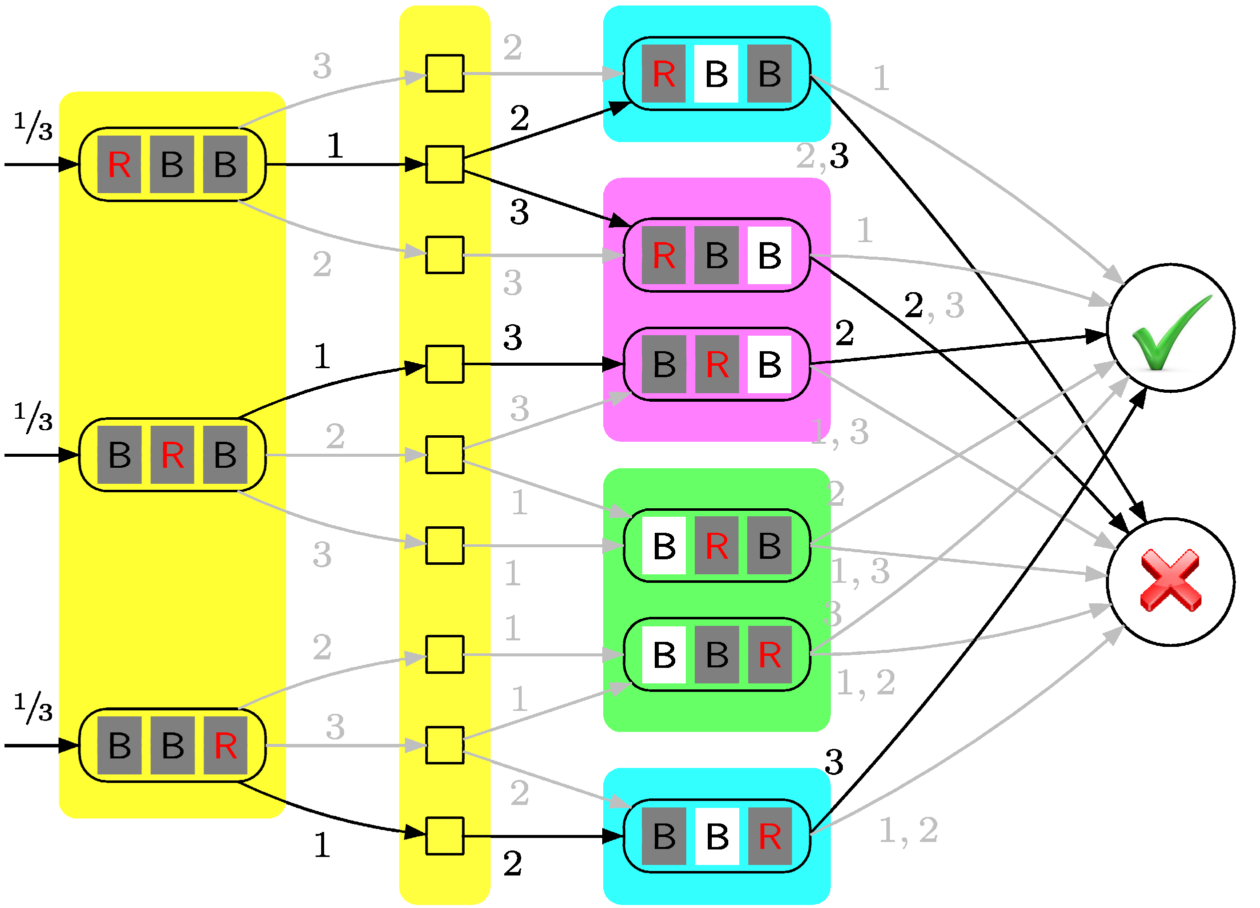


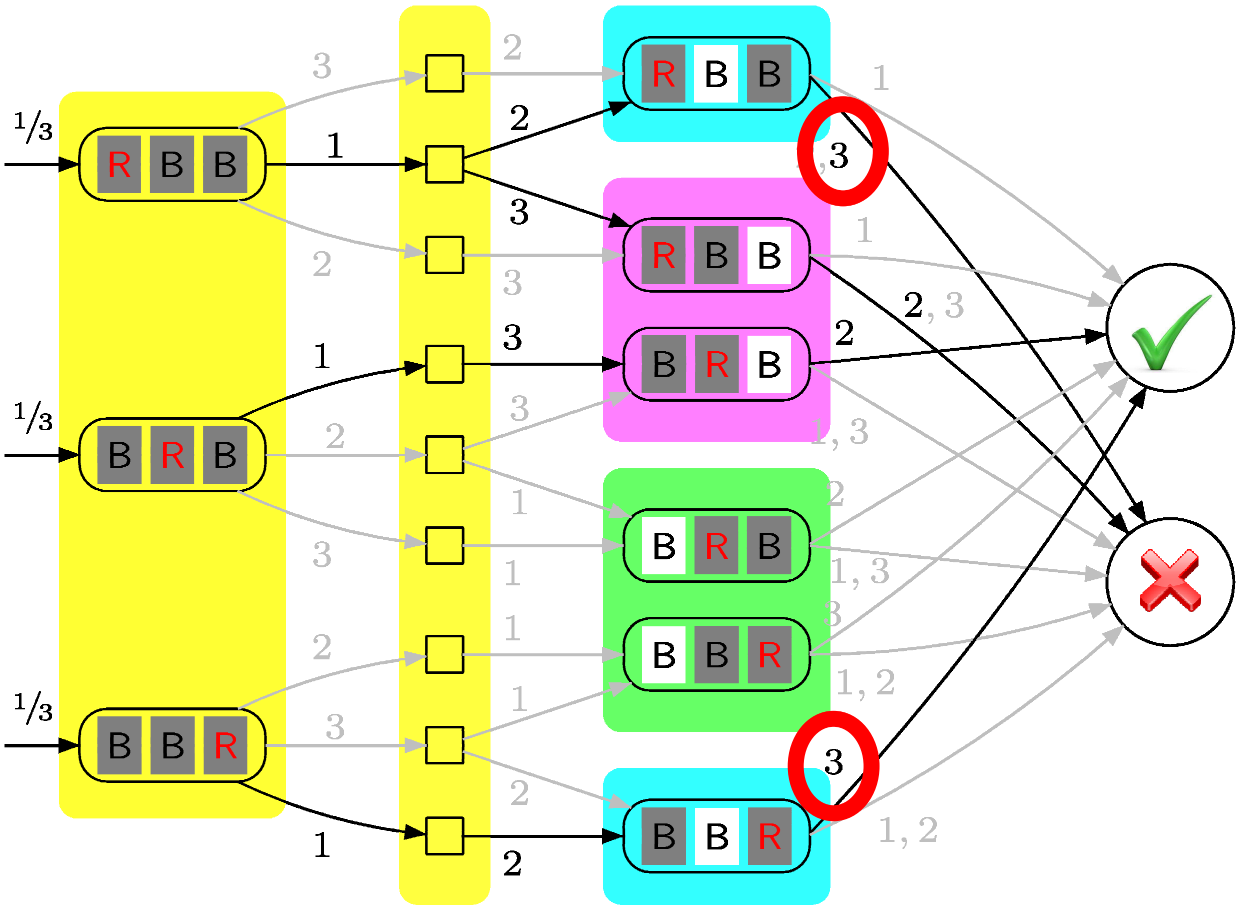
Observation-based strategy



This strategy is observation-based,

e.g. after , , it plays 3

Observation-based strategy



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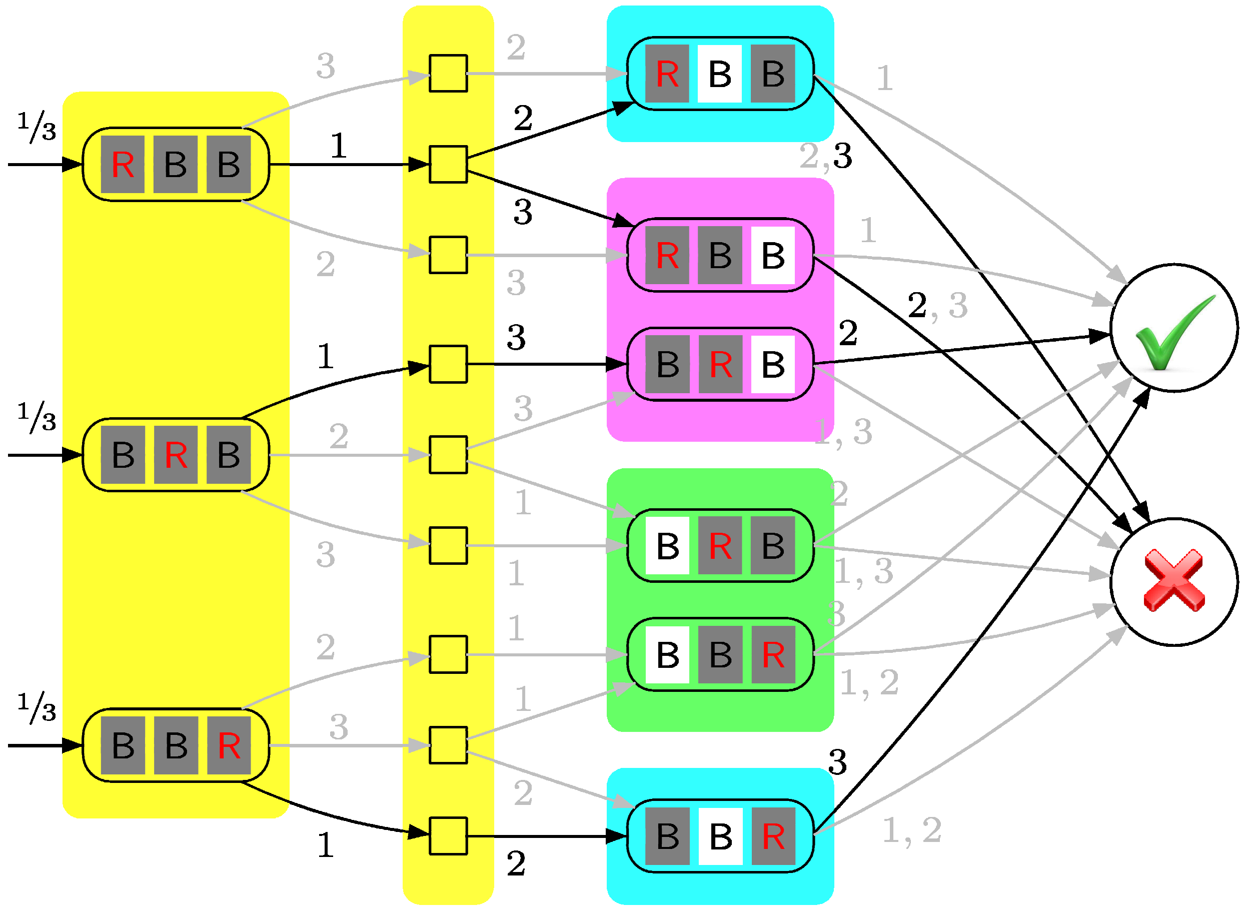
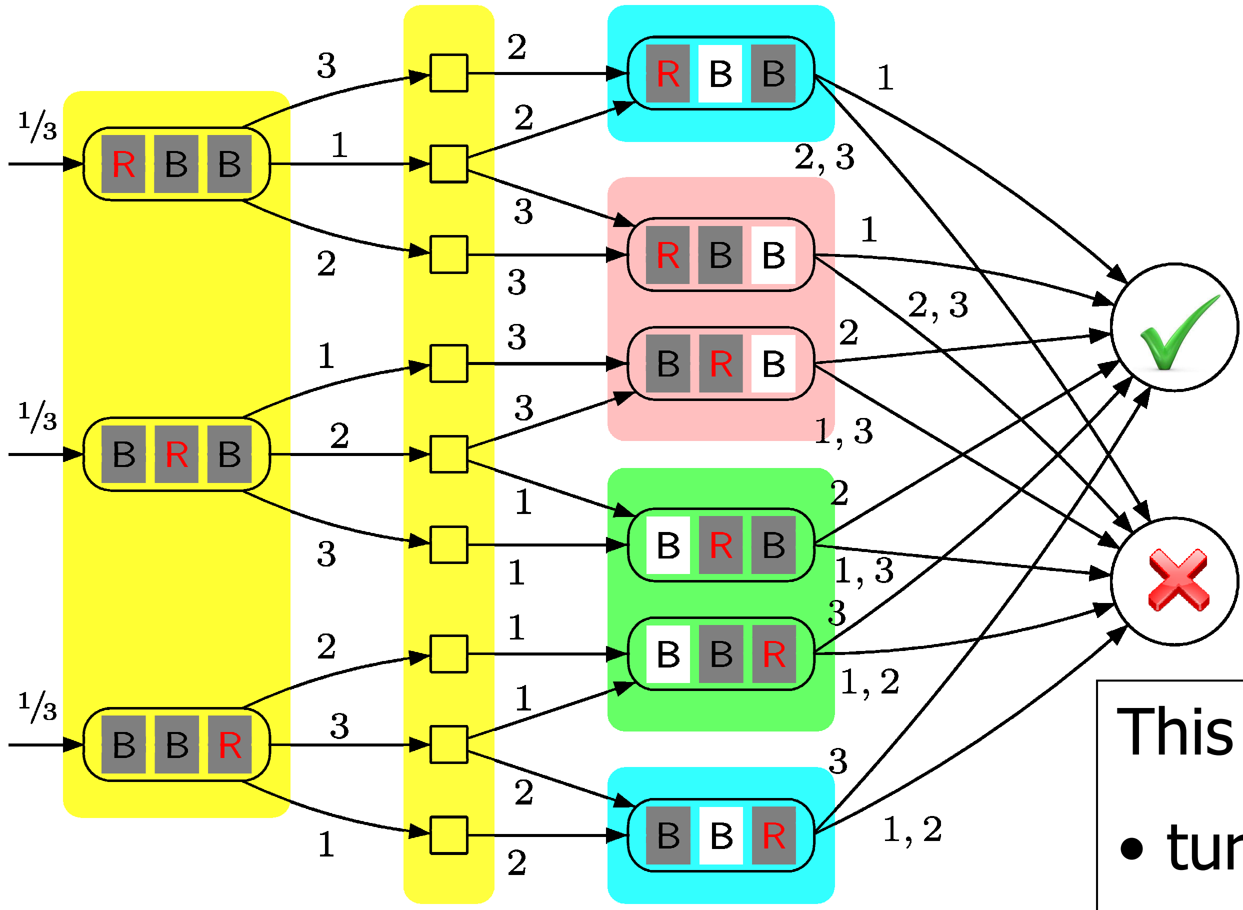
Optimal observation-based strategy



This strategy is winning with probability 2/3

Game Model



This game is:

- turn-based
- (almost) non-stochastic
- player 2 has perfect observation

Interaction

General case: concurrent & stochastic

$$\delta : Q \times A_1 \times A_2 \to \mathcal{D}(Q)$$

Player 1's move

Player 2's move

Players choose their moves simultaneously and independently

Interaction

General case: concurrent & stochastic

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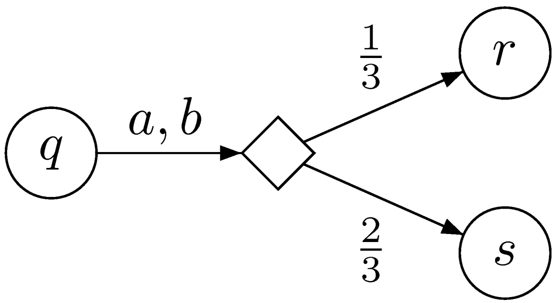
Player 1's move

Player 2's move

Probability distribution on successor state

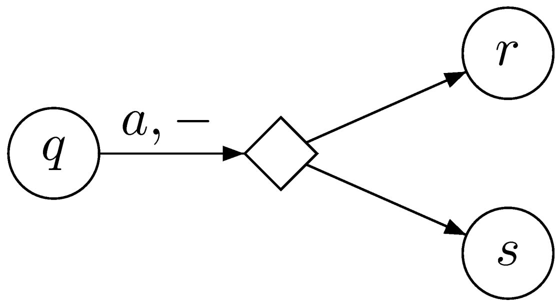
$2\,{}^1\!/_2$-player games



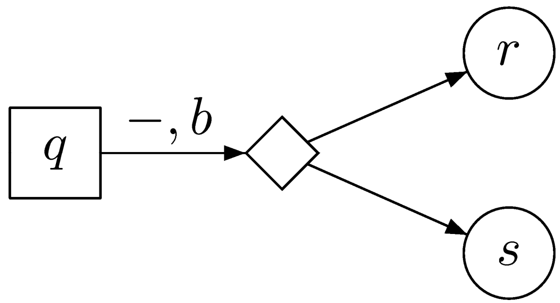
Interaction

Special cases: Turn-based games

- player-1 state



- player-2 state

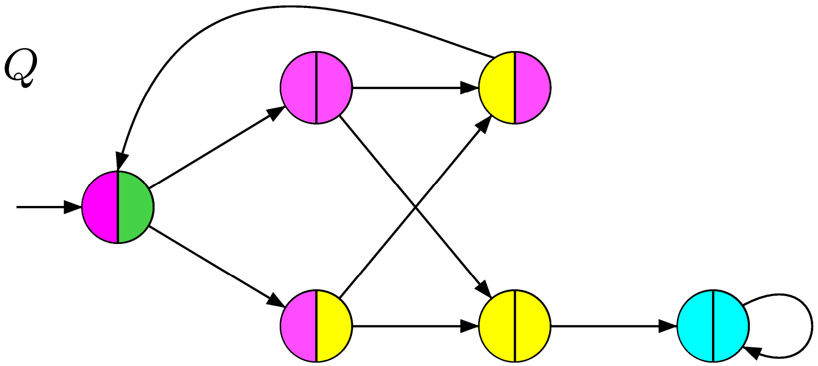


Partial-observation

Observations: partitions induced by coloring

General case: 2-sided partial observation

Two partitions $\mathcal{O}_1 \subseteq 2^Q$ and $\mathcal{O}_2 \subseteq 2^Q$



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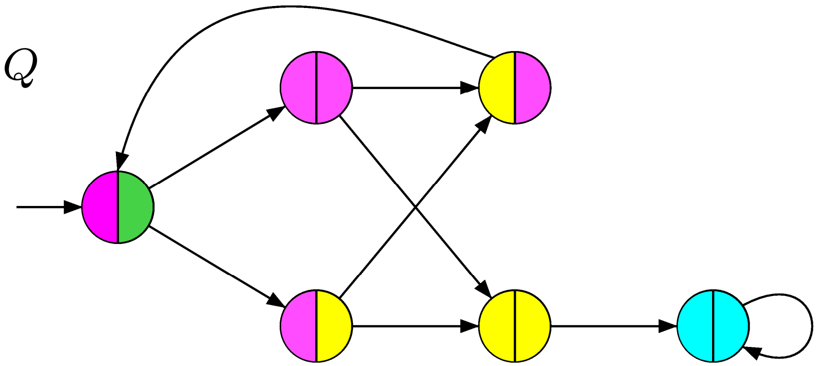
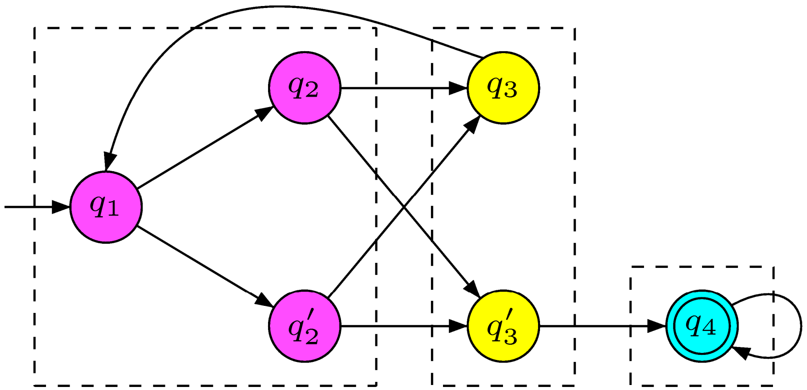
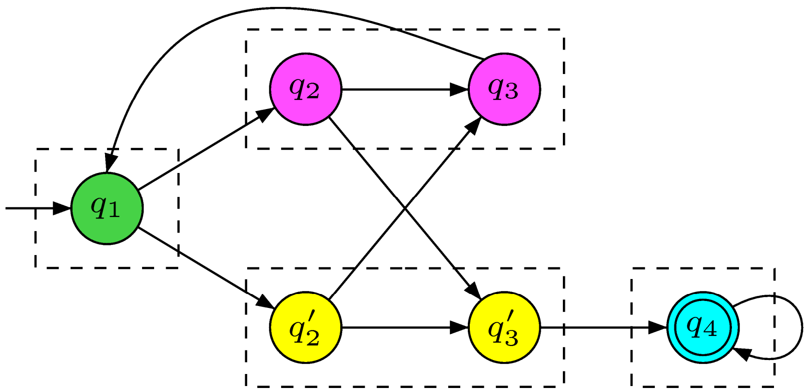




Player 1's view



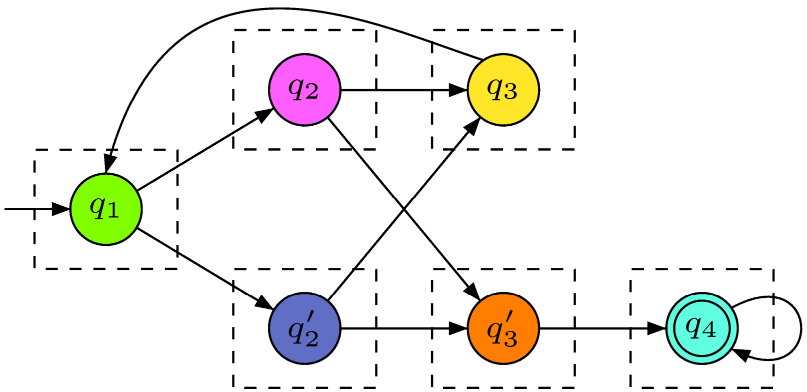
Player 2's view

Partial-observation

Observations: partitions induced by coloring

Special case: 1-sided partial observation

$\mathcal{O}_1 = \{\{q\} \mid q \in Q\}$ or $\mathcal{O}_2 = \{\{q\} \mid q \in Q\}$



Strategies & objective

A strategy for Player i is a function $\sigma_i : \mathcal{O}_i^+ \to \mathcal{D}(A_i)$ that maps histories (sequences of observations) to probability distribution over actions.

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History-depedent randomized

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Reachability objective: $\mathcal{T} \subseteq Q$

Winning probability: $\inf_{\sigma_2} Pr_{q_0}^{\sigma_1,\sigma_2}(\exists i \geq 0 : q_i \in \mathcal{T})$

Qualitative analysis

The following problem is undecidable:
(already for probabilistic automata [Paz71])

Decide if there exists a strategy for player 1 that is winning with probability at least 1/2

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(already for probabilistic automata [Paz71])

Decide if there exists a strategy for player 1 that is winning with probability at least 1/2

Qualitative analysis:

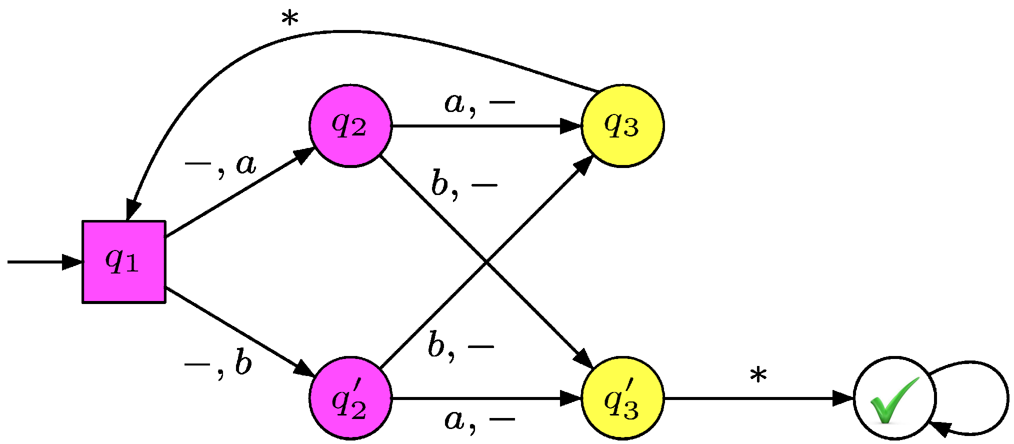
- Almost-sure: … winning with probability 1
- Positive: … winning with probability > 0

$$\exists \sigma_1 \cdot \forall \sigma_2 : Pr_{q_0}^{\sigma_1,\sigma_2}(\exists i \geq 0 : q_i \in \mathcal{T}) \begin{cases} = 1 \\ > 0 \end{cases}$$

Example 1

Player 1 partial, player 2 perfect

$\sigma_i : \mathcal{O}_i^+ \to \mathcal{D}(A_i)$



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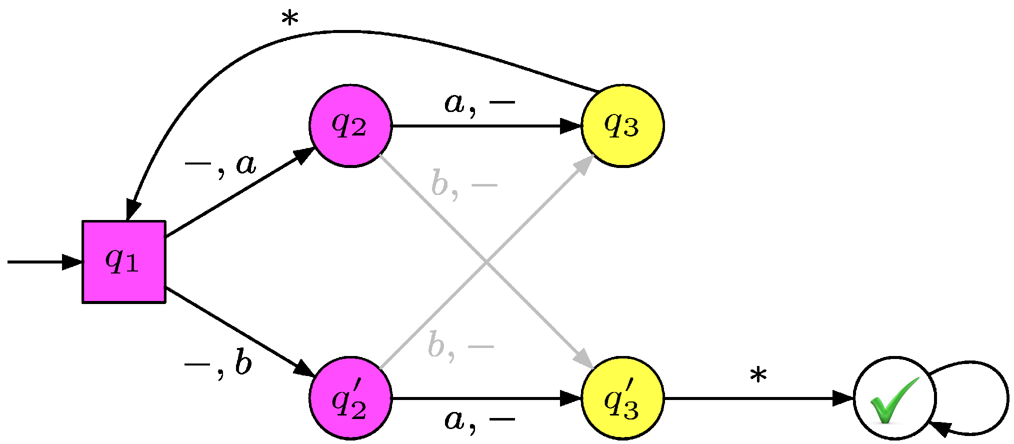

No pure strategy of Player 1 is winning with probability 1

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Player 1 partial, player 2 perfect

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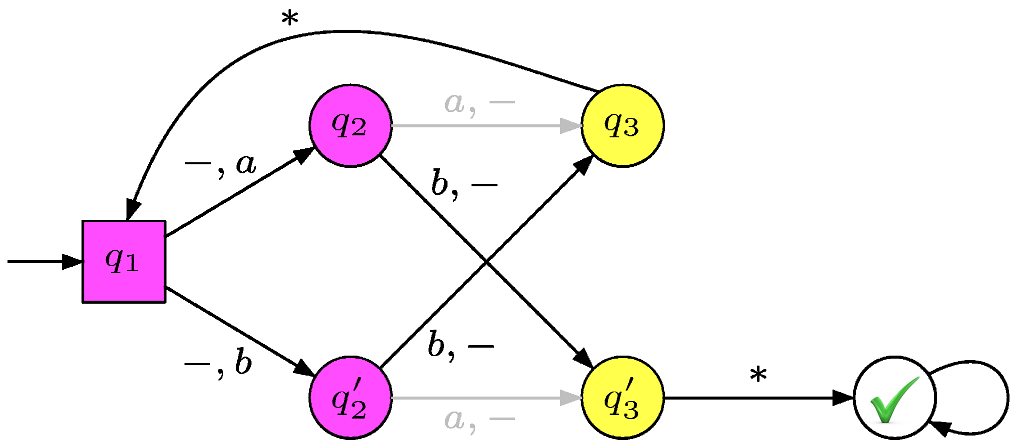

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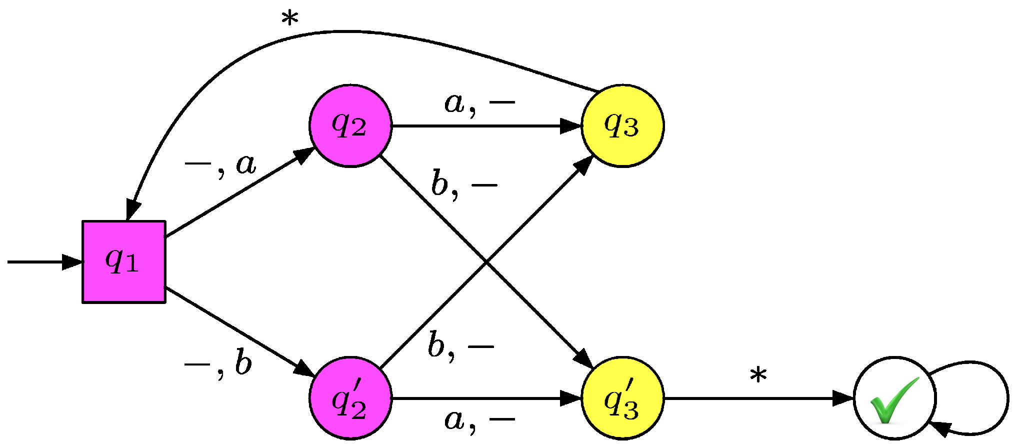
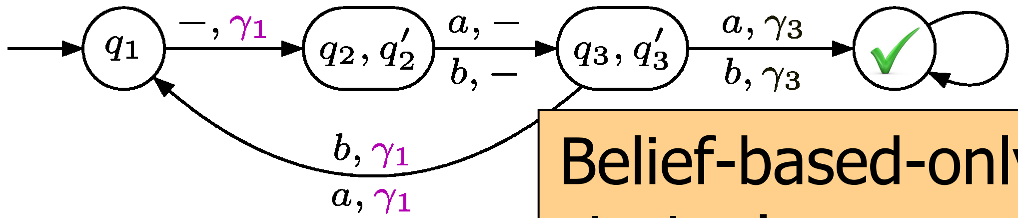


Player 1 wins with probability 1, and needs randomization

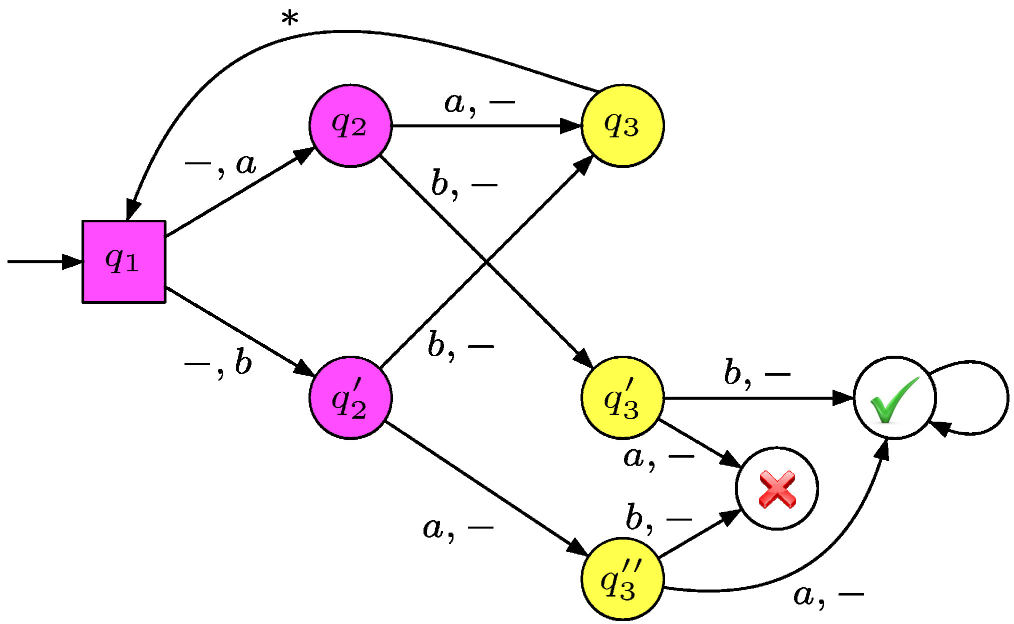


Belief-based-only randomized strategies are sufficient

Example 2

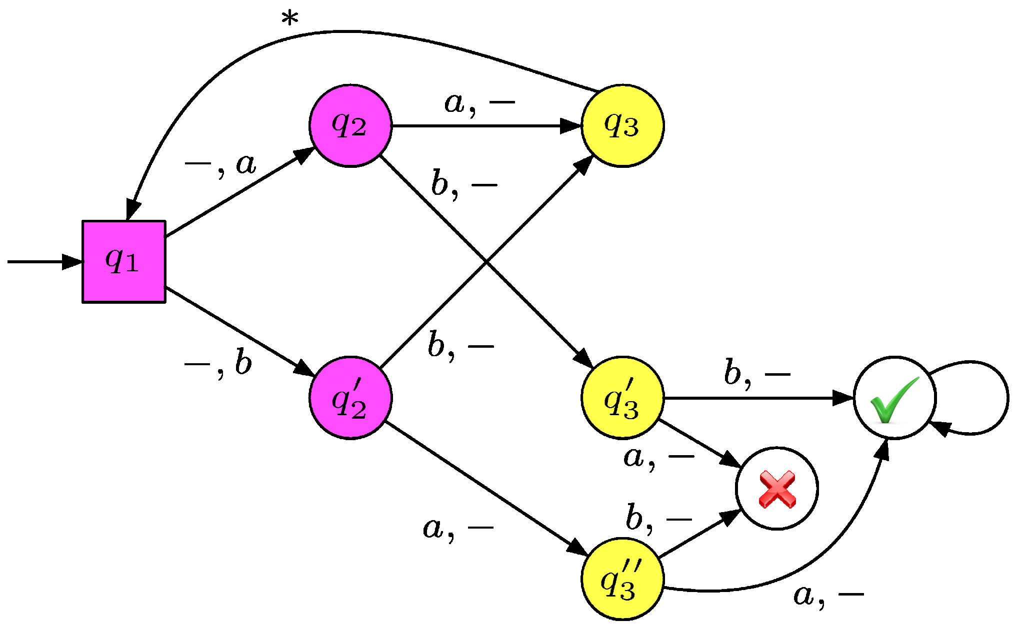
Player 1 partial, player 2 perfect

$\sigma_i : \mathcal{O}_i^+ \to \mathcal{D}(A_i)$



Example 2

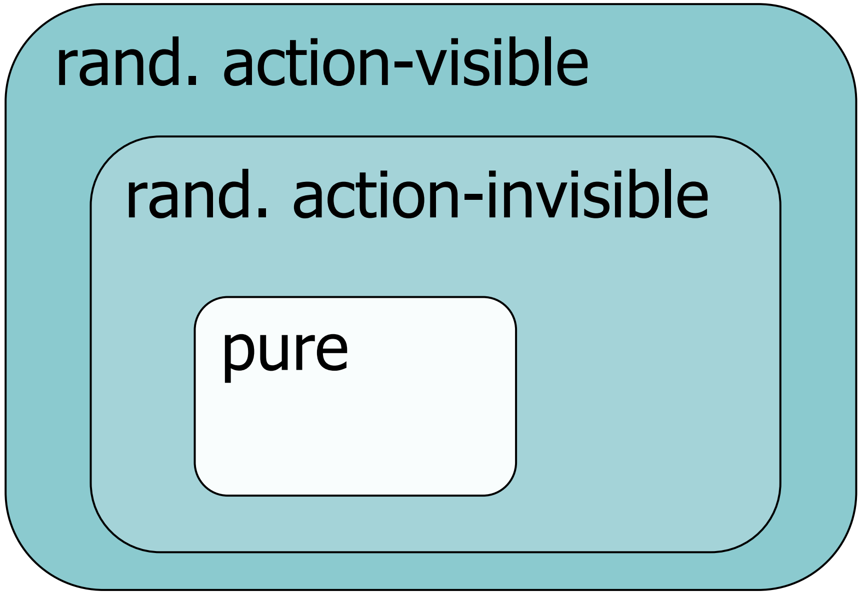
Player 1 partial, player 2 perfect



To win with probability 1, player 1 needs to observe his own actions.

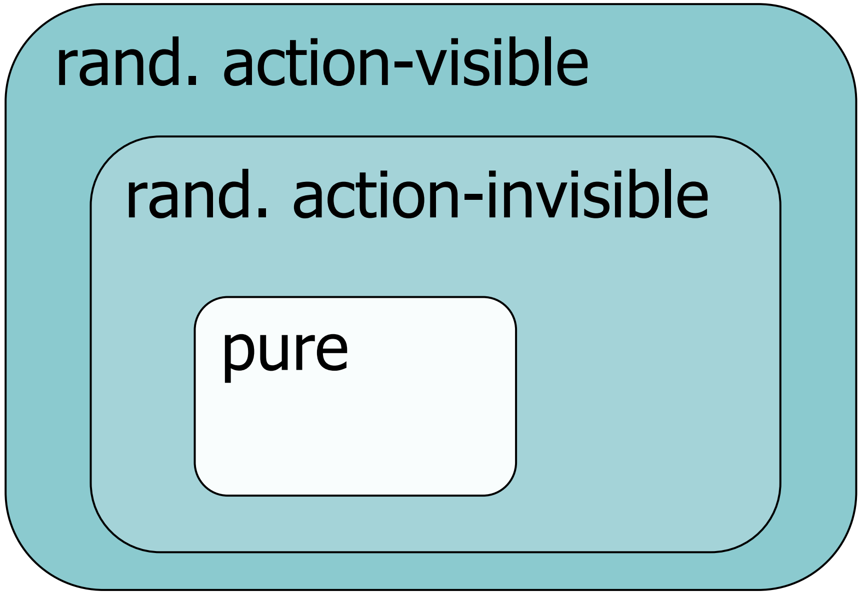
Randomized action-visible strategies: $\sigma_i : (\mathcal{O}_i A_i)^* \mathcal{O}_i \to \mathcal{D}(A_i)$

Classes of strategies



Classification according to the power of strategies

Classes of strategies

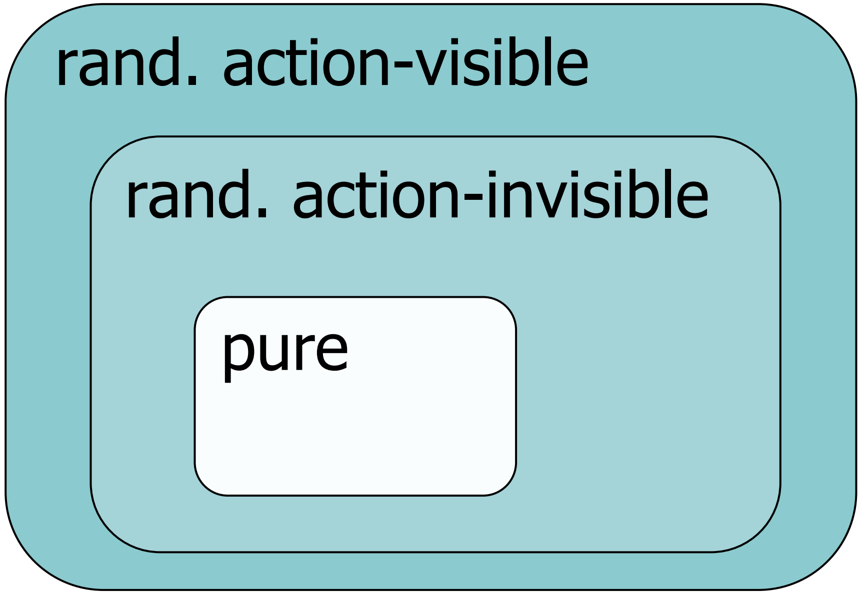


Classification according to the power of strategies

Poly-time reduction from decision problem of rand. act.-vis. to rand. act.-inv.

The model of rand. act.-inv. is more general

Classes of strategies



Classification according to the power of strategies

Computational complexity (algorithms)

Strategy complexity (memory)

Known results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.			
rand. act.-inv.			
pure			

Known results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (belief) **[CDHR'06(remark), GS'09]**		exponential (belief) **[GS'09]**
pure	?	?	?

[BGG09] Bertrand, Genest, Gimbert. ***Qualitative Determinacy and Decidability of Stochastic Games with Signals*. LICS'09.**
[CDHR06] Chatterjee, Doyen, Henzinger, Raskin. ***Algorithms for ω-Regular games with Incomplete Information*. CSL'06.**
[GS09] Gripon, Serre. ***Qualitative Concurrent Stochastic Games with Imperfect Information*. ICALP'09.**

Known results

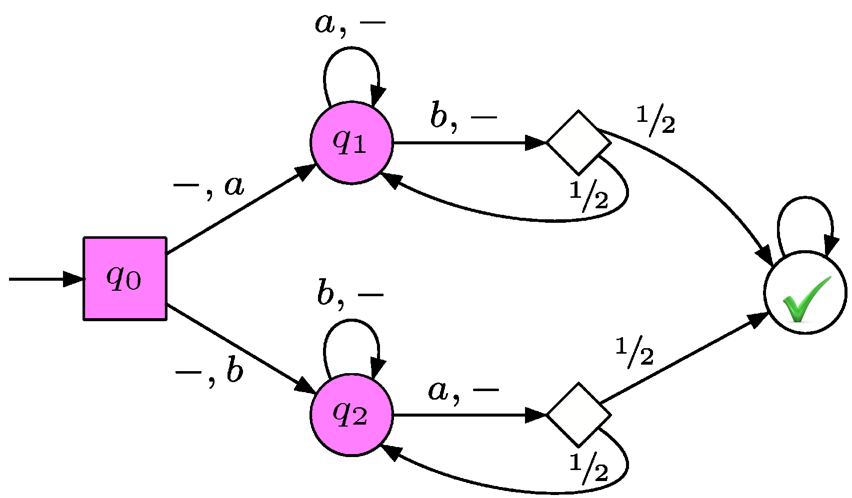
Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (belief) **[CDHR'06(remark), GS'09]**		exponential (belief) **[GS'09]**
pure	?	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	?	?	?

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

player 2 perfect

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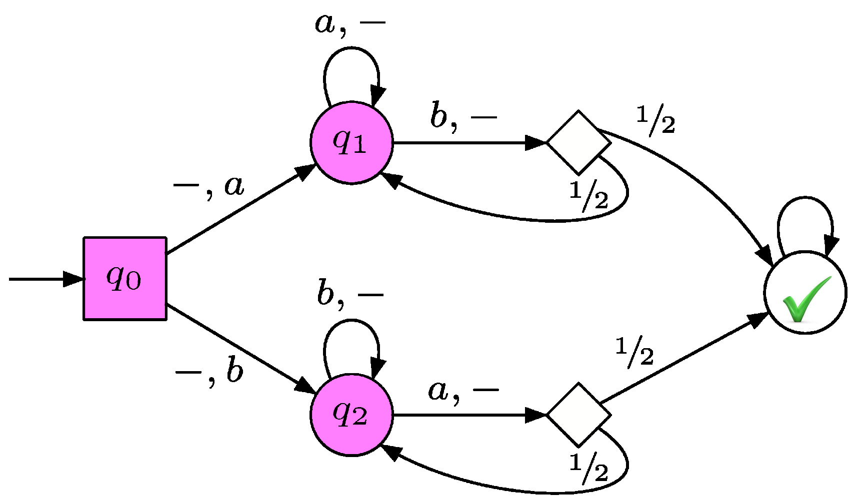


player 1 partial

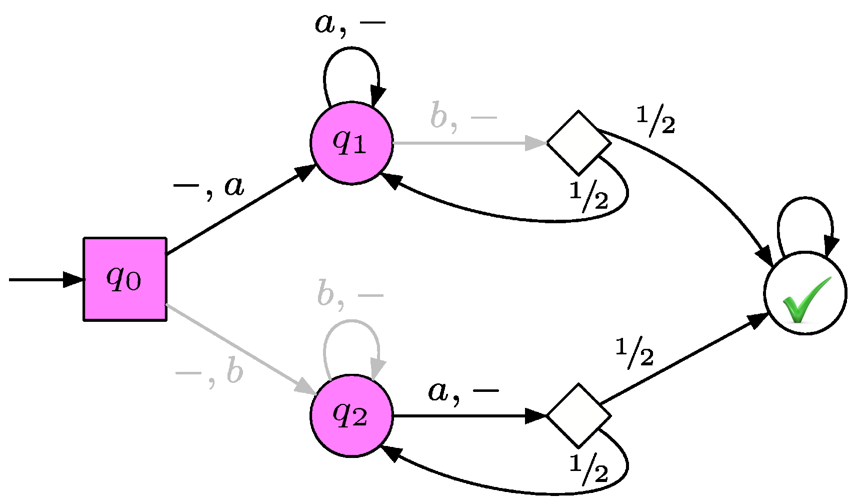
player 2 perfect

There are two belief-based-only pure strategies:

1. When belief is $\{q_1, q_2\}$, play a
2. When belief is $\{q_1, q_2\}$, play b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

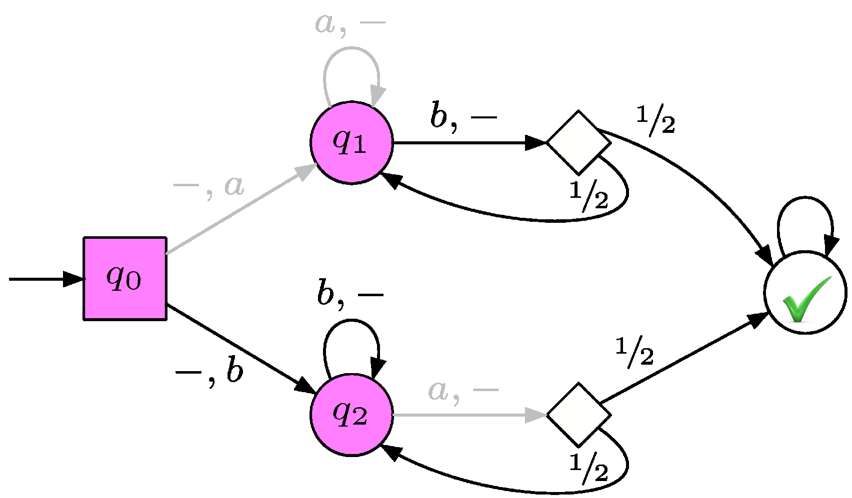
player 2 perfect

There are two belief-based-only pure strategies:

1. When belief is $\{q_1, q_2\}$, play a ⇐ not winning
2. When belief is $\{q_1, q_2\}$, play b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



player 1 partial

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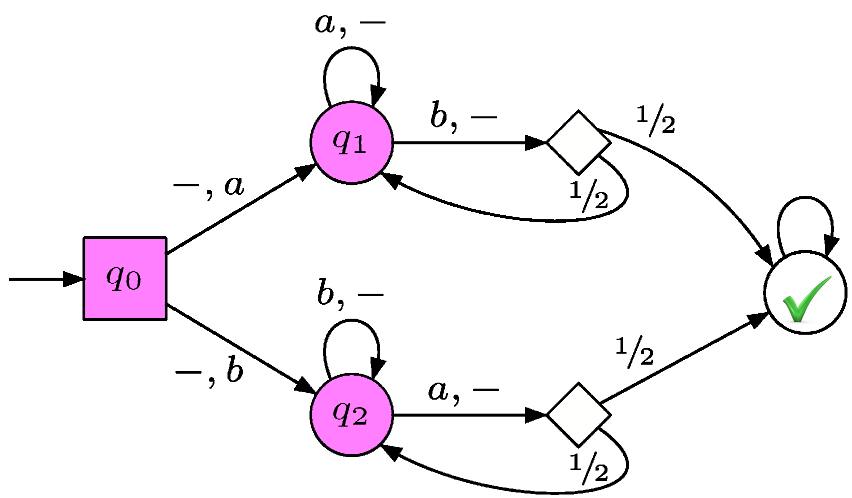


player 1 partial

player 2 perfect

There are two belief-based-only pure strategies:

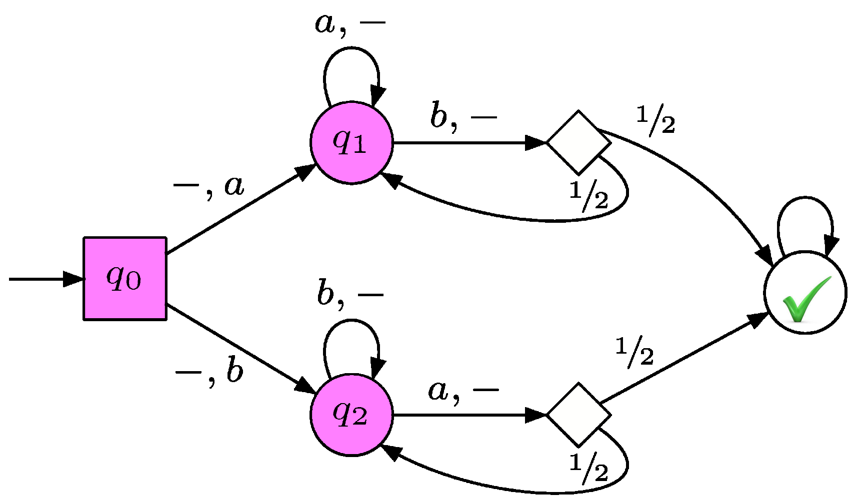
1. W

Neither is winning !

2. W

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



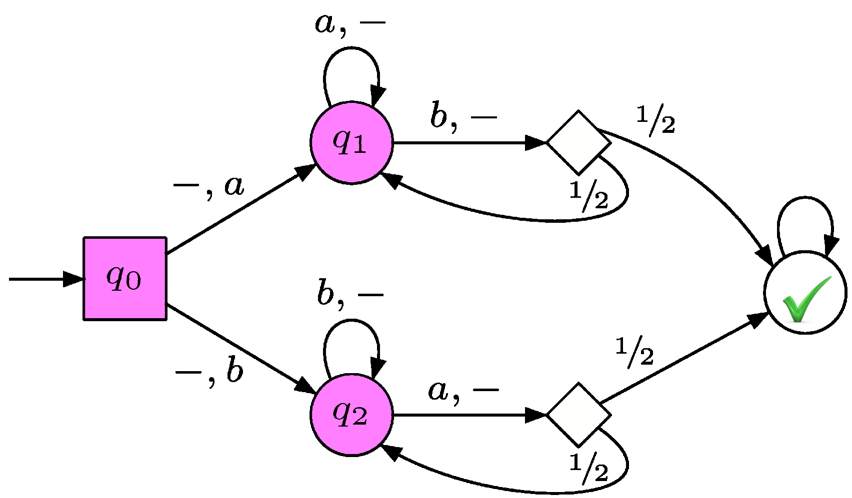
player 1 partial

player 2 perfect

When belief is $\{q_1, q_2\}$, alternate a and b

When belief fails (1/2)

Belief-based-only pure strategies are not sufficient, both for positive and for almost-sure winning



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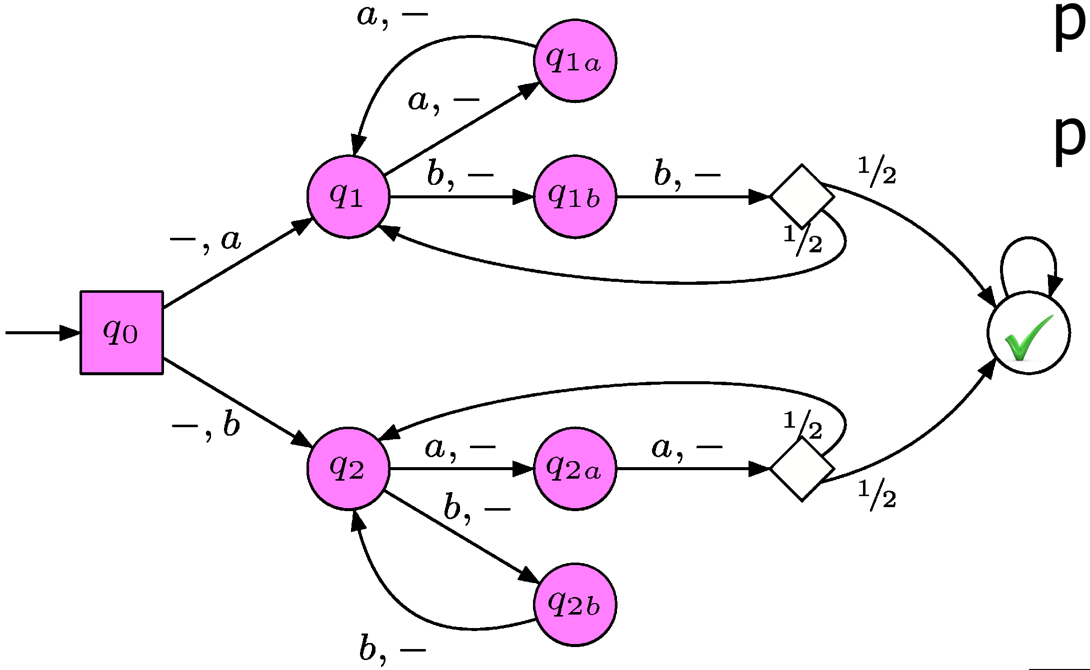
player 2 perfect

When belief is $\{q_1, q_2\}$, alternate a and b

This strategy is almost-sure winning !

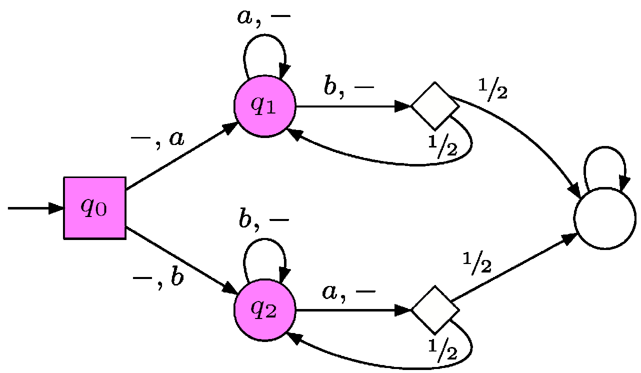
When belief fails (2/2)

Using the trick of "repeated actions" we construct an example where belief-only randomized action-invisible strategies are not sufficient (for almost-sure winning)



player 1 partial

player 2 perfect



When belief fails (2/2)

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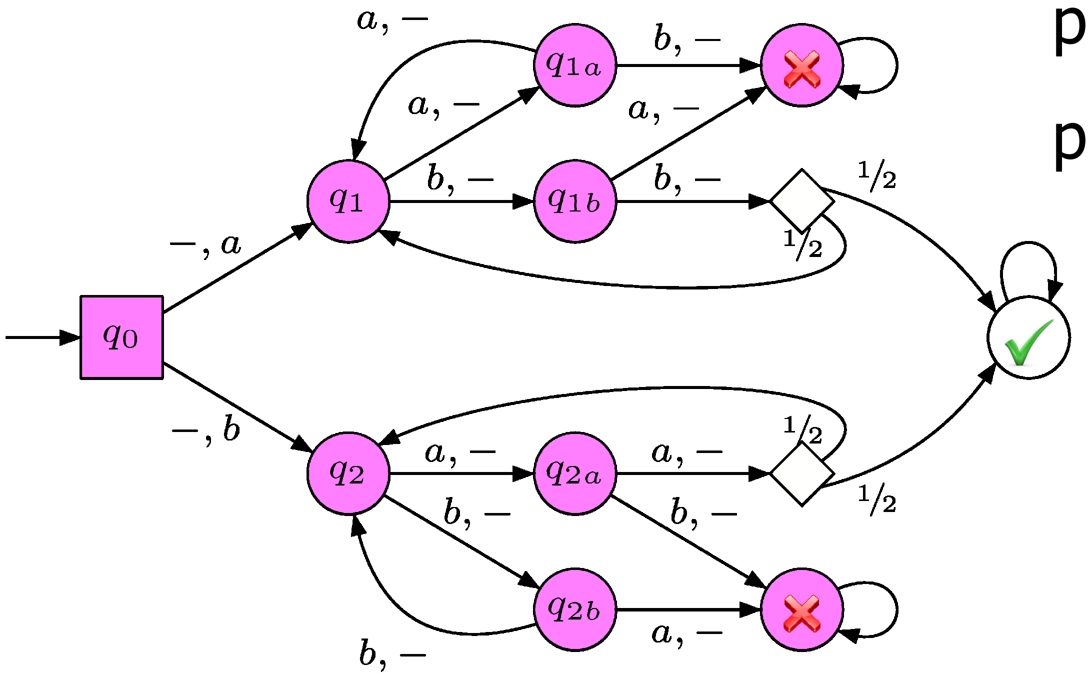
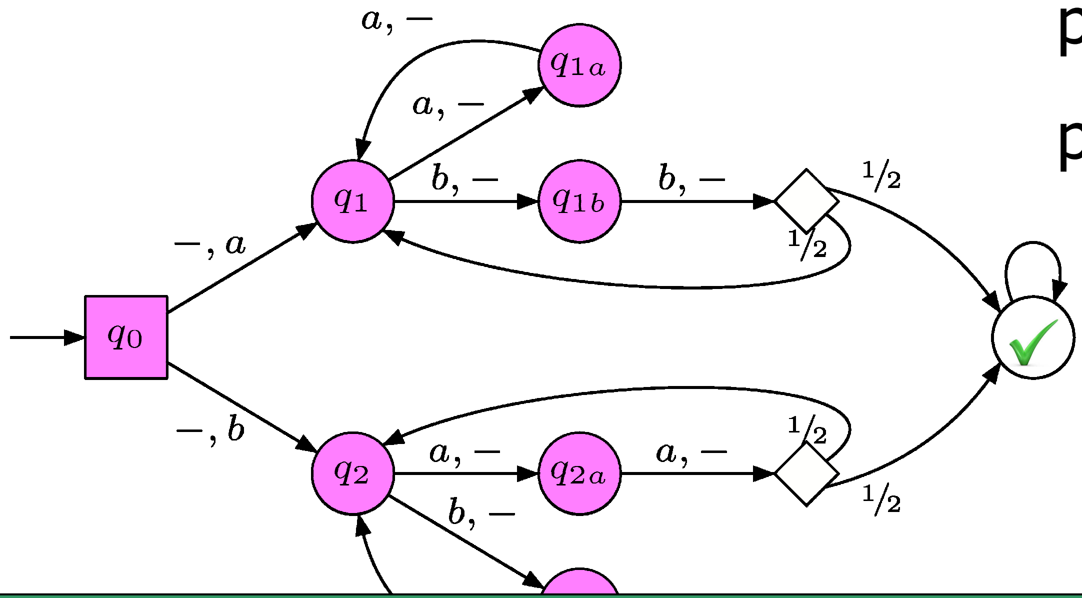


player 1 partial

player 2 perfect

When belief fails (2/2)

Using the trick of "repeated actions" we construct an example where belief-only randomized action-invisible strategies are not sufficient (for almost-sure winning)



player 1 partial

player 2 perfect

Almost-sure winning requires to play pure strategy, with more-than-belief memory !

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	~~exponential (belief) **[CDHR'06(remark), GS'09]**~~		exponential (belief) **[GS'09]**
pure	?	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	?	?	?

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (more than belief)		exponential (belief) **[GS'09]**
pure	exponential (more than belief)	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (more than belief)	?	?

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (more than belief)		~~exponential (belief)~~ ~~**[GS'09]**~~
pure	exponential (more than belief)	?	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (more than belief)	?	?

New results

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Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
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rand. act.-inv.	exponential (more than belief)		~~exponential (belief) **[GS'09]**~~
pure	exponential (more than belief)	non-elementary complete	?

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
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New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (more than belief)		~~exponential (belief)~~ ~~**[GS'09]**~~
pure	exponential (more than belief)	non-elementary complete	?

Positive	player 1 partial player 2 perfect		
rand. act.-vis.	memoryless		
rand. act.-inv.	memoryless		memoryless
pure	exponential (more than belief)	non-elementary complete	?

Player 1 wins from more states, but needs more memory !

Player 1 perfect, player 2 partial

Memory of **non-elementary** size for pure strategies

- lower bound: simulation of counter systems with increment and division by 2

- upper bound:
  positive: non-elementary counters simulate randomized strategies
  almost-sure: reduction to iterated positive

Counter systems with $\{+1, \div 2\}$ require non-elementary counter value for reachability $2^{2^{\cdot^{\cdot^{2}}}} \Big\}$ height n

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (more than belief)		~~exponential (belief)~~ ~~**[GS'09]**~~
pure	exponential (more than belief)	non-elementary complete	finite (at least non-elementary)

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (more than belief)	non-elementary complete	finite (at least non-elementary)

Player 1 perfect, player 2 partial

Equivalence of the decision problems for almost-sure reach with **pure** strategies and **rand. act.-inv.** strategies

- Reduction of rand. act.-inv. to pure
  choice of a subset of actions (support of prob. dist.)
- Reduction of pure to rand. act.-inv.
  repeated-action trick (holds for almost-sure only)

It follows that the memory requirements for pure hold for rand. act.-inv. as well !

New results

Reachability - Memory requirement (for player 1)

Almost-sure	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	exponential (belief) **[CDHR'06]**	memoryless **[BGG'09]**	exponential (belief) **[BGG'09]**
rand. act.-inv.	exponential (more than belief)		finite (at least non-elementary)
pure	exponential (more than belief)	non-elementary complete	finite (at least non-elementary)

Positive	player 1 partial player 2 perfect	player 1 perfect player 2 partial	2-sided both partial
rand. act.-vis.	memoryless	memoryless	memoryless
rand. act.-inv.	memoryless		memoryless
pure	exponential (more than belief)	non-elementary complete	finite (at least non-elementary)

Summary of our results

Pure strategies (for almost-sure and positive):

- player 1 partial: exponential memory, more than belief
- player 1 perfect: non-elementary memory (complete)
- 2-sided: finite, at least non-elementary memory

Randomized action-invisible strategies (for almost-sure) :

- player 1 partial: exponential memory, more than belief
- 2-sided: finite, at least non-elementary memory

More results & open questions

Computational complexity for 1-sided:

- Player 1 partial: reduction to Büchi game, EXPTIME-complete
- Player 2 partial: non-elementary complexity

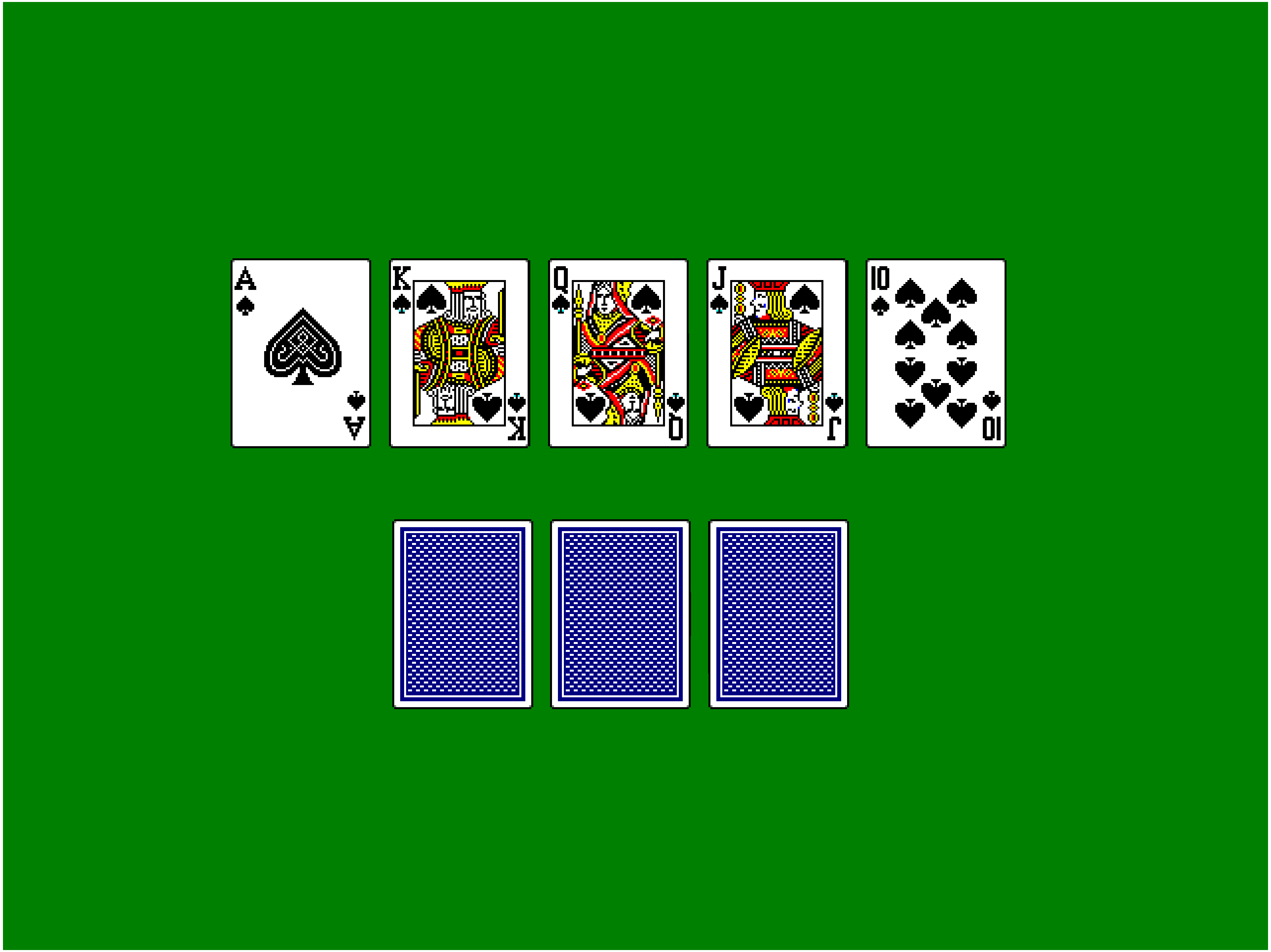
Open questions:

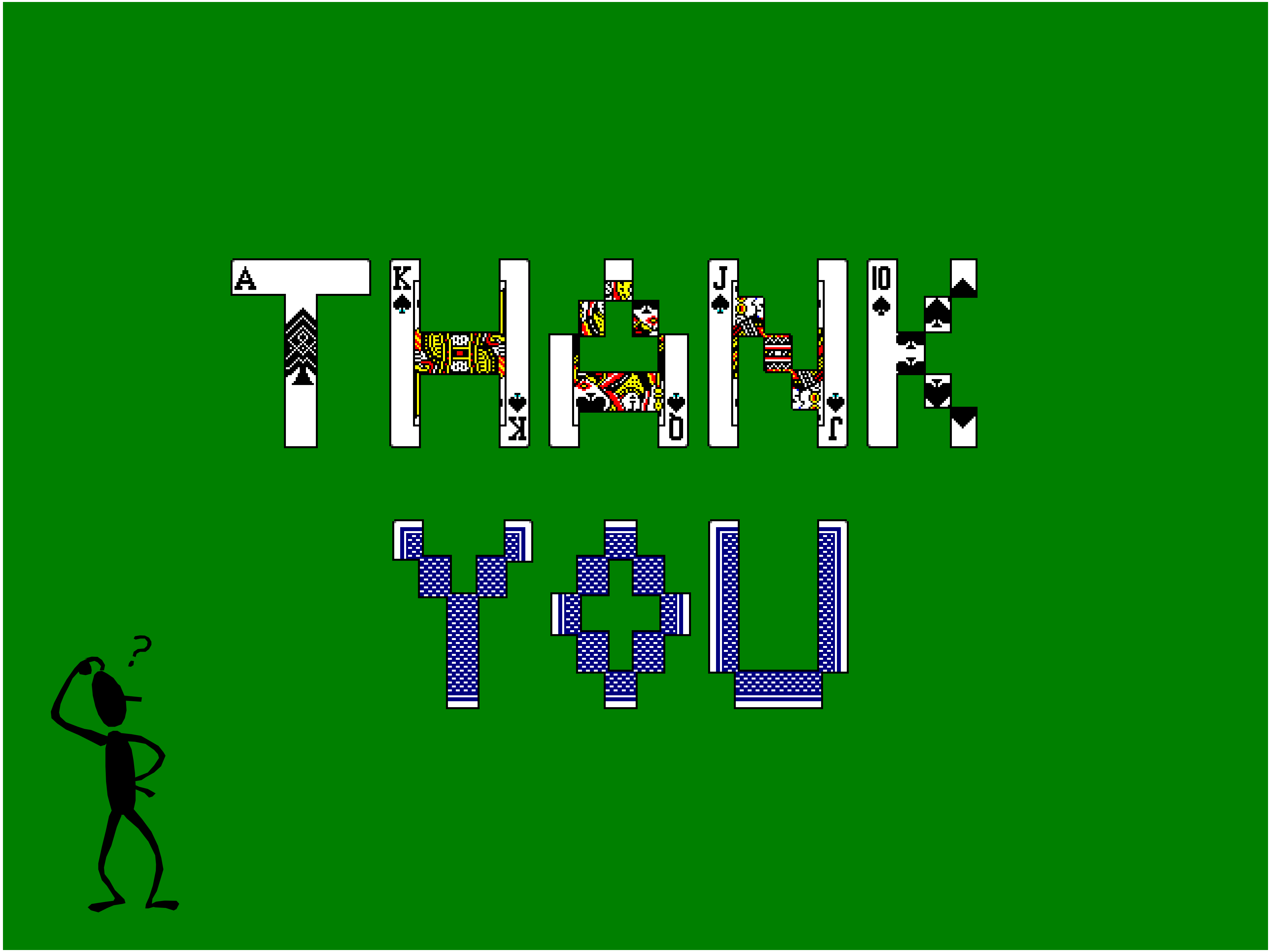
- Whether non-elementary size memory is sufficient in 2-sided
- Exact computational complexity

Details

Details can be found in:

[CD11] Chatterjee, Doyen. ***Partial-Observation Stochastic Games: How to Win when Belief Fails*****. CoRR abs/1107.2141, July 2011.**




THANK
YOU

References

Details can be found in:

[CD11] **Chatterjee, Doyen.** ***Partial-Observation Stochastic Games: How to Win when Belief Fails*****. CoRR abs/1107.2141, July 2011.**

Other references:

[BGG09] **Bertrand, Genest, Gimbert.** ***Qualitative Determinacy and Decidability of Stochastic Games with Signals*****. LICS'09.**

[CDHR06] **Chatterjee, Doyen, Henzinger, Raskin.** ***Algorithms for ω-Regular games with Incomplete Information*****. CSL'06.**

[GS09] **Gripon, Serre.** ***Qualitative Concurrent Stochastic Games with Imperfect Information*****. ICALP'09.**

[Paz71] **Paz.** ***Introduction to Probabilistic Automata*****. Academic Press 1971.**